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STUDIES RELATED TO THE DESIGN AND IMPLEMENTATION OF A MAGNETIC SUSPENSION AND BALANCE SYSTEM

by

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A Thesis Submitted to the Faculty of
Old Dominion University in Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

ELECTRICAL ENGINEERING

OLD DOMINION UNIVERSITY
December 2000

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ABSTRACT

STUDIES RELATED TO THE DESIGN AND IMPLEMENTATION OF A MAGNETIC SUSPENSION AND BALANCE SYSTEM

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Old Dominion University, 2000
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This thesis presents studies related to the magnetic suspension and balance system (MSBS) of the Princeton/ONR High Reynolds Number Testing Facility (HRTF). The main motivation for developing the MSBS is to provide interference free aero/hydrodynamic testing of submersible models, which will lead to more accurate measurements. From the controls point of view, the main specification of the MSBS is to robustly control the position of a submersible model in five degrees of freedom (DOF), including three translational as well as pitching and yawing positions. The MSBS should not only regulate the submersible model’s position but should also allow for small motions around the specified 5 DOF position in the presence of aerodynamic wind tunnel disturbances. The MSBS consists of ten electromagnets, a cooling system for the coils, power amplifiers, a dSpace real-time controller hardware, and laser sensors. The MSBS software consists of a graphical user interface (GUI) developed using tools from Matlab/Simulink and dSpace control system software. A unique feature of this MSBS is that the electromagnets are placed outside a two inches thick stainless steel pipe. This arrangement, required for economic reasons, imposes limitations on the achievable performance by limiting the magnetic field and introducing eddy currents in the stainless steel pipe. These limitations are included in the design of the controller.
A linear quadratic multiple input multiple output (MIMO) LQ compensator is designed using loop transfer recovery (LTR) techniques to stabilize and regulate the model inside the test section of the HTRF. Procedures for tuning of the MIMO controller are developed. Practical concerns, i.e., amplifier dynamics, eddy current effects, disturbances due to aerodynamic forces and torques, and B-coefficient uncertainties, are taken into account during the design phase of the controller. The stability robustness of the LQG/LTR compensator is validated by time domain analysis of the closed-loop system consisting of the controller and a complete nonlinear model of the MSBS.

An interactive graphical user interface is developed for operation and maintenance of the HTRF-MSBS. Important system components, including hardware interfacing, software workbench, protection interlocking, operational modes, and operational procedures, are defined and implemented.

The design and implementation of a 1 DOF system that was used to validate the hardware and software is also presented. A classical linear lead-lag controller and a sliding mode controller are designed, analyzed, and implemented for the 1 DOF magnetic suspension and balance system.
This thesis is dedicated to my Grandma.
ACKNOWLEDGMENTS

I would like to thank Dr. Steven Gray and Dr. Colin Britcher for their patience in guiding my research and in the editing of this manuscript, and I would like to especially thank my advisor, Dr. Oscar González, for his guidance and support.

This project has been supported by the Office of Naval Research under Grant No. N00014-99-1-0298, by Princeton University under Grant No. 150-67555-1, and by NASA Langley Research Center under Cooperative Agreement No. SAA #450.
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CHAPTER I

INTRODUCTION

Over the centuries, the idea of magnetic suspension systems that overcome gravity has challenged humans. More recently, magnetic suspension systems have been shown to be practical for use as bearings or gyroscopes, as plasma containers in nuclear fusion electric power generators, as supports for high speed ground transportation systems, and as interference-free model support systems for use in wind tunnels.

The basic objectives of this thesis\(^1\) are to establish procedures for the design and implementation of magnetic suspension and balance system for the Princeton/ONR High Reynolds Number Testing Facility (HRTF). The High Reynolds Number Test Facility (HRTF) is a specialized wind tunnel aero/hydrodynamic testing of submersible shapes. The facility will operate at very high pressures, up to 230 atmospheres, and relatively low velocities. More information about the wind tunnel and the test facility can be obtained from the official web site of the HTRF [33]. The objective of the MSBS for HRTF is to reliably suspend and position in six degrees of freedom a test model with a cylindrical permanent magnet core. Only a five degrees of freedom magnetic suspension system are addressed in this thesis. A unique feature of this MSBS is that the electromagnets are placed outside a two inches thick stainless steel pipe. This arrangement, required for economic reasons, imposes limitations on the achievable performance by limiting the magnetic field and introducing eddy currents in the stainless steel pipe.

\(^1\) The journal model for this work is *IEEE Transactions on Control Applications.*
This chapter begins with a brief review of magnetic suspension technology. After a review of important features of basic technologies, aerodynamic measurements with magnetic suspension and balance systems are discussed in detail. This chapter concludes with the thesis objectives and the overview of the thesis.

