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**Research note**

# On the linear relation between $m_b$ and $M_s$ for discrimination between explosions and earthquakes

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**Summary**. The statistical capability of the $m_b:M_s$ discriminant for the discrimination of earthquake and explosion populations is examined by application of discriminant functions to a group of 83 explosions and 72 earthquakes in Eurasia. Equations are derived for the probability that an event is an earthquake or an explosion. The positive sign of DIS in the decision index equation,

$$\mathrm{DIS}_i = 34.3383 - 11.9569\, mb_i + 7.1161\, M_{si},$$

indicates that the event $i$ is an earthquake. Its negative sign indicates that event $i$ is an explosion. The probability of correct classification for an event, $P_i$, is related to its $\mathrm{DIS}_i$ value, by

$$P_i = [1 + \exp(\mathrm{DIS}_i)]^{-1},$$

where a large, positive DIS indicates a high probability that an event is an earthquake and a large, negative DIS indicates a high probability that an event is an explosion. The discrimination line $M_s = 1.680\, m_b - 4.825$, or $m_b = 0.595\, M_s + 2.872$ very successfully separates the explosion population from the earthquake population. The points on this line have an equal chance of being an earthquake or an explosion; moreover, for any event, the distance parallel to the $M_s$-axis from the point representing that event in the $m_b:M_s$ plane to this line is a measure of the probability for the correct classification of that event.

**Key words**: discrimination, explosions, earthquakes, linear relation between $m_b$, $M_s$

## Introduction

The success of the $m_b:M_s$ magnitude discriminant is thought to be based on the relative efficiency of 20 s Rayleigh waves and 1 s $P$-waves. The differences between the source mechanism, source dimension, source spectral content, near source elastic properties, interference of $pP$ phase with $P$ phase, and seismic rise time are recognized to be the underlying causes (Brune, Espainosa & Oliver 1963; Press, Dewart & Gilman 1963; Douglas, Hudson & Kembhavi 1971; Peppin 1976; Douglas 1981; Stevens & Day 1985). Many observational data on $m_b$ and $M_s$ have shown that, in general, the explosions generate about 1–1½ magnitude unit greater $m_b$ values for the same $M_s$ values (Weichert & Basham 1973; Tatham, Forsyth & Sykes; and many others). In a recent paper, Stevens & Day (1985) estimated the contribution of source spectra, focal mechanism, near source elastic properties, and interference of *pP* to the success of the $m_b:M_s$ discriminant. Although the reasons for the success of $m_b:M_s$ discriminant are not completely known, the $m_b:M_s$ diagram is easily constructed and successfully discriminates earthquakes from explosions in many cases.

There are cases, however, for which the $m_b:M_s$ method alone may incorrectly classify an event. For example, a deep focus earthquake that has generated a very small 20 s Rayleigh wave but has produced a relatively large 1 s *P*-wave may be classified as an explosion by this method. For these cases, as well as for other anomalous cases (Landers 1972; Nuttli & Kim 1975; Tatham *et al.* 1976), additional discriminant parameters such as location, focal depth, first motion, complexity, spectral ratio and others may be employed and multidimensional discrimination techniques can be used. A review of these techniques is discussed by Tjostheim (1980).

When the $m_b:M_s$ method is applied, the differences due to regional attenuation, earth structure, the effect from tectonic strain release on explosion magnitudes and other effects that cause variation in $m_b:M_s$ should be considered. In this work, however, I have used uncorrected published $m_b$ and $M_s$ values. In addition, I have followed many preceding authors and adopted a linear relationship between $m_b$ and $M_s$, although this relationship may be non-linear over the entire magnitude range.

Many workers have assumed a linear relationship of the form:

$$M_s = a + bm_b \tag{1}$$

or

$$m_b = a' + b'M_s \tag{2}$$

between $m_b$ and $M_s$ (see Tables 1 and 2); then the coefficient pairs, $a$, $b$ or $a'$, $b'$, are estimated. In order to obtain a measure of the separation between earthquakes and explosions, a linear discriminant relationship such as:

$$D = a' + b'M_s - m_b, \tag{3}$$

or

$$D = a + bm_b - M_s \tag{4}$$

is formed (Ericcson 1970; Weichert & Basham 1973; Bungum & Tjostheim 1976).

Weichert & Basham (1973) have proposed three intuitive geometric procedures for measuring the discriminant. It could be measured perpendicularly to a line fitted to: (a) the combined earthquake and explosion population, (b) the earthquake population alone, or finally, (c) the explosion population alone. Their data as well as other data (Stevens & Day 1985; Sandvin & Tjostheim 1978; Basham 1969) indicate that the $m_b:M_s$ diagram for earthquake populations have greater scatter about their average trend lines. Thus Weichert & Basham (1973) rejected possibilities (a) and (b) and measured the discriminant distances from the explosion trend line. However, the relatively small scatter of the explosions data may be deceptive, as they pointed out, since earthquake populations have a much wider range of epicentral distances, $M_s$ magnitudes, focal mechanisms, depths and azimuthal

