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Solutions for Fermi Questions, October 2018: Question 1: Automobile Air Conditioning; Question 2: Falling Leaves

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Solutions for Fermi Questions, October 2018

Question 1: Automobile air conditioning

How much does it cost to air condition all American automobiles for a year? (*Thanks to Alex Godunov of Old Dominion University for suggesting the question.*)

Answer: To estimate this, we need to estimate the number of cars, the amount of time that they use AC each year, and the cost using the AC. There are 3×10^8 Americans and not quite one car per person, giving 2×10^8 cars. The average car is driven about 10^4 miles per year (based on car warranties) at an average speed of 30 mph (faster than 10 and slower than 100 mph or faster than 20 and slower than 60 mph [faster in Montana and slower in Los Angeles]). Thus, the average car is driven about 300 hours per year (which is reasonable, since that is about one hour per day).

We could try to estimate the fraction of driving time spent using AC by considering drivers in Kansas City, Seattle, New Orleans, etc., but that is needlessly complicated. The fraction of driving time spent using AC will be more than 10% and less than 100% (even in New Orleans), giving an estimate of 30%. This means that each car uses its AC 100 hours per year.

Now let's estimate the cost per hour of running the AC. We'll start by estimating its power consumption. A typical window air conditioner for a home consumes a maximum of 1500 W (or it's likely to overload the electrical circuit) and cools an entire room. Therefore, a car air conditioner will use between 10% and 100% of that, for an estimated power consumption of 500 W. (That's about 2/3 Horsepower.)

That means that the AC will consume a total energy $E_{\text{total}} = NPt = (2 \times 10^8)(500 \text{ W})(100 \text{ hr}) = 10^{10} \text{ kW-hr}$. At typical electrical rates of \$0.1/kW-hr, that is \$1 billion of energy. Wow! That's a lot of money!

Or is it? That's the cost for all of the 200 million cars in the entire country. The cost per car is only \$5. Even if gasoline energy is twice as expensive as electrical energy, that would only be \$10 of gasoline per car.

Who cares? Compared to the cost of operating a car, the cost of its air conditioning is negligible. If you turn off the AC and open the windows, increased drag will probably cost at least as much (but we'll estimate that in a future column).

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Question 2: Falling leaves

How many leaves will fall in the United States this fall?

Answer: To estimate this, we need to estimate the area of the United States, the fraction of that area covered by trees, and the number of leaves per area. The United States is about 3000 miles (5000 km) from East to West (based on either 6 hours of airplane flight at 500 mph or 3 time zones which is 3/24 of the Earth's circumference). It is about half that from North to South, giving an area of

$$A = lw = (5 \times 10^3 \text{ km})(2 \times 10^3 \text{ km}) = 10^7 \text{ km}^2.$$

We could add another 30% for Alaska (more than 10%, less than 100%), but why bother?

The fraction covered by trees will be more than 10% and less than 100%, so we will estimate 30%, giving a forested (or at least treed) area of $3 \times 10^6 \text{ km}^2$.

Now we need to estimate the leaf density. We could estimate the tree density, the size of each tree, the number of limbs, branches, twigs, etc, and the number of leaves per twig, but that is needlessly complicated. Let's simplify this. Go stand under a tree and look up. It's OK, I'll wait. What is the average number of leaves over any point?

The number of leaves over any point covered by the tree will be more than one and less than ten, giving an average leaf coverage of three.

Now we only need to estimate the size of the average leaf. The average leaf will have an area of more 10 cm^2 (2 in^2) and less than 100 cm^2 (the palm of my hand), giving an estimated area of 30 cm^2 . Thus, the total number of leaves on all the trees in the United States is

$$N = \frac{3(3 \times 10^6 \text{ km}^2)}{30 \text{ cm}^2} (10^{10} \text{ cm}^2 / \text{km}^2) = 3 \times 10^{15}.$$

That's a LOT of leaves (3 quadrillion). Does this make sense?

Let's check this number for sanity. There are an estimated 3×10^{12} trees in the world. With 5% of the surface area, this would give the U.S. 2×10^{11} trees, with about 10^4 leaves per tree. This is not crazy. Alternatively, if you own a house on a half-acre (1/4 hectare) lot and it is 30% treed, then you will end up raking

$$n = \frac{3(10^3 \text{ m}^2)}{30 \text{ cm}^2} \left(\frac{10^4 \text{ cm}^2}{\text{m}^2} \right) = 10^6 \text{ leaves every fall.}$$

This is also not crazy. Wow. Three quadrillion leaves really do fall on the United States every fall. Who knew?

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