1.1 Magnetic Suspension Technology

Research into magnetic suspension and balance system (MSBS) started in the early 1950's and continues today [1]. Significant advances have been made in the past forty years. Scientists and engineers are starting to look at magnetic suspension as a viable technology with overwhelming benefits applicable to several industries and research areas alike. Some of the most important applications are:

1. MagLev trains
2. Wind tunnel MSBS
3. Microgravity and vibration isolation system
4. Magnetic bearings
5. Space applications
6. Applications in medicine

In the late 1960's, research started on the improvement of land-based transportation systems [1]. MagLev trains provide one solution to this problem. Due to their frictionless nature, higher speeds can be achieved. They have been successfully built in Germany and Japan. MagLev train technology is divided into two sub categories: feedback controlled electromagnetic levitation (EML), and superconductor-based electrodynamic levitation (EDL).
Wind tunnel MSBS alleviates some of the problems encountered during wind tunnel testing of an aircraft. The first MSBS for a hypersonic wind tunnel was constructed in the early 1950’s by researchers at Office National d’Etudes et de Recherches Aérospatiales (ONERA) in France. Research continues in this area.

Magnetic levitation is used successfully in different areas of medicine. It is used as a “flying” device in which a cylindrical body is inserted in the blood stream of an animal test subject in order to assess the tendency of blood to clot on materials intended for use in the manufacture of artificial organs [2].

Magnetic levitation applications in space include flywheels, space-docking mechanisms, and weapon development. Space may become the ideal site for applications of large MSBSs, since the maintenance of superconducting magnets is much easier in space than on earth due to the low temperatures [2].

1.2 Types of Magnetic Suspension

There are nine possible electromagnetic methods of supporting masses known at this time and each one of them could be considered a technology in its own right [1]. These are:

(a) repulsion between magnets of fixed strength
(b) levitation by forces of repulsion and diamagnetic materials
(c) levitation with superconducting magnets or superconducting surfaces
(d) levitation by repulsive forces due to eddy currents induced in a conducting surface or body principally at power frequencies
(e) levitation by a force acting on a current carrying a linear conductor in a magnetic field
(f) suspension by a tuned LCR circuit

(g) mixed permeability systems

(h) suspension by controlled DC magnets and the force of attraction between magnetized bodies

1.2.1 Review of Basic Technologies

(a) Suspension with permanent magnets

Earnshaw’s theorem [4] and Braunbeck’s analysis shows that it is not possible to maintain permanent magnets in stable suspension. To levitate permanent magnets, the condition of Earnshaw’s theorem need to be changed, for example, by introducing diamagnetic material (relative permeability \( \mu_r < 1 \)) or a superconductor (\( \mu_r = 0 \)). In addition, feedback control with electromagnets can also be used for levitating permanent magnets.

(b) Levitation with diamagnetic materials

Diamagnetic materials can be levitated by sufficiently high magnetic fields. However, the two materials that exhibit the most pronounced diamagnetic properties, bismuth and graphite, are so weakly diamagnetic that only small pieces can be levitated. Braunbeck suspended two small pieces of bismuth weighing 8 mg and pieces of graphite weighing 75 mg in a magnetic field of 2 Tesla [1]. It is clear, however, that diamagnetic levitation phenomena can only be of academic interest.

(c) Levitation with superconductors

Absence of electrical resistance at temperatures approaching 0 K is known as superconduction. The phenomenon of rejection of magnetic flux in the superconducting body, the Meissner effect, is not well known [1]. This effect causes the superconducting
body to behave as a perfect diamagnetic material ($\mu_r = 0$) and, therefore, stable suspensions using permanent magnets in the proximity of superconductors are possible. A scheme for levitating a passenger-carrying vehicle over two parallel rails was proposed for the first time by Powell in 1963. The basic principle of most of the subsequent work on levitated vehicles consists of attaching superconducting magnets to a vehicle body which rides over conducting loops laid in the track without touching it.

(d) **Levitation with induced currents**

A coil carrying alternating currents when placed in the proximity of a conducting surface or a sheet will induce eddy currents in it. The interaction of these two currents will produce a force of repulsion between the coil and the plate. This effect can be used for levitation of conducting objects. The same principle has been used for simultaneous levitation and melting of specimens such as in zone refining of metals [1].

(e) **Levitation with forces acting in current carrying conductors situated in magnetic fields**

The force on a conductor of length $l$, carrying a current $i$ and situated transversely in a magnetic field of density $B$, is given by $Bil$ and acts in a direction normal to both the conductor and the magnetic field. This principle has been used to support molten portions of a metal rod undergoing zone melting by locating the molten portions in a transverse magnetic field and passing a current through the rod [1]. Heating of this molten zone is done by induction thus keeping the functions of levitation and melting separate.

(f) **Suspension with tuned LCR circuits**

Electrostatic as well as electromagnetic forces of attraction have been used to produce suspension systems in conjunction with tuned LCR circuits [1]. In electrostatic
systems, the body together with a fixed electrode forms the capacitive element of a tuned LCR circuit. The potential difference between the two electrodes is arranged to increase as the distance between them increases, i.e., the circuit is tuned to resonate at values of capacitance less than those at suspension gap. The application to this principle has been applied to a vacuum gyroscope. This technique suffers from all the disadvantages of the electromagnetic method as well as a very high voltage across the electrodes.