**Table 1.** Relations between $m_b$ and $M_s$ for earthquakes.

| Number | Equation | Remarks | Reference |
|---|---|---|---|
| 1 | $M_s = 1.59\ m_b - 3.97$ | World-wide events, California records | Gutenberg & Richter (1956) |
| 2 | $M_s = m_b + 0.44$ | Unspecified, North America records | Romney (1964) |
| 3 | $M_s = 1.01\ m_b - 0.23$ | World-wide events, Eskalemuir records | Marshall *et al.* (1966) |
| 4 | $M_s = 1.27\ m_b - 1.78$ | Central Asian events, Lasa records | Capon, Greenfield & Lacoss (1967) |
| 5 | $M_s = 2.63\ m_b - 11.09$ | European events, world-wide records | Karnik (1969) |
| 6 | $M_s = 1.93\ m_b - 4.8$ | Sino–Soviet events, USSR records | Marshall (1970) |
| 7 | $M_s = 1.07\ m_b - 0.23$ | 2.5 yr, world-wide records | Gupta & Rastagi (1972) |
| 8 | $M_s = 1.89\ m_b - 4.62$ | 2 yr of world-wide records, $M_s > 5.73$ WWSSN records | Nagamune (1972) |

coverage. If the number of the test sites increases, it is probable that the scatter of the explosion population around the mean trend increases as well although the explosion sources are limited in focal depth and have simpler focal mechanism.

Various methods have been proposed for the calculation of coefficients $a$, $b$ or $a'$ and $b'$. The least square technique and maximum likelihood estimation procedure have been used by Basham (1969) and Bungum & Tjostheim (1976). Weichert & Basham (1973) have pointed out the uncertainties in the estimated values of $M_s$ and $m_b$, and argued that, because of these uncertainties the standard least square procedure should not be employed. They used, instead, a simpler intuitive grouping method proposed by Wald (1940), Bartlett (1949) and Madansky (1959), where *a priori* estimates of the variance in values of the coordinates, $m_b$, $M_s$, are not required. Because of the variation in near source and near receiver elastic properties, and the various methods of estimation used in the analysis, the reported coefficients contain a large scatter. Therefore, there are many different types of linear relationships between $M_s$ and $m_b$ in the literature; Tables 1 and 2 give a summary for earthquakes and explosions respectively. A more complete table which includes relationships for other types of magnitude is presented by Bath (1981).

Ideally, a classification rule is desirable where the probability of misclassification can be

**Table 2.** Relation between $m_b$ and $M_s$ for explosions.

| Number | Equation | Remarks | References |
|---|---|---|---|
| 1 | $M^P = 1.24\ m_b - 1.76$ | Nevada and New Mexico events, Canadian records | Basham (1969) |
| 2 | $M^B = 1.00\ m_b - 1.20$ | LONGSHOT and others, Eskdalemuir records | Marshall *et al.* (1966) |
| 3 | $M_s = 1.43\ m_b - 2.87$ | Central Asia events, Canadian records | Basham (1969) |
| 4 | $M^B = 1.17\ m_b - 2.87$ | Central Asia events, Lasa records | Capon *et al.* (1967) |
| 5 | $M_s = 0.89\ m_b - 0.55$ | World-wide events, USSR records | Passechnik *et al.* (1970) |
| 6 | $M_s = 1.06\ m_b - 1.46$ | Asian events, Indian records | Gupta, Sitaram & Narain (1972) |
| 7 | $M_s = 1.73\ m_b - 6.03$ | Eurasia event, NORSAR, $M_s$ | Bungum & Tjostheim (1976) |
| 8 | $M_s = 1.39\ m_b - 2.59$ | Eurasian event, NORSAR, $M_s$ | Sandvin & Tjostheim (1978) |

$M^P$ = Pasadena's magnitude.
$M^B$ = Bath's magnitude.

estimated. In this work the discriminant functions are used, and a linear function for the separation of explosions and earthquakes and the probability of their correct classification is derived. The linear discriminant line, when plotted on the $m_b:M_s$ diagram, separates the explosions from the earthquakes. Moreover, the distance between the line and position of an event in the $m_b:M_s$ plane is a measure of the probability of that event being an explosion or an earthquake. For example, events on the line have a 50 per cent probability of being an earthquake and when events are farther away from this line classification can be made and its probability can be calculated by using the sign and value of a parameter defined as the decision index.

## Discriminant functions

Discriminant functions have been used for discrimination between underground nuclear explosions and earthquakes by Booker & Mitronovas (1964), Bell (1978), Sandvin & Tjostheim (1978), Rivers *et al.* (1980) and Nowroozi (1984). Detailed discussions of this technique are given by Anderson (1958) and Davis (1973). In the discriminant function technique several discriminant parameters can be used; the process involves the formation of the pooled variance–covariance matrix of both explosions and earthquakes discriminant parameters. Let $[S^2p]$ and $[d]$ be the pooled variance–covariance matrix, and the multivariate mean differences of various discriminant parameters, respectively. Then the discriminant coefficient $[\lambda]$ can be obtained from:

$$[S^2p]\cdot[\lambda]=[d]. \tag{5}$$

The dimension of the column vector $[\lambda]$ is equal to the number of discriminant parameters. This technique requires a training data set. For this purpose the database reported by Sandvin & Tjostheim (1978) was selected. Their data consist of 83 presumed explosions and 72 earthquakes in Eurasia. In this database the variables that can be used are $m_b$, $M_s$, and focal depth. Unfortunately, all the focal depths for explosions are given as zero, and many focal depths for earthquakes are indicated as normal. Therefore, only $m_b$ and $M_s$ can be used. The increase of discriminant parameters may potentially increase the resolving power and improve the discrimination result.

