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Solutions for Fermi questions, February 2019: Question 1: Driving Rain, Question 2: Cooking Off Ammo

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Solutions for Fermi Questions, February 2019

► Question 1: Driving rain

How much does our highway gas mileage decrease in a heavy rain? (*Thanks to Alex Godunov of Old Dominion University for suggesting the question.*)

Answer: At highway speed, our gas mileage is primarily determined by air resistance. A heavy rain will decrease our gas mileage by increasing “air” resistance, i.e., by increasing the mass of the “air” the car pushes aside. Therefore we need to estimate the amount of water in the air as we drive through it. To do that, we need to know both the rainfall rate (in inches per hour or furlongs per sidereal month) and the speed of the falling rain drops. The faster the drops fall, the less water is in the air for a given rainfall rate.

A heavy rain will fall at more than 0.1 inch per hour and less than 10 inches per hour, so we will take the geometric mean and estimate 1 inch per hour. Since there are 2.5 cm in an inch and 3600 s in an hour, let’s arbitrarily increase the rainfall by 40% so that our rain falls at 3.6 cm per 3600 s, or 10^{-5} m/s.

We can estimate the speed of large falling rain drops in a few ways. The simplest is to observe that their speed must be more than 1 m/s (2 mph) since then we could easily track individual falling rain drops and it must be less than the speed of sound ($c = 330$ m/s) since otherwise rainstorms would be filled with sonic booms. Taking the geometric mean gives us a speed of 20 m/s. The second method only works with sedans, cars that have an angled rear window. If you drive a sedan in the rain, you can notice that at low speeds (below 30 mph or 15 m/s) the back window gets wet and at highway speeds (60 mph or 30 m/s) the back window stays dry. If the back window is angled at 45 degrees, this implies that the terminal velocity of the raindrops is between 15 and 30 m/s. Since both methods agree, we will use 20 m/s for the terminal velocity.

The fraction of water in the air will simply be the ratio of the flow rate, 10^{-5} m/s, and the raindrop terminal velocity, 20 m/s, or

$$f = \frac{\text{rate}}{v_{\text{term}}} = \frac{10^{-5} \text{ m/s}}{20 \text{ m/s}} = 5 \times 10^{-7}.$$

Thus, less than 1 part per million of the air is replaced

with water. That seems remarkably little when our windshield wipers can’t keep up with the flow. Let’s take a brief detour and see how much water hits our windshield and check if that seems reasonable.

The volume of air per time swept out by our windshield will be its area times its speed. A windshield is about 2 m wide by 0.5 m high so that, at highway speed,

$$\dot{V} = Av = (2 \text{ m})(0.5 \text{ m})(30 \text{ m/s}) = 30 \text{ m}^3/\text{s}.$$

Of that, about 10^{-6} will be water, or $30 \text{ cm}^3/\text{s}$. This is about 1 ounce per second or 1 quart or liter every 30 s. That seems reasonable. If you’ve ever dropped a liter of liquid on the floor, it makes a BIG mess (as I remember from the time I knocked a hammer off the counter and it hit a glass two-liter soda bottle).

Anyway, back to our regularly scheduled estimation question. The density of water is 10^3 times the density of air, so that adding one part per million of water to the air will only increase its fractional density by

$$\Delta\rho/\rho = f(\rho_{\text{water}}/\rho_{\text{air}}) = (10^{-6})(10^3) = 10^{-3}.$$

Since the force of air resistance is linearly proportional to the density of the displaced fluid, this will only increase the force of air resistance by 10^{-3} .

We should probably also consider the extra work needed for our car’s tires to push any accumulated water out of their way. In a heavy rain, there is probably about 1 to 2 mm of water on the road. If our car’s tires are 8 in wide (20 cm), then the mass of water displaced per unit time will be

$$\dot{M}_{\text{water}}^{\text{tire}} = Av\rho = 2(0.2 \text{ m})(10^{-3} \text{ m})(30 \text{ m/s})(10^3 \text{ kg/m}^3) = 10 \text{ kg/s},$$

where the factor of two comes from our two front tires. Let’s compare that to the mass of air pushed aside each second:

$$\dot{M}_{\text{air}}^{\text{car}} = Av\rho = (2 \text{ m})(1.5 \text{ m})(30 \text{ m/s})(1 \text{ kg/m}^3) = 10^2 \text{ kg/s}.$$

Our car will have to push away about 10% more mass in a heavy rain, due primarily to the extra water on the road. This will decrease our gas mileage about 10%.

Thus a heavy rain will decrease our highway gas mileage by only about 10%, although it may be suicidal to drive at highway speeds in a so-called “driving” rain.

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► Question 2: Cooking off Ammo

Guns have long barrels to take full advantage of the force of the expanding gases to propel their projectiles. What would be the initial velocity of a bullet if the propellant in its cartridge exploded outside of a gun?

Answer: We can treat this as a very, very short-barreled gun. A typical cartridge has a metal casing containing the propellant and a projectile (the bullet). The propellant will detonate, converting instantly to a gas with a much larger volume (at STP) and hence with a much larger pressure (at the same volume). This will exert a force on the bullet while it is contained in the metal casing. Once the bullet leaves the casing, the gas can expand in all directions, dramatically decreasing the force it exerts on the bullet.

We can estimate this problem in two ways. We can take a detailed look at a cartridge, estimate its propellant, the mass of the bullet, and the length of the bullet in the casing, or we can scale down from a full-size pistol barrel to a barrel the length of a casing. The second option sounds much easier.

A typical pistol bullet has a muzzle velocity of a few hundred m/s (more than the speed of sound of 330 m/s, but not too much greater since a lot of pistols can be fired subsonically). Let's estimate 400 m/s. A typical pistol barrel is about 10 cm long (more than 1 cm and less than 100 cm). A typical pistol cartridge is 2 to 3 cm long, with the bullet extending about 0.3 cm into the casing (more than 0.1 cm and less than 1 cm). Thus, the pistol barrel is about 30 times longer than the effective barrel length of the casing.

If the force of the propellant remains constant, then the work done on the bullet in the casing will be 30 times smaller than that done in the pistol barrel, giving a kinetic energy 30 times smaller and a velocity $\sqrt{30} \approx 5$ times smaller.

However, the pressure (and hence the force) due to the expanding gas will decrease as L^{-1} where L is the distance traveled by the bullet along the barrel. We could do the integrals and solve the problem exactly, but that would be far too much work. Instead, let's estimate the ratio of the average pressures. The average pressure in the casing will be greater than the average pressure in the pistol barrel by a factor of more than 1 (it's not constant!) and less than 30 (the ratio of the lengths), giving an estimate of $r = \sqrt{30} \approx 5$. Since the force will be five times larger and the distance will be 30 times smaller, this means that work done on the bullet in the casing will be only six times smaller than that done in the pistol barrel.

This means that the kinetic energy of a bullet fired from its casing will only be about six times smaller than if it was fired from a barrel. Therefore its speed will be $\sqrt{6}$ times smaller than 400 m/s, or about 150 m/s. This assumes that the casing does not move. If the casing can move, then it will share the kinetic energy with the bullet, reducing bullet's kinetic energy by a factor of two and its speed by a factor of $\sqrt{2}$. The direction, of course, will be rather unpredictable.

That speed is still quite dangerous. Ammunition needs to be treated with great respect!

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