Theoretically, a stable setpoint is possible with the tuned system but LCR circuits have a large time constant and even a slight displacement at the state setpoint is enough to cause a divergent oscillation leading to failure of the suspension. Some means to control and force current changes as well as means of damping are necessary [1]. The stiffness of AC suspensions tends to be low for many applications. The main drawback, however, is the large reactive power required and because the magnetic field is alternating, all the magnetic structures such as coil cores and the suspended body need to be laminated.

(g) Mixed permeability system of levitation

It is observed that Earnshaw’s and Braunbeck’s analyses of the stability of bodies in inverse square law fields leads to the conclusion that stable equilibrium in such fields for bodies with fixed charges per current distributions is not possible. However, where the permeability of the system is slightly less than that of free space, stability is possible. Bevir has shown that stable suspension of conducting bodies is possible in mixed systems where permeability in some places is less than that of free space but is greater in others [1]. A lot of work is needed not only on the technique but also on the control aspects of the system. Nonetheless, it is an extremely ingenious technique.
(h) **Suspension with controlled DC electromagnets**

This technology is by far the most advanced and is the subject of worldwide investigation, not only for advanced ground transportation schemes but also for application in contact-less bearings for high and low speeds and support-less objects in wind tunnels. In this technology, position of the object is sensed by the position sensing system and the feedback signal is used to control the position by changing the current(s) of the DC electromagnet(s). This technology is applied in this research work and is discussed throughout the thesis.

1.3 **Aerodynamic Measurements with MSBS**

Several aerodynamic measurements can be improved with the aid of magnetic suspension or balance systems. For example, flow field measurements in the wake of the model are improved since there is no support interference. In making wake measurements care must be taken to avoid interference between probes and the position sensing system and to insure electromagnetic compatibility with transducers, hot-wire anemometers, and other instruments. Care must be taken to insure that the probe drive mechanism is compatible with the environment in which it is to be used [5].

1.4 **Thesis Objectives**

The principle focus of this thesis is to investigate the controller design and implementation issues for a real-time 5-DOF magnetic suspension and balance system. The main objectives of this thesis are as follows:

1. To design and analyze controllers that regulate the position of a submarine model inside the Princeton/ONR High Reynolds number Test Facility (HRTF)
2. To test tracking of small displacements from the equilibrium position
3. To validate the controllers via simulations
4. To develop a Matlab/Simulink model used for dSpace real time application
5. To build an intuitive graphical user interface with dSpace's ControlDesk software
6. To compose a User's Guide for operating the graphical user interface

In order to achieve these objectives, a 1 DOF MSBS was assembled as a prototype. Chapter II describes the 1 DOF MSBS in detail, while the design and optimization of stabilizing controllers for this system are explained in Chapter III. Chapter IV develops the 5 DOF MSBS model and the stabilizing controller is discussed in Chapter V. Simulation and results are given in Chapter VI, and conclusions are given in Chapter VII. The user's guide for the operation and maintenance of the 5 DOF system and the control system software is given in the appendices.
CHAPTER II

1 DOF MAGNETIC SUSPENSION AND BALANCE SYSTEM

A 1 degree of freedom (DOF) magnetic suspension and balance system (MSBS) is described in this chapter. The 1 DOF system will control the position of a 1” diameter ferromagnetic-bearing suspended below an electromagnet that is controlled by a feedback system. In this 1 DOF system only air separates the electromagnet from the levitated object. The purpose of the 1 DOF system was to test the real-time controller and to gain understanding of the MSBS. In this chapter, various aspects of a 1 DOF magnetic suspension and balance system are discussed. This chapter begins with the basic description of the system. After explaining the position sensing scheme, a detailed description of the real time controller is given.

2.1 Basic Description of MSBS

The 1 DOF MSBS was assembled to test system components in a simple integrated environment. It is composed of three subsystems: the electromagnet, the position detection system, and the real time control system, as shown in Figure 2.1. The position of the model is sensed by the position detection system and the feedback signal is fed to the real time controller to control the position by changing the current of the electromagnet. The electromagnet is comprised of a DC magnet and a Copley amplifier power supply. The DC magnet was constructed by David Lane at Drexel University in Pennsylvania. It was formed with 3800 turns of gauge-22 magnet wire wrapped around a 1-inch low carbon steel core, 4.5 inches in length [6]. The power supply system was constructed by Daniel J. Neff at Old Dominion University in Virginia. It is a 303 series
amplifier manufactured by Copley Controls Corporation and is equipped with fault indicator lamps, normal operation lamp, reset switches, and ON/OFF buttons on the front panel [7]. The position detection system consists of a laser sensor pair SUNX LA 511 and a ±15 volt power supply. An off-the-shelf power supply is used to supply the voltage to the laser sensor transmitter and receiver. A plywood sensor frame was built to mount the sensor pair. The sensor frame is made per actual dimensions of the test section of the HTRF magnetic suspension and balance system; its outer diameter is 19 inches and inner diameter is 18 inches.

![Diagram of 1-DOF MBSB](image)

**Figure 2.1** Schematic diagram of 1-DOF MBSB.

The real-time control system is comprised of a dSpace controller board in a Gateway PC E-5200 with a Pentium 500 MHz microprocessor. The model number of the controller board is DS 103 PPC, which was included in dSpace’s Advance Control Kit
1103. The kit includes software that allows the direct implementation of controllers designed in Matlab/Simulink.