Using only $m_b$ and $M_s$ parameters for the population discussed above, the matrix of pooled variance–covariance, $[S_p^2]$, is:

$$[S_p^2]=\begin{bmatrix}0.26597 & 0.34838\\ 0.34838 & 0.65707\end{bmatrix}. \tag{6}$$

The multivariate mean difference, or column vector $[d]$ is:

$$[d]=\begin{bmatrix}0.7010\\ -0.5102\end{bmatrix}. \tag{7}$$

The discriminant coefficient $[\lambda]$ can be obtained from:

$$[\lambda]=[S_p^2]^{-1}\cdot[d]. \tag{8}$$

where $[S_p^2]^{-1}$ is the inverse of $[S_p^2]$.

The value of $\lambda$s are $\lambda_1=11.9569$ and $\lambda_2=-7.1161$ respectively, and the value for the discriminant index $R_0$, the centroids of the explosion population $R_X$ and the earthquake population $R_E$ are: $R_0=34.3383$; $R_X=40.345$; and $R_E=28.333$ respectively.

Thus, the discriminant function equation $DS_i$ is:

$$DS_i=11.9569m_{b_i}-7.11615M_{s_i} \tag{9}$$

where $m_{b_i}$ and $M_{s_i}$ are referring to the $m_b$ and $M_s$ values of event $i$, and $DS_i$ is the discriminant score of that event. The value of $DS_i$ now can be compared with the discriminant index $R_0$ to declare whether event $i$ is an explosion or an earthquake by using the decision index equation:

$$DIS_i = R_0 - (\lambda_1 m_{b_i} + \lambda_2 M_{si}). \tag{10}$$

For our case the decision index is defined as:

$$DIS_i = 34.3383 - (11.9569\, m_{bi} - 7.1161\, M_{si}). \tag{11}$$

The negative value of DIS indicates that the event is an explosion, and its positive value indicates that the event is an earthquake. For a detailed discussion of this technique refer to Anderson (1958) and Davis (1973). If other parameters were used instead of $m_b$ and $M_s$, the form of equation for DIS would still have been similar to that of equation (10). However, the value of $R_0$ would have been different and one additional term for each additional parameter would have been introduced.

The values of DIS are easier to understand in terms of the probability of an explosion event. Let $P_{X,i}$ be the probability of an event, $i$, to be an explosion; then $1-P_{X,i}$ is the probability $P_{E,i}$ for that event to be an earthquake. $P_{X,i}$ is related to $DIS_i$ by:

$$P_{X,i} = [1 + (\exp(DIS_i)]^{-1}. \tag{12}$$

From equations (10) and (12) it follows:

$$M_{si} = \frac{R_0}{\lambda_2} - \frac{\log(P_E, i/P_{X,i})}{\lambda_2} - \frac{\lambda_1}{\lambda_2} m_{bi}. \tag{13}$$

This is the equation of the discriminant line in terms of $M_s$ and $m_b$, and $P_{X,i}$. The slope of the line depends on $\lambda_1/\lambda_2$, or the ratio of the two discriminant functions, and the intercept with the $M_s$-axis is

$$\left(R_0 - \log \frac{P_{E,i}}{P_{X,i}}\right)\lambda_2^{-1},$$

where the values of the intercept depend on $P_{X,i}$. For $P_{X,i} = ½$, the intercept is $R_0/\lambda_2$, thus equation (13) reduces to:

$$M_{si} = \frac{R_0}{\lambda_2} - \frac{\lambda_1}{\lambda_2} m_{bi}. \tag{14}$$

This line separates the $m_b : M_s$ plane into two earthquake and explosion regions depending on the sign of the DIS. Equation (13) can separate the explosions from the earthquakes with any desired probability. The distance parallel to the $M_s$-axis from a point representing an event on the $m_b : M_s$ diagram to this line is a measure of the probability for the correct classification of that event (see Fig. 1).

From equation (12), for DIS = 0, it follows that $P_X = ½$ and, therefore, $P_E = ½$ as well. Thus if an event is on the borderline, it could be either an explosion or an earthquake. If DIS is a very large negative number, $P_X$ will approach 1, and the event has a very high probability of being an explosion, and a very low probability of being an earthquake. On the other hand, if DIS is a very large positive number, $P_X$ will approach zero, and then the event has a very low probability of being an explosion, and a very high probability of being an earthquake. Fig. 2 gives a plot of equation (12) which can be used for reading the probability as a function of DIS.

