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Solutions for Fermi Questions, October 2014: Question 1: Accelerating The Flash; Question 2: Flashing Through the Air

Larry Weinstein
Old Dominion University, lweinste@odu.edu

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Fermi Questions

***Larry Weinstein,* Column Editor**
Old Dominion University, Norfolk, VA 23529;
weinstein@odu.edu.

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Question 1: Acclerating The Flash

How much force does The Flash need to apply as he accelerates? (*Thanks to Scott Whittington of Old Dominion University for suggesting the question.*)

Answer: In order to uphold our reputations as physicists and make the appropriate snarky comments when we watch the new Flash TV series, we should understand some of the underlying physics. To estimate the force applied to accelerate The Flash, we need to estimate his mass, speed, and time to accelerate. Since The Flash appears to use normal human furniture, we can assume he has a normal human mass of about 100 kg (more than 10 kg and less than 10^3 kg).

Estimating The Flash's speed is more complicated. In the DC comics, he can travel at almost light speed and accelerate almost instantaneously. If he accelerates to $0.7c$ (where estimators can ignore special relativity because $\gamma = (1 - v^2/c^2)^{-1/2} \approx 1.4 << 10$) in one second, then he needs a force:

$$\begin{aligned} F &= ma = m\frac{\Delta v}{\Delta t} \\ &= (10^2\,\text{kg})\frac{0.7\times 3\times 10^8\,\text{m/s}}{1\ \text{s}} \\ &= 2\times 10^{10}\,\text{N} \end{aligned} .$$

This corresponds to the weight of 2×10^9 kg = 2×10^6 tons or about 20 aircraft carriers. Even if The Flash could withstand that force, the pavement he is running on certainly could not.

Let's consider this from the television trailer. The Flash is visible in one frame and then gone in the next, having travelled at least two city blocks. Since there are 22 movie frames per second and 20 city blocks in a mile (at least in Manhattan), this implies a minimum speed of 2 mi/s, or in more prosaic units:

$$\begin{aligned} v &= \frac{\Delta x}{\Delta t} \geq \frac{(0.1\ \text{mi})\times(2\times 10^3\,\text{m/mi})}{5\times 10^{-2}\,\text{s}} \\ &\geq 4\times 10^3\,\text{m/s} \end{aligned}$$

and a force of

$$\begin{aligned} F &= ma = m\frac{\Delta v}{\Delta t} \\ &\geq (10^2\,\text{kg})\frac{4\times 10^3\,\text{m/s}}{5\times 10^{-2}\ \text{s}} \\ &\geq 10^7\,\text{N} \end{aligned} .$$

The minimum value corresponds to the weight of only 10^3 tons. If The Flash obeys Newton's third law, this force will still make a serious impression on the pavement.

Air resistance, of course, will pose its own unique problems.

Question 2: Flashing Through the Air

How much force does The Flash need to run at constant velocity near the Earth's surface? How much power? (*Thanks to Scott Whittington of Old Dominion University for suggesting the question.*)

Answer: While it is important to suspend disbelief when reading about or watching comic book characters, some phenomena just cry out to be analyzed. Since 100 mph winds can pick up humans, cars, and trailers, the effects of air resistance at Flash-like speeds should be rather remarkable.

In the previous problem we estimated The Flash's speed at either $0.7c$ (from the comic books) or greater than 4×10^3 m/s (from the TV trailer). Let's estimate the effects of air resistance at both speeds. We'll start with the lower speed.

Our standard formula for the force of air resistance on a moving body, $F = (1/2)C\rho Av^2$, where ρ is the density of air, A is the cross sectional area of the body, C is a shape-related constant, and v is the relative speed of the body with respect to the air, only applies at low speeds. Since The Flash moves much faster than sound, this approximation fails.

At worst, The Flash will need to accelerate all of the air molecules he encounters up to his speed. In time t, he will encounter a mass of air $m = \rho Avt$ and accelerate it to speed v. Thus,

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt}$$
$$= \frac{\rho A v t v}{t} = \rho A v^2$$

Huh. That's the same equation that we just discarded. On the other hand, it's not surprising because dimensional constraints greatly restrict the possible equations.

At the lower speed, a mere 4×10^3 m/s, this will result in a force

$$F \geq (1 \text{ kg/m}^3)(1 \text{ m}^2)(4\times10^3 \text{ m/s})^2$$
$$\geq 2\times10^7 \text{ N},$$

where we used a cross-sectional area of A = 1 m^2 for the Flash, and $\rho_{air} = 10^{-3}\ \rho_{water}$. At these speeds The Flash needs more force to maintain his speed than to achieve it. He should be easy to track by following the path of ruined pavement he leaves behind.

Exerting that much force to maintain this speed will take a lot of power.

$$P = Fv \geq (2\times10^7 \text{N})(4\times10^3 \text{ m/s})$$
$$\geq 10^{11} \text{ W},$$

or the output of every single nuclear power plant in the entire United States.

If we increase the speed by about 10^5 from $v \geq 4\times10^3$ m/s to $v = 0.7c = 2\times10^8$ m/s, then the force increases by a factor of about 10^{10}, the power needed increases by a factor of 10^{15} (although the time needed decreases by a factor of 10^5) and the results get even sillier. And that is not even considering the effects of nuclear fusion as nitrogen and oxygen in the air fuse with hydrogen, carbon, and oxygen in The Flash (see http://what-if.xkcd.com/1/ for a description).

We clearly need to set safe and sane speed limits for intra-atmospheric travel.