The plant modeling and controller design is done with Simulink and Control System toolbox. The controller’s Simulink model is then compiled with Real-Time Workshop (RTW) for real-time application in dSpace’s ControlDesk. A detailed description of dSpace software and hardware is given in Section 2.3.

2.2 Position Detection

The model position sensing scheme is one of the most important elements of magnetic suspension and balance system for closed loop operation. Since no mechanical contact exists with a magnetically suspended model, the position sensor makes it possible to stabilize a suspended model and to achieve the performance specifications.

The SUNX LA 511, A Class 1,780nm laser beam sensor is used. It is comprised of a transmitter and a receiver. The transmitter generates a 15mm wide collimated beam, which is sensed by the receiver. An APC (Automatic Power Control) circuit in the transmitter is used to maintain stable emission strength. The resolution of the sensor system is less than or equal to 10μm.

The sensor frame is made per actual dimensions of the test section of the HTRF 6-DOF MSBS with the sensors mounted for measuring displacement in 1 dimension. Mirrors attached at 45° view attachments are used on the receiver and transmitter in order to increase the effective area of airflow inside the wind tunnel. The side view attachments bend the laser beam at right angles. They consist of a piece of glass and a bottom-coated aluminum-polished mirror. Since scattering of the glass and the mirror is less than unity and their absorption is not zero, their presence in the light path results in a signal loss. To
get the desired output voltage range, the emission strength span must be adjusted using
the span screw. Span adjustment can be done by turning the span adjuster until the analog
voltage reaches +5 V under no obstruction. As the span adjuster is turned clockwise, the
analog voltage increases.

Laser beam alignment is done with the help of four alignment LEDs present on
the receiver. For beam alignment, the emitter and the receiver should be placed so that
they face each other in a line and then their positions should be aligned until all yellow
LEDs turn off and the green LED lights up.

The maximum sensing range of the LA511 is 500 mm and the laser beam width is
15 mm as shown in Figure 2.2. The range of output voltage is 1 V (full obstruction) to 5
V (no obstruction). For control purposes, it should be noted that the response time of the
sensor system is 0.5 ms, which imposes a maximum closed-loop bandwidth limitation of
about 2kHz.

![Figure 2.2: SUNX LA511 sensing range.](image)

2.3 **Real Time Controller**

This section gives a detailed description of the dSpace hardware. The dSpace
Advance Control Kit 1103 is used in this project. It includes Real Time Interface (RTI),
Microtec C compiler, DS1103 PPC controller board, MLIB, MTRACE, and
ControlDesk.
The control system hardware is comprised of two components:

- DS 1103 – PPC controller board
- CP1103 – interconnection box.

2.3.1 DS1103 – PPC Controller Board

The DS1103 is a single board computer system with digital and analog I/O’s. The main processor for control is a Motorola PowerPC 604e processor running at 333 MHz. This processor executes a real-time controller designed and compiled in Simulink.

For special I/O tasks, a DSP controller unit built around a Texas Instruments TMS320F240 DSP is integrated as a subsystem. A Siemens CAN microcontroller, along with serial interfaces and incremental encoders, is also integrated in the DS1103 for additional tasks. Hardware interface programming is done via Simulink, however, it can also be done directly in C language. A Real Time Interface library (RTII103) is used for hardware interface programming and it supports all I/O modules of the PowerPC, the TMS320F240 and the CAN microcontroller.

The DS1103 is plugged into a 16-bit ISA slot of the host PC, Gateway E-5200, its picture is shown in Figure 2.3.

![Figure 2.3 DS1103 processor board.](image)
2.3.2 CP1103 – Interconnection Tool

The CP1103 connector panel provides easy access to all input and output signals on the dSpace controller board DS1103. It is connected to the controller board with three 100-line high-density ribbon cables. Analog signals are accessed via BNC connectors and digital signals are accessed via sub-D connectors. A LED panel indicates the current level of digital signals. The slave DSP, CAN microcontroller, serial interfaces, and incremental encoders can also be accessed via sub-D connectors. A picture of the CP1103 is shown in Figure 2.4.

![CP1103 - I/O interconnection box.](image)

Figure 2.4 CP1103 - I/O interconnection box.

The analog voltage proportional to the position signal is connected to a 12-bit analog to digital converter (ADC) via a BNC connector. This ADC allows sample rates with periods as small as 800 nanoseconds. The commanded output voltage signal is connected to a 14-bit digital to analog converter (DAC). Wiring interconnections of the sensor pair and the power supply are given in Appendix C.
2.3.3 Software Work Bench

The software workbench is comprised of the MathWork’s Matlab/Simulink and the dSpace control system software. The dSpace control software includes a Real Time Interface (RTI) that automates the implementation of controllers in Simulink in the real-time controller board DS1103.