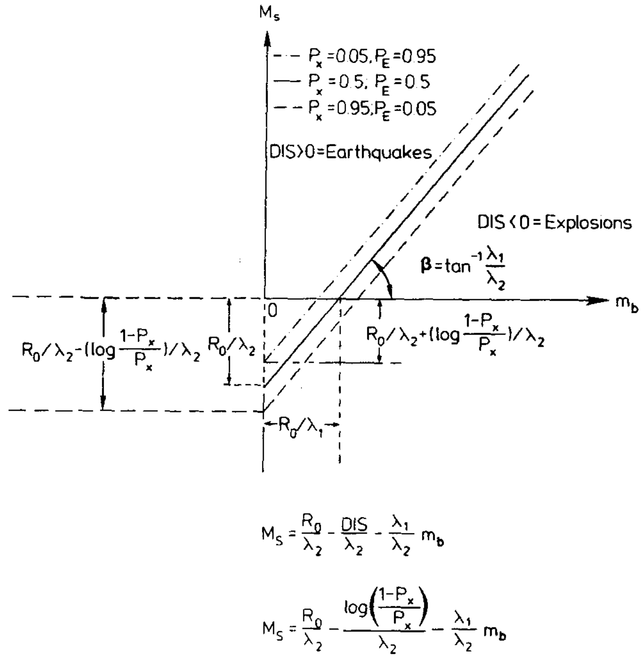


**Figure 1.** Plot of the discriminant line equation, $M_S = R_0/\lambda_2 - \mathrm{DIS}/\lambda_2 - (\lambda 1/\lambda_2)m_b$. Note DIS = $\log((1 - P_X)/P_X)$, where $P_X$ is the probability of the correct classification of an explosion. When $D = 0$, $P_E = P_X = 0.5$ where $P_E$ = the probability of the correct classification of an earthquake; $R_0$, $\lambda_1$ and $\lambda_2$ are the discriminant index and the first and second discriminant functions respectively. The upper line separates earthquakes at the 95 per cent level, thus the events that are classified and remain in the earthquake population have a 5 per cent chance of being an explosion. Similarly, the lower line separates the explosions at the 95 per cent level. Events which are not classified have a 5 per cent chance of being an earthquake.

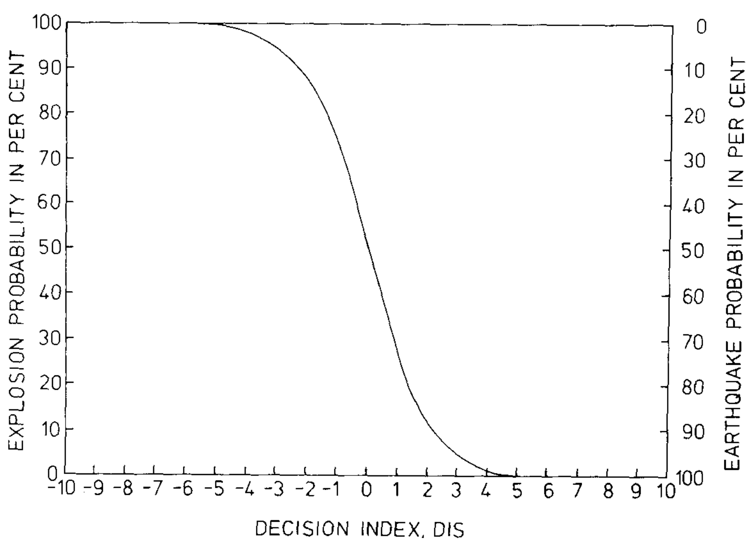


**Figure 2.** The probability for correct classification of an event as a function of the decision index DIS. Because there are only two types of classification, earthquakes and explosions, the sum of the two probabilities is always 1. When DIS > 0, the event is an earthquake; when DIS < 0, the event is an explosion.

Letting DIS = 0 in equation (10), a linear relationship between $m_b$ and $M_s$ appears. The relationship can be expressed as either:

$$M_s = -4.825 + 1.680\,m_b, \tag{15}$$

or

$$m_b = 2.872 + 0.595\,M_s. \tag{16}$$

Note that both equations indicate the same line, which separates the explosions from the earthquakes on an $m_b:M_s$ diagram. For a classification of an explosion with probability $P_X = 0.95$, and an earthquake probability of 0.05, from equation (12), we obtain DIS = $-2.9444$. Thus from equation (13), the line:

$$M_s = -5.239 + 1.680\,m_b \tag{17}$$

or

$$m_b = 3.119 + 0.595\,M_s \tag{18}$$

separates the explosions and the earthquakes at the $P_X = 95$ per cent level. Similarly, for DIS = 2.9444, or an explosion probability $P_X = 0.05$ and an earthquake probability of $P_E = 0.95$, the line:

$$M_s = -4.412 + 1.680\,m_b \tag{19}$$

or

$$m_b = 2.626 + 0.595\,M_s \tag{20}$$

separates the explosions from the earthquake population.

## Application

Many authors have used $m_b$ and $M_s$ parameters for discrimination. In this section we plot our linear equations on their $m_b:M_s$ plots for comparison. It should be noted that the probability is used here in a limited statistical sense, because many other discriminant parameters that could have been used for classification of an event were not included. I have demonstrated the effectiveness of this technique for $m_b$ and $M_s$, but this method is general and all the known discriminant parameters can be included. The inclusion of all available parameters is desirable and potentially may improve the discrimination results.

Given $m_b$ and $M_s$ of an event *i*, the sign of DIS in equation (11) gives the classification of that event. If DIS is positive the event is classified as an earthquake; if DIS is negative, that event is classified as an explosion. Moreover, equation (12) or Fig. 2 can be used to estimate the probability of the correct classification of that event. For example, assume $m_b = 5$ and $M_s = 4$, then from equation (11) DIS = 3.0184, and the event has a probability of about 4.6 per cent of being an explosion and a probability of about 95.4 per cent of being an earthquake. But if $m_b = 6$ and $M_s = 4.0$ then DIS = $-8.9387$, and the event has a probability of about 99.98 per cent of being an explosion and a probability of 0.02 per cent of being an earthquake. An event with $m_b = 5.25$ and $M_s = 4$ has about a 50 per cent chance of being an explosion and a 50 per cent chance of being an earthquake.

