11-2018

Solutions for Fermi Questions, November 2018: Question 1: Kale; Question 2: Hurricane Angular Momentum

Larry Weinstein
Old Dominion University, lweinste@odu.edu

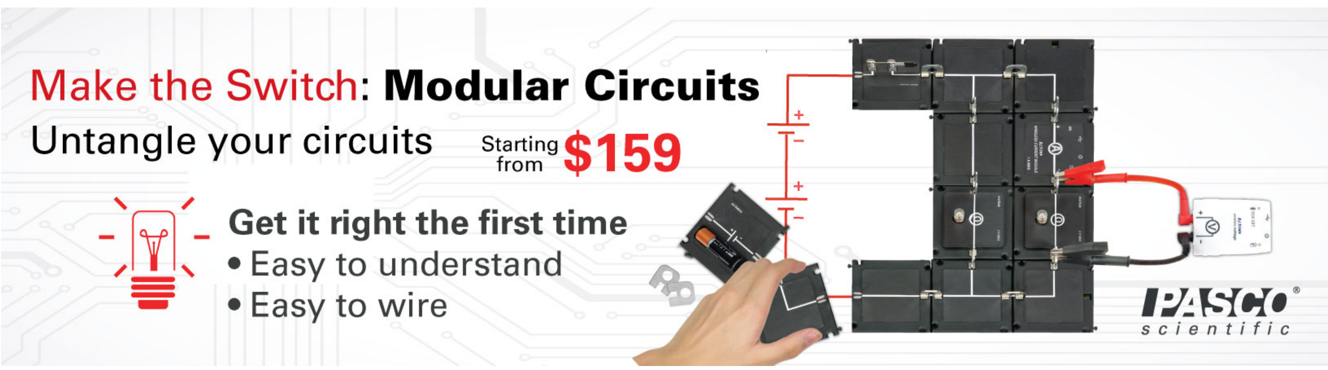
Original Publication Citation

Weinstein, L. (2018). Solutions for Fermi questions, November 2018: Question 1: Kale; question 2: Hurricane angular momentum. *Physics Teacher, 56*(8), A534-A534. doi:10.1119/1.5064551

Solutions for Fermi Questions, November 2018: Question 1: Kale; Question 2: Hurricane angular momentum

Larry Weinstein

Citation: The Physics Teacher **56**, A534 (2018); doi: 10.1119/1.5064551
View online: https://doi.org/10.1119/1.5064551
View Table of Contents: http://aapt.scitation.org/toc/pte/56/8
Published by the American Association of Physics Teachers



Fermi Questions

Larry Weinstein, **Column Editor**
Old Dominion University, Norfolk, VA 23529
weinstein@odu.edu

Solutions for Fermi Questions, November 2018

Question 1: Kale

How much kale would be needed to provide enough calories for a person for an entire year? (*Thanks to Michael Briggs and Beverly Sher of William & Mary for suggesting the question.)*

Answer: We will cheerfully ignore the fact that kale does not provide enough nutrients and just estimate how to provide enough calories. Kale does provide lots of fiber, but that is not very digestible and will not provide the calories we need. We can delve into the details and try to estimate the relative amounts of water, fiber, complex carbohydrates and sugar in kale, but that task would be almost as unpalatable as kale itself. Let's simplify things. The digestible fraction of kale (sugars and other carbohydrates) is more than 1% and less than 100%, so we will estimate 10%.

Now we need to estimate our caloric needs and the energy density of carbohydrate. We might remember from ad campaigns that sugar is "only" 16 cal/tsp (or 3 cal/g). On the other hand, we might remember that gasoline has an energy density of 5×10^7 J/kg (at 4×10^3 J/cal, this is 10^4 cal/kg or 10 cal/g). We could then estimate that carbohydrate has between 10% and 100% of the energy density of gasoline, giving 3 cal/g. On the third hand, we could start from the formula for sugar ($C_6H_{12}O_6$ or something like that). The H and O are already bonded to make hydrates so we only gain energy by oxidizing the C to make CO_2. At an estimated 1.5 eV per energetic chemical reaction (from the voltage of standard AA battery or from 10% of the binding energy of the H atom), we can calculate the energy density of sugar. Go ahead.

We need about 2×10^3 cal/day or about 7×10^5 cal/year. If kale has 10% of the useful energy density of pure carbohydrate, then it has 0.3 cal/g. (And, in fact, according to Wikipedia, that is exactly how many calories cooked kale contains!) This means that we will need to consume a mass

$$M = \frac{7\times10^5\,\text{cal/yr}}{3\times10^{-1}\,\text{cal/g}} = 2\times10^6\,\text{g/yr} = 2\times10^3\,\text{kg/yr}.$$

That is 2 tons of kale per year, or about 6 kg of kale per day. We hope that you really really like kale!

Question 2: Hurricane angular momentum

How much angular momentum did Hurricane Florence have when it hit land?

Answer: To calculate this, we need to know the wind speed as a function of radius for the hurricane, as well as the height of the hurricane. However, to estimate this, we only need to estimate the average wind speed, the effective area, and the height. The angular momentum goes as

$$L = I\omega \propto Mr^2\omega \propto Mrv$$

(ignoring factors of ½), and the mass is

$$M = \rho V = \rho\pi r^2 h$$

so the angular momentum will be

$$L = \rho\pi r^3 hv$$

and it will therefore be dominated by the speed at large radius.

I've been looking at maps of hurricane Florence all week, so I know that the diameter of the storm is about 250 miles or 400 km. If you weren't glued to the weather reports, then you can estimate that the radius of a large hurricane is more than 10 km and less than 10^3, so we will estimate 10^2 km. The height will be more than 1 km and less than 100, so we will estimate 10 km. The wind speed will be at least that of a tropical storm, 39 mph or about 20 m/s, and the density of air is 1 kg/m^3. Therefore, the angular momentum is:

$$L = \rho\pi r^3 hv = (1\ \text{kg/m}^3)3(2\times10^2\ \text{km})^3(10\ \text{km})(20\ \text{m/s})(10^3\ \text{m/km})^3 = 5\times10^{18}\ \text{kg}\cdot\text{m}^2/\text{s}$$

which seems like a lot.

Let's compare it to something else large. If all the cars in the U.S. drove at highway speed around the perimeter of the U.S., they would have angular momentum

$$L = Mrv$$
$$L = (2\times10^8\ \text{cars})(2\ \text{tons/car})(10^3\ \text{km})(30\ \text{m/s})$$
$$L = 10^{19}\ \text{kg}\cdot\text{m}^2/\text{s}$$

or about the same as the hurricane! Wow. That is a LOT of angular momentum. It takes a very large country-sized effort to create as much angular momentum as one hurricane.