Control system modeling, design, analysis, simulation, and optimization is done in Simulink and then it is compiled with the Real-time Workshop (RTW) for real-time application. The RTI includes a block library for Simulink that provides all of the digital and analog I/Os for I/O-interface programming. It generates and optimizes C code for real-time implementation automatically, before calling a compiler to compile and link the code. During a build process, controller code is compiled, linked, and downloaded to the processor for execution. Microtec PowerPC cross compiler generates the executable object code for Motorola’s PowerPC 604e. Once the controller code is downloaded in the board, the ControlDesk, an integrated tool, is used for the control, monitoring and automation of real-time experiments. The ControlDesk has a graphical user interface (GUI) for experiment and hardware management.

In this chapter, the main components of the 1 DOF MSBS have been described, including real time controller hardware interfacing, the sensing system, the software workbench, and the electromagnet and power supply. In the next chapter, controller design for stabilization and regulation of this system is discussed.
CHAPTER III

STABILIZATION OF 1 DOF MSBS

In this chapter, stabilization and regulation of a 1 DOF MSBS using two control algorithms are presented. The first algorithm is based on classical techniques of linear controller design using a lead and lag compensator. The second algorithm is based on nonlinear techniques of controller design using a sliding mode controller.

3.1 Plant Model

To analyze the behavior and to systematically design controllers, a mathematical model of the dynamical system needs to be developed and verified. To develop the dynamical equation of motion for the 1 DOF system, the gravitational and magnetic forces acting on the model are considered, as shown in Figure 3.1. The magnetic force is proportional to the magnetic field strength, which in turn is proportional to the model location below the electromagnet, \( x \), and the coil current, \( i \). The basic system can be modeled by the dynamical equations for the current-voltage relationship of the coil and the equation of the ball’s motion

\[
\frac{di}{dt} = \frac{-R}{L}i + \frac{1}{L}V
\]

\[
m\ddot{x} = f(i, x) - g
\]

A magnetic force model was found using direct measurements of the magnetic field and constant force data as described in [8]. The nonlinear relation for the magnetic force is

\[
f(i, x) = \frac{i}{(a_0 + a_1x)^2}
\]

where the system variables are summarized in Table 3.1.
Figure 3.1  Schematic diagram of a 1 DOF system.

Table 3.1  System variables for 1 DOF plant model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i(t) )</td>
<td>Coil current (A)</td>
</tr>
<tr>
<td>( x(t) )</td>
<td>Distance from the bottom of the magnet to the top of the ball (m)</td>
</tr>
<tr>
<td>( V )</td>
<td>Coil voltage (V)</td>
</tr>
<tr>
<td>( f(t) )</td>
<td>Electromagnetic force (N)</td>
</tr>
<tr>
<td>( a_i )</td>
<td>Constant in force vs. distance relationship ((a_1 = 1.2084, a_2 = 460.16))</td>
</tr>
<tr>
<td>( g )</td>
<td>Gravitational acceleration ((g = 9.8 \text{ m/sec}^2))</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass of the ball ((m = 0.0682 \text{ kg, calculated in [2]})</td>
</tr>
<tr>
<td>( R )</td>
<td>Resistance of the coil ((R = 43.1 \Omega))</td>
</tr>
<tr>
<td>( L )</td>
<td>Inductance of the coil ((L = 2.29 \text{ H}))</td>
</tr>
</tbody>
</table>

For linear controller design, the behavior of the nonlinear 1 DOF plant model is linearized around an operating range to extract the transfer function. The linearization is
performed using a Taylor series expansion of \( f(i,x) \), where the higher order terms are neglected. The equilibrium position is selected to be 10 mm below the coil and equilibrium current is found to be 0.54 amps. The state space representation is given by

\[
A = \begin{bmatrix} 0 & 1 \\ 1769.36 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ -34.84 \end{bmatrix}
\]

\[
C = [0 \ 1]; \quad D = [0]
\]

where ball position and velocity are the states. The model transfer function represents a second order system with one unstable pole. The pole-zero map of the linearized plant is shown in Figure 3.2.

![Pole-zero map of 1 DOF linearized plant.](image)

Figure 3.2 Pole-zero map of 1 DOF linearized plant.
3.2 Linear Controller Design

Since the system is inherently unstable [8], a closed loop system is designed to stabilize the system. Of several methods for determining and analyzing the operational form of the control stabilizing networks, the root-locus method is chosen because of its ability to picture the entire character of a system in terms of system loop gain and component time constants. The basic control structure is presented in Figure 3.3.

![Figure 3.3 Basic linear control structure.](image)

The general form of the controller's transfer function must necessarily contain at least one zero to stabilize the system and a pole at the origin (or fairly close to the origin) since integral feedback is required to drive the steady-state error down to zero.