In this section equations (15), (17) and (19) are plotted on several $m_b:M_s$ diagrams which are published in the literature. The line for $P_E = P_X = 50$ per cent, very successfully separates the earthquake population from the explosion population in all cases. Fig. 3 indicates a plot

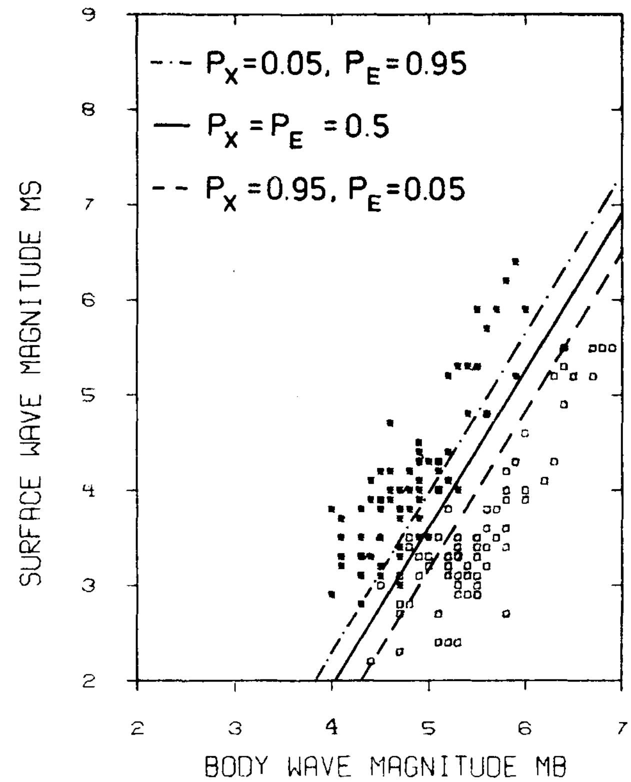


**Figure 3.** $m_b : M_s$ diagram for the data set of Sandvin & Tjostheim (1978). This set includes 83 explosions (open squares) and 72 earthquakes (solid stars). Discriminant line with $P_E = P_X = 0.5$ misclassifies three earthquakes and seven explosions, but at $P_X = 0.05$, all the explosions have been classified correctly. However, there are 13 misclassified earthquakes or false alarms. On the other hand, when $P_X = 0.95$, all the earthquakes are classified correctly, but 15 explosions are misclassified. Note that the majority of misclassified explosions have $m_{b,s}$ less than five.

of $m_b : M_s$ for the data set of Sandvin & Tjostheim (1978). The set includes 83 explosions and 72 earthquakes; $m_b$ magnitudes are from PDE and $M_s$ magnitudes are recorded at NORSAR. The line given by equation (15), $P_E = P_X = 50$ per cent, very successfully separates the events. Only seven explosions are in the earthquake population and only three earthquakes are in the explosion population. However, at the $P_X = 95$ per cent, $P_E = 5$ per cent probability level as many as 15 explosions are misclassified, eight events are between the lines for $P_X = 0.5$ and $P_X = 0.95$, thus this technique gives a probability of 0.5–0.05 that these events may be earthquakes. This line gives a low false alarm rate, but many explosions may not be identified. On the other hand, at the $P_X = 5$ per cent; $P_E = 95$ per cent probability level, about 13 earthquakes are misclassified, 10 events are between the lines for $P_E = 0.5$ and 0.95, thus this technique gives a probability of 0.5–0.05 that these earthquakes may be explosions. This line gives a high false alarm rate, but many suspicious events may be declared. Fig. 4 is a plot of $m_b : M_s$ from the data set of Weichert & Basham (1973). The data set includes 100 explosions and 611 earthquakes. Magnitudes are from various sources such as WWSSN, CNS, LASA, OGD, NOAA, and the UK arrays, and the sources include Eurasia, WN America, the Aleutians, the Solomon Islands and the Sahara.

Again the line given by equation (15), $P_E = P_X = 50$ per cent, rather successfully separates