Controller design techniques are primarily based on reshaping of the root locus. The classical lead-lag design techniques given in [9] are used for root locus reshaping. The design considerations are as follows:

1. Stabilize the system by moving the unstable RHP pole to the LHP
2. Achieve a transient response with minimum overshoot and fast rise time

3. Achieve a steady state response with minimum error

4. Maximize the commanded dynamic range in both directions (towards the magnet and away from the magnet)

First, a lead compensator is designed to stabilize the plant by pulling the unstable RHP pole to the open LHP. The generalized transfer function of a lead compensator is given by

\[
G_{\text{Lead}}(s) = K \frac{s + \frac{1}{T_{\text{LEAD}}}}{s + \frac{1}{\alpha_{\text{LEAD}} T_{\text{LEAD}}}}
\]

The lead compensator introduces a zero at \( s = -1/T_{\text{LEAD}} \) and a pole at \( s = -1/\alpha_{\text{LEAD}} T_{\text{LEAD}} \). By making \( \alpha \) sufficiently small, the location of the pole is far to the left of the zero and has small effect on the important part of the root locus. The best locations of the zero and the pole, which stabilize the system and achieve acceptable transient response, are determined by trial and error and found to be 42.06 and 110, respectively, at the gain of 210. The corresponding \( T_{\text{LEAD}} \) is 0.02377 and \( \alpha_{\text{LEAD}} \) is 0.382. Therefore, as \( \alpha_{\text{LEAD}} \) decreases, the sensitivity increases. Root loci of the uncompensated and the lead-compensated network are presented in Figure 3.4.
Figure 3.4  Root loci of the uncompensated and the lead-compensated systems.

Since the lead-compensated loop gain is Type 0, steady state error is associated with it. In order to remove steady state error, a lag compensator is designed in cascade with the lead compensator. The generalized transfer function of a lag compensator is

\[ G_{\text{Lag}}(s) = A \frac{1 + T_s}{1 + \alpha_{\text{LAG}} T_s} = A \frac{s + \frac{1}{\alpha T_{\text{LAG}}}}{s + \frac{1}{\alpha T_{\text{LAG}}}} \]

where \( \alpha_{\text{LAG}} > 1 \). The pole \( s = -1/T_{\text{LAG}} \) is therefore to the right of the zero \( s = -1/\alpha_{\text{LAG}} T_{\text{LAG}} \). The locations of the pole and zero of the lag compensator are determined by trial and error and found to be 3 and .01, respectively. Figure 3.5 shows the root locus of lead-lag compensated network.
A unity DC gain prefilter is added to cancel the undesirable zero of the lead-lag compensated closed loop transfer function. As a result, overshoots are reduced, initial values of the control input are reduced, and the closed loop system is slowed down. The prefilter's transfer function is given by

$$G_{\text{PREFILTER}}(s) = \frac{T_{\text{LAG}}}{s + T_{\text{LAG}}}$$

where $T_{\text{LAG}}$ is pole of the prefilter which is located at 3.

![Root locus of lead-lag compensated system.](image)

Figure 3.5 Root locus of lead-lag compensated system.

A saturation block is also added to limit the commanded current to 1.5 amps because the maximum designed current for the coil is 1.5 amps, which is based on a wire gauge. The controller's block diagram with prefilter and saturation block is given in Figure 3.6.
The simulation plots of the nonlinear model and linear model are given in Chapter VI. The maximum dynamic range is found to be 27 mm, 9 mm towards the magnet and 18 mm away from the magnet.

3.3 Stabilization with Nonlinear Controller

The main objective to design a nonlinear controller was to increase the dynamic range or the region of operation of the 1 DOF MSBS. Variable structure control is a nonlinear control technique, which was initially pioneered by researchers in the Soviet Union. A survey by Utkin [10] summaries their work. A brief introduction to the sliding mode control approach of Slotine [11] and its application to the 1 DOF MSBS are given here.
3.3.1 Review of Variable Structure Control (VSC)

A variable structure control is a high-speed switched feedback control resulting in a sliding mode. The gain is switched between two values according to a rule that depends on the value of the state at each instant. The purpose of the switching control law is to drive the nonlinear plant’s state trajectory onto a pre-specified surface in the state space and to maintain the plant’s state trajectory on this surface for all subsequent time. This surface is called a switching surface. When the plant’s state trajectory is “above” the surface, a feedback path has one gain and a different gain if the trajectory drops “below” the surface. This surface defines the rule for proper switching. The surface is also called a sliding surface (sliding manifold) because, ideally speaking, once the state trajectory intersects it, the switched control will maintain the state trajectory on this surface. The plant dynamics restricted to this surface represent the controlled system behavior.

Variable structure control design breaks down into two phases. The first phase is to design or choose a switching surface so that the states restricted to the surface have desired dynamics. The second phase is to design a switched control that drives the states to the surface. A Lyapunov approach is used to characterize this second design phase [12].

3.3.2 Control Design

The theory of sliding mode control is applicable to systems of order n, but only a second order system is used in this case. As derived in [13], the SISO system can be written as
\[
\ddot{x} = g - \frac{1}{m(a_0 + a_1x(t))^2} i(t)
\]
\[
\frac{1}{b} \ddot{x} = -\frac{1}{b} g + u
\]

where

\[u = \text{control input} = \text{coil current} = i(t)\]

and

\[b = \text{control gain} = \frac{1}{m(a_0 + a_1x(t))^2}\]

The control problem is to get the system state \(x(t)\) to track a specific time varying state \(x_d(t)\) in the presence of model imprecision. Let a time-varying surface \(S(t)\) be defined in the state space by the scalar equation \(s(x; t) = 0\)

\[
x(t) = [x, \dot{x}]^T
\]
\[
x_d(t) = [x_d, \dot{x}_d]^T
\]

where \(x_r = \text{model position}\) and \(\dot{x}_r = \text{model velocity}\)

\[s(x; t) = \left( \frac{d}{dt} + \lambda \right) \tilde{x}\]

where \(\tilde{x} = \text{tracking error} = x_r - x_d\)

and \(\lambda\) is a strictly positive constant that determines the bandwidth of the system.