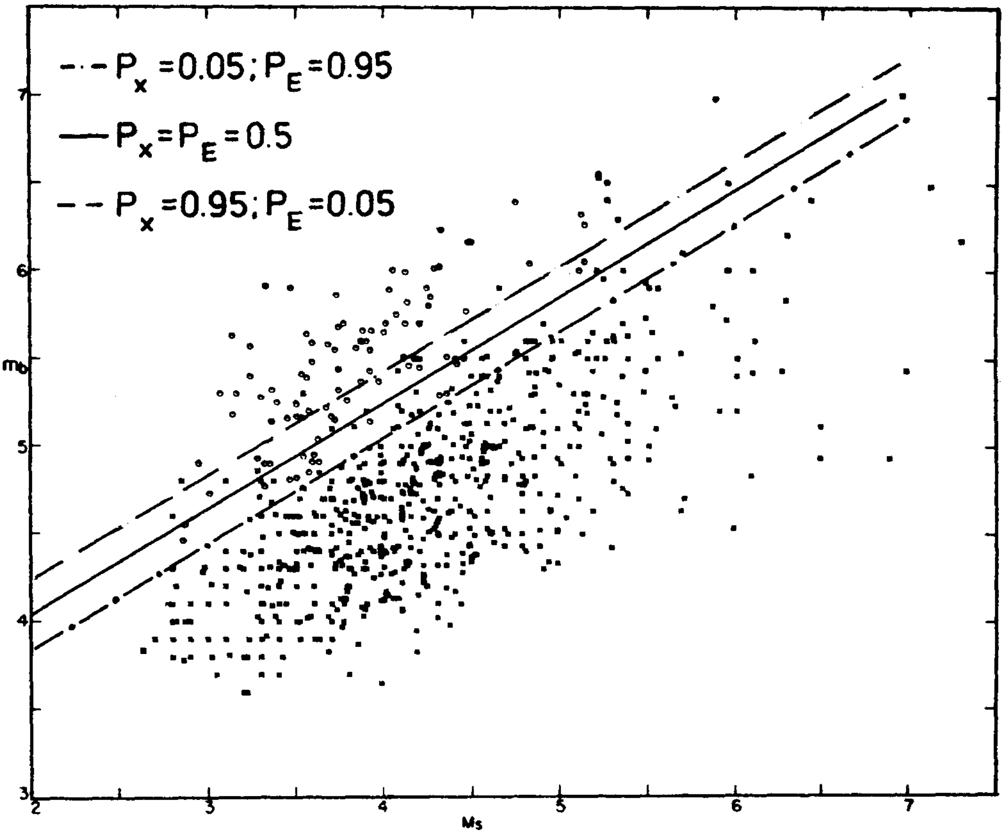


**Figure 4.** $m_b : M_s$ diagram of 100 explosions (circles) and 611 earthquakes (stars) (Weichert & Basham 1973). The discriminant line with $P_E = P_X = 0.5$ misclassifies 12 earthquakes and 16 explosions. At $P_X = 0.05$ and $P_E = 0.95$ all the explosions are classified correctly. However, about 44 earthquakes are misclassified. At $P_X = 0.95$ and $P_E = 0.05$ only two earthquakes but also 34 explosions are misclassified.

the explosion population from the earthquake population. About 16 explosions are either in the earthquake population or on the line, thus about 84 per cent of explosions are separated correctly. Furthermore, only about 12 earthquakes are in the explosion population, thus about 98 per cent of earthquakes are correctly separated, although the probability of correct classification for some events may be too low. The line for $P_E$ = 95 per cent, $P_X$ = 5 per cent separates all the explosions, but it has a high false alarm rate. It misclassifies 44 earthquakes, but these events have a 5–50 per cent chance of being explosions. On the other hand the line for $P_X$ = 95 per cent, $P_E$ = 5 per cent misclassified only two earthquakes but as many as 34 explosions are also misclassified. It is interesting to note that 23 misclassified explosions in this test have $m_b$ magnitude less than 4. Fig. 5(a), from Stevens & Day (1985), indicates a population of 1926 earthquakes for which $m_b$ and $M_s$ were reported by NEIS during 1980 and 1981, and Fig. 5(b) shows recorded explosions through 1982 for which $m_b$ and $M_s$ are available. The area of the circle (earthquakes) or square (explosions) indicates the number of events with the same $m_b$ and $M_s$ coordinates. Again the $P_E = P_X$ = 50 per cent line successfully separates explosions from earthquakes: only about 16 data points from the earthquake populations and about eight data points from the explosion population are on the wrong side of the line. At the $P_X$ = 5 per cent, $P_E$ = 95 per cent level only one explosion, but 63

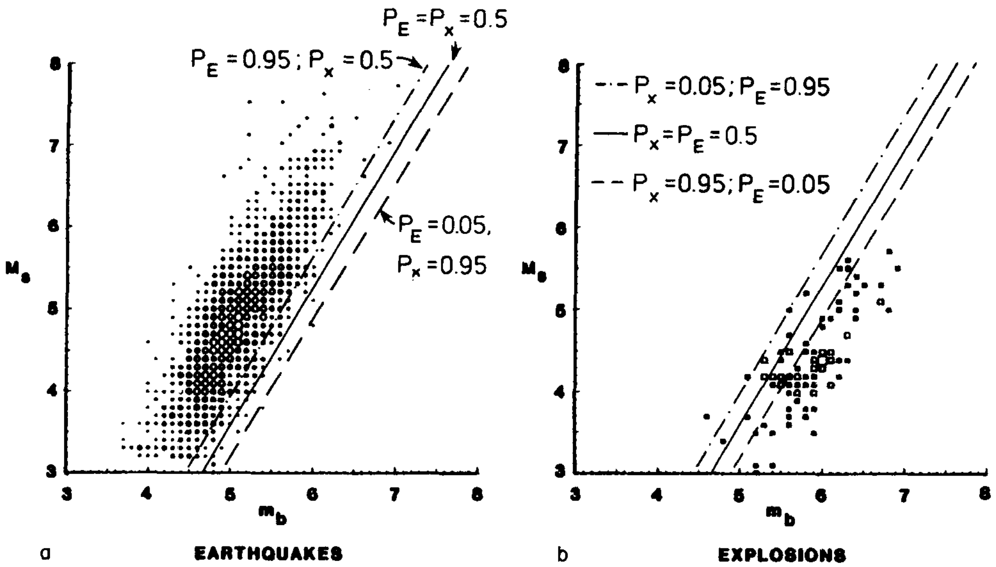


**Figure 5.** $m_b : M_s$ diagram of 1926 earthquakes and all recorded explosions with known $m_b$ and $M_s$ (Stevens & Day 1985). The discriminant line with $P_E = P_X = 0.5$ misclassifies eight explosion data points and 16 earthquake data points. When $P_E = 0.95$ and $P_X = 0.05$, only one explosion with $m_b \leqslant 4.5$ is misclassified. However, there are many false alarms because there are about 63 earthquake data points that are misclassified. When $P_E = 0.05$ and $P_X = 0.95$, only two earthquakes are misclassified but the number of misclassified explosions increases by 19 data points.

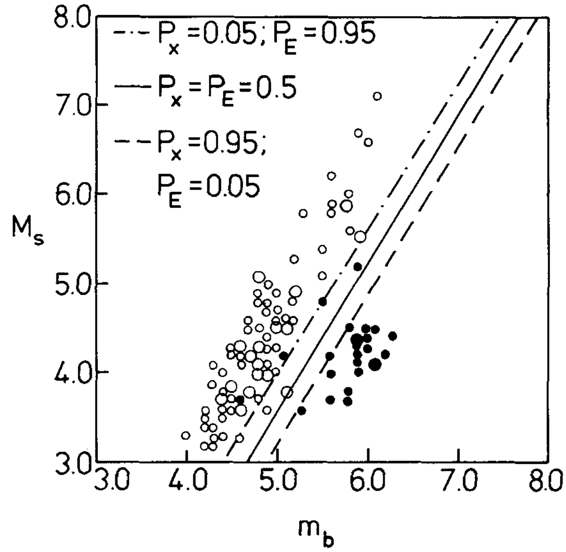


**Figure 6.** $m_b : M_s$ diagram for 26 explosions (solid circles) and 99 earthquakes (open circles) in Eurasia. Larger circles refer to the events with the same coordinates. The discriminant line with $P_E = P_X = 50$ per cent only misclassifies four explosions. All the earthquakes are classified properly. When $P_E = 95$ per cent; $P_X = 0.05$, three earthquake data points and two explosions are misclassified. However, when $P_E = 0.05$; $P_X = 0.95$, all the earthquakes are classified properly but five explosions are misclassified.

earthquake data points are misclassified. Also, at the $P_X = 95$ per cent, $P_E = 5$ per cent level, 19 explosion data points and two earthquakes are misclassified. Fig. 6 gives a plot of $m_b : M_s$ for 26 explosions and 99 earthquakes in Eurasia. Where events have the same coordinates, a larger circle is indicated. The explosions occurred between 1976 and 1981, and the earthquakes occurred during 1979. At the $P_E = P_X = 50$ per cent level, the line given by equation (15) separates all the earthquakes, but four explosions are misclassified. At the $P_X = 5$ per cent, $P_E = 95$ per cent level three earthquakes' data points are misclassified, as well as two explosions. However, at the $P_X = 95$ per cent, $P_E = 5$ per cent level, all the earthquakes are classified properly but five explosions are misclassified. The lines have been plotted on many other reported data sets and in all cases, the earthquake and explosion populations are separated very effectively.

## Conclusion

For discrimination between explosions and earthquakes a method is desirable that can yield the probability of misclassification of either event. Previously, the linear discriminant $D = a + b M_s - m_b$ or $D' = a' + b' m_b - M_s$ was used, but the probability of the misclassifications was often not reported or else neglected. In addition, there are uncertainties associated with the selection of an appropriate $a$, $b$ or $a'$, $b'$ because of regional variations in the near source and near receiver elastic properties, azimuthal distribution of receiver, and method of calculation. Therefore, there are considerable variations in the reported coefficients. In this paper a linear equation,

$$M_s = 1.680\, m_b - 4.825 \quad \text{or} \quad m_b = 2.872 + 0.595\, M_s,$$

is derived by application of the discriminant function. The line very successfully separates the explosions and the earthquakes in an $m_b : M_s$ plot. Moreover, the sign of DIS in the linear equation,

$$\mathrm{DIS}_i = 34.3388 - 11.9569\, m_{bi} + 7.11616\, M_{si},$$

indicates the classification of the event and $P_i = [1 + \exp(\mathrm{DIS}_i)]^{-1}$ gives the probability of correct classification. The positive sign of the decision index, DIS, indicates that the event is an earthquake while a negative DIS indicates that the event is an explosion. The probability of correct classification depends on the value of the decision index parameters. A large, positive DIS indicate that the event has a high probability of being an earthquake and a low probability of being an explosion. On the other hand, a large, negative DIS indicates that the event has a high probability of being an explosion and a low probability of being an earthquake. The discriminant approach adopted here gives the probability of correct classification in a statistical sense at any desired level. For example, the line $M_s = 1.680\, m_b - 5.239$ separates the explosions from the earthquakes at the 95 per cent level, and the line $M_s = 1.680 m_b - 4.412$ separates the earthquakes from the explosions at the 95 per cent level. The method presented can be used to derive the equation of discriminant line for any training data set and at any desired probability level. Although in this paper only $m_b$ and $M_s$ were used, this method can be expanded to include any number of additional discriminant parameters.

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## References


Anderson, T. W., 1958. *An Introduction to Multivariate Statistical Analysis*, Wiley, New York.

Bartlett, M. S., 1949. Fitting a straight line when both variables are subject to error, *Biometrics*, **5**, 207–212.

Basham, P. W., 1969. Canadian magnitudes of earthquakes and nuclear explosions in southwestern North America, *Geophys. J. R. astr. Soc.*, **17**, 1–13.

Bath, M., 1981. Earthquake magnitude – recent research and current trends, *Earth Sci. Rev.*, **17**, 315–398.

Bell, A. G. R., 1978. Application of two multi-variate classification techniques to the problem of seismic discrimination, *MS thesis*, Department of Geosciences, Pennsylvania State University.

Booker, A. & Mitronovas, W., 1964. An application of statistical discrimination to classify seismic events, *Bull. seism. Soc. Am.*, **54**, 961–971.