The problem of tracking \(x_d(t)\) is equivalent to that of remaining on the surface \(S(t)\) for all \(t > 0\) and implies that the scalar quantity \(s\) is kept at zero. For a second order system the switching surface is a line and different control structures are applied according to which side of the line the tracking error vector is. The control \(u(x,t)\) is designed so that the system state trajectory is attracted to the switching surface and, once intercepted, remains on the switching surface for all subsequent time. The state trajectory can be viewed as sliding along the switching surface and thus the system is in sliding
A sliding mode exists if, in the vicinity of the switching surface, $S(t)$, the tangent or the velocity vectors of the state trajectory point toward the surface, the value of the state trajectory or “representative point” remains within an $\epsilon$ neighborhood of $S(t)$. Note that the interception of the surface $s(x; t) = 0$ does not guarantee sliding on the surface for all subsequent time, as illustrated in the Figure 3.8, although it is possible.

The problem of keeping the scalar $s$ at zero can be achieved by choosing the control law of $u$ such that outside of $S(t)$

$$\frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s|$$

where $\eta$ is a strictly positive constant. The above equation states that the squared “distance” to the surface, as measured by $s^2$, decreases along all system trajectories. Thus, it constrains trajectories to point towards the surface $S(t)$. In particular, once on the surface the system trajectories remain on the surface and the tracking error tends exponentially to zero, with a finite time constant as shown in Figure 3.7. In other words, satisfying the above condition (sliding condition), makes the surface an invariant set.

A feedback control law $u(t)$ is selected so as to satisfy the sliding condition; however, in order to account for the presence of modeling imprecision and of disturbances the control law has to be discontinuous across $S(t)$ and may require infinitely fast switching. In real systems, a switched controller has imperfections, such as delay, hysteresis, etc., limit switching to a finite frequency. The representative point then oscillates within a neighborhood of the switching surface. This oscillation, called chattering, is illustrated in Figure 3.9.
Figure 3.7  2-dimensional illustration of the domain of a sliding mode.

Figure 3.8  Phase diagram of a sliding surface for a single-state system.
If the frequency of the switching is very high relative to the dynamic response of the system, the imperfections and the finite switching frequencies are often but not always negligible. Chattering is undesirable in practice, since it involves high control activity and may excite high frequency dynamics neglected in the course of modeling.

Slotine proposed an approach [14] to the reduction of chattering of the system states, by introducing a boundary layer of width $\Phi$ on either side of the switching surface

$$B(t) = \{ x, |s(x;t)| \leq \Phi \} \quad \Phi > 0$$

where $\Phi$ is the boundary layer thickness and $\varepsilon = \Phi/\lambda^{n-1}$ is the boundary layer width. Figure 3.10 illustrates for the case $n = 2$. In other words, outside of $B(t)$, we choose the control law $u(t)$ as before (i.e., satisfying the sliding condition), which guarantees that the boundary layer is attractive, hence invariant: all trajectories starting inside $B(t=0)$ remain inside $B(t)$ for all $t \geq 0$; and we then interpolate $u(t)$ inside $B(t)$. 

Figure 3.9  Chattering as a result of imperfect control switching.
The control law can then be derived as follows:

\[ s(x,t) = \left( \frac{d}{dt} + \lambda \right) \ddot{x} = \ddot{x} + \lambda \dddot{x} \]
\[ \dot{s} = \ddot{x} - \ddot{x}_d + \lambda \dddot{x} = g - bu - \ddot{x}_d + \lambda \dddot{x} \]

Thus, the best approximation of a continuous control law is:

\[ \hat{u} = \begin{bmatrix} g - \ddot{x}_d - \lambda \dddot{x} \end{bmatrix} \]

Now, in order to satisfy the sliding condition, we add a term discontinuous across the surface \( s = 0 \):

\[ u = \frac{1}{b} \left[ \hat{u} - k \text{sat}(\gamma') \right] \]

where \( k \) is chosen in a manner that the control law satisfies the sliding condition as \( k \) must satisfy:
\[
\frac{1}{2} \frac{d}{dt} s^2 = \dot{s} s = \left( g - bu - \dot{x}_d + \lambda \ddot{x} \right) s \\
= \left( g - b\hat{b}^{-1} \left[ \hat{u} - k \text{sat}(\phi) \right] - \dot{x}_d + \lambda \ddot{x} \right) s \\
= \left( g - b\hat{b}^{-1} \left[ g - \ddot{x}_d + \lambda \ddot{x} - k \text{sat}(\phi) \right] - \dot{x}_d + \lambda \ddot{x} \right) s \\
k \geq \left| b\hat{b}^{-1} g - g + \left( b\hat{b}^{-1} - 1 \right) \left[ - \dot{x}_d + \lambda \ddot{x} \right] \right| - \eta b\hat{b}^{-1}
\]

Rearranging the terms and letting \( b\hat{b}^{-1} = \beta \), results in:

\[
k \geq (\beta - 1) \left| \dot{u} \right| - \eta \beta
\]

The inequality sign can be changed to equality by dropping the last term. So

\[
k = (\beta - 1) \left| g - \ddot{x}_d + \lambda \ddot{x} \right|
\]

allows the control system to operate in the presence of model uncertainty.