Brune, J., Espainosa, A. & Oliver, J., 1963. Relative excitation of surface waves by earthquakes and undergound explosions in the California–Nevada region, *J. geophys. Res.*, **68**, 3501–3513.

Bungum, H. & Tjostheim, D., 1976. Discrimination between Eurasian earthquakes and underground explosions using the $m_b:M_s$ method and short-period autoregressive parameters, *Geophys. J. R. astr. Soc.*, **45**, 371–392.

Capon, J., Greenfield, R. J. & Lacoss, R. T., 1967. Surface vs body-wave magnitude results, *Semi-annual Tech. Summary – Seismic Discrimination*, pp. 3–5, June 30, Lincoln Laboratory, MIT, Lexington.

Davis, J. C., 1973. *Statistics and Data Analysis in Geology*, Wiley, New York.

Douglas, A., 1981. Seismic source identification: a review of past and present research efforts, in *Identification of Seismic Sources – Earthquakes or Underground Explosions*, pp. 1–47, eds Huseye, E. & Mykkelveit, S., Reidel, Boston.

Douglas, A., Hudson, J. A. & Kembhavi, V. K., 1971. The relative excitation of seismic surface and body wave by point sources, *Geophys. J. R. astr. Soc.*, **23**, 451–460.

Ericsson, U., 1970. Event identification for Test Ban control, *Bull. seism. Soc. Am.*, **60**, 1521–1546.

Gubta, H. K. & Rastagi, B. K., 1972. Earthquake $m_b$ vs $M_s$ relations and source multiplicity, *Geophys. J. R. astr. Soc.*, **28**, 65–89.

Gupta, H. K., Sitaram, M. V. D. & Narain, H., 1972. Surface wave and body wave magnitudes of some Sino-Soviet explosions and earthquakes, *Bull. seism. Soc. Am.*, **62**, 509–517.

Gutenberg, B. & Richter, C. F., 1956. Magnitude and energy of earthquakes, *Annls Geophys.*, **9**, 1–15.

Karnik, V., 1969. *Seismicity of the European Area*, Reidel, Dordrecht, Holland.

Landers, T., 1972. Some interesting central Asian events on the $M_s: m_b$ diagrams, *Geophys. J. R. astr. Soc.*, **31**, 329–339.

Madansky, A., 1959. The fitting of straight lines when both variables are subject to errors, *Am. stat. Ass. J.*, **54**, 173–212.

Marshall, P. D., 1970. Aspects of the spectral differences between earthquakes and the underground explosions, *Geophys. J. R. astr. Soc.*, **20**, 397–416.

Marshall, P. D., Carpenter, E. W., Douglas, A. & Young, J. B., 1966. *AWRE Rep. No. 0-67166*, I, Atomic Weapons Research Establishment, UKAEA, Aldermaston.

Nagamune, T., 1972. Magnitudes estimated from body waves for great earthquakes, *Q. J. seism. Tokly J.*, **37**, 1–8.

Nowroozi, A. A., 1984. Discrimination between underground explosions and earthquakes using discriminant functions: examples for Eurasia and North America, *Abstr., Ann. Meet. Seismological Society of America*, Anchorage, Alaska.

Nuttli, O. W. & Kim, S. G., 1975. Surface wave magnitudes of Eurasian earthquakes and explosions, *Bull. seism. Soc. Am.*, **65**, 693–709.

Passechnik, I. P., Dashikov, G. G., Polikaropova, L. A. & Gamburtseva, N. G., 1970. The magnitude method for identification of underground nuclear explosion (English trans.), *Acad. Sci. USSR, Phys. Solid Earth*, **1**, 19–24.

Peppin, W. A., 1976. *P* wave spectra of Nevada Test Site events at near and very near distances: implications for near regional body wave surface discriminant, *Bull. seism. Soc. Am.*, **66**, 803–825.

Press, F., Dewart, G. & Gilman, R., 1963. A study of techniques for identifying earthquakes, *J. geophys. Res.*, **68**, 2909–2938.

Rivers, D. W., Von Seggern, D. H., Elkins, B. L. & Sproules, H. S., 1980. A statistical discrimination experiment for Eurasian events using a twenty-seven station network, *SDAC-TR-79-5*, Teledyne Geotech, Alexandria.

Romney, C. F., 1964. *Univ. Michigan Rep. No. 4410-71-X*, p. 83, Institute of Science and Technology.

Sandvin, O. & Tjostheim, D., 1978. Multivariate autoregressive representation of seismic *P* wave signals with application to short period discrimination, *Bull. seism. Soc. Am.*, **68**, 735–756.

Stevens, J. L. & Day, S. M., 1985. The physical basis of $m_b : M_s$ and variable frequency magnitude methods for earthquake/explosion discrimination, *J. geophys. Res.*, **90**, 3009–3020.

Tatham, R. H., Forsyth, D. W. & Sykes, L. R., 1976. The occurrence of anomalous events in eastern Tibet, *Geophys. J. R. astr. Soc.*, **85**, 451–481.

Tjostheim, D., 1980. Multidimensional technique – theory and application, in *Identification of Seismic Sources – Earthquakes or Underground Explosions*, pp. 663–694, eds Husebye, E. S. & Mykkelt-veit, S., Reidel, Boston.

Wald, A., 1940. The fitting of straight lines if both variables are subject to error, *Ann. math. Stat.*, **11**, 284–300.

Weichert, D. H. & Basham, P. W., 1973. Deterrence and false alarms in seismic discrimination, *Bull. seism. Soc. Am.*, **63**, 1119–1133.