Tuning parameters, i.e., \( \beta \): the control gain, \( \Phi \): the boundary layer, and \( \lambda \): the control bandwidth, are used to tune the sliding mode controller. Maximum dynamic range of 27 mm (9 mm towards the magnet and 18 mm away from the magnet) is obtained with \( \beta = 65 \), \( \Phi = 0.0125 \), and \( \lambda = 5\pi \). Simulation results are given in Chapter VI.

In this chapter, two control algorithms for the stabilization and regulation of a 1 DOF MSBS were outlined; this concludes the discussion of 1 DOF system. The 5 DOF HRTF magnetic suspension and balance system is addressed in the next two chapters.

Chapter IV describes 5 DOF modeling aspects while Chapter V gives a detailed description of MIMO control algorithm.
CHAPTER IV

5 DOF MAGNETIC SUSPENSION AND BALANCE SYSTEM

This chapter begins with a detailed description of a 5 DOF MSBS for the Princeton/ONR High Reynolds Number Testing Facility (HRTF). After discussing the modeling issues, open loop characteristics of the system are explained. A unique feature of this MSBS is that the coils are placed on the outside of a 2.5 in. stainless steel pipe to levitate a test model inside. The limitations imposed by this configuration are also investigated.

4.1 Basic Description of the MSBS

The High Reynolds Number Test Facility (HRTF) is a specialized wind tunnel for aero/hydrodynamic testing of submersible shapes. The facility will operate at very high pressures, up to 230 atmospheres, and relatively low velocities. The objective of the MSBS for HRTF is to reliably suspend and position in six degrees of freedom a test model with a cylindrical permanent magnet core. In addition to regulating the position in six degrees of freedom, the system should also allow tracking of slow time-varying signals.

For the initial phase of the design of the MSBS for the HRTF, a permanent magnet core, magnetized along the principal test section axis, will be used. This configuration prevents roll control. Thus, only a five degrees of freedom MSBS will be discussed, from now on.

The 5 DOF magnetic suspension and balance system is composed of four subsystems: the electromagnets and power supplies, the cooling system, the position
detection system, and the real-time control system. Details of the electromagnets and power supplies, and the cooling system are not discussed in this thesis. After describing the HRTF test section, the position sensing system will be discussed. Since the real-time controller hardware for the system is the same as 1 DOF case, only the real time control software will be discussed later in this section.

4.1.1 The HRTF Test Section

The HRTF test section is made of stainless steel (304L) and measures 95.5 inches in length with an average inside diameter (ID) of 18.897 inches and an outside diameter (OD) of approximately 24 inches. The nominal thickness of the wall is 2.5515 inches. The test section was fabricated by rolling a plate and welding a seam. For metallurgical reasons, the weld material is stainless steel with 10% ferrous addition [15]. This results in the weld being weakly magnetic. Figure 4.1 shows important dimensions of the test section.

![Figure 4.1 HRTF test section schematic](image)

---

2 Courtesy of Dr. Colin P. Britcher
For five degrees of freedom, at least five coils are required. For the MSBS, twice as many coils are used due to the introduction of a high level of symmetry. After analyzing a number of possible configurations, an “X” configuration was chosen because it results in eight equal-size lift coils lowering the overall cost. Figure 4.2 shows the “X” configuration of electromagnets. It consists of eight coils at 45 degrees from the vertical and horizontal axes and 90 degrees of each other. Two axial coils are wound around the test section close to the ends. For initial phase of the MSBS, the ten coils are divided into five pairs where each coil pair is wired in series to provide an equal amount of attractive and repulsive force. This connection requires only five power amplifiers to provide four lift coil currents and an axial coil current. The four lift coil pairs provide lift, lateral, pitch, and yaw capabilities. Figure 4.2 (b) shows coils and their respective currents. The remaining pair provides control in the axial direction.

(a) Front view of the ‘X’ configuration
4.1.2 Position Sensing

The position detection system includes five SUNX LA 511 laser sensor pairs, a ±15 Volts power supply with contactors, and a sensor framework. An off-the-shelf power supply with contactors will be used to provide the supply voltage to the five sensor pairs, and contactors will be used to provide remote control and monitoring from the control system. Feedback signal from the sensor power supply is used in the protection interlocking of the control system software. Figure 4.3 shows a schematic diagram for the power supply interface. Analog signals from these five sensor pairs will be manipulated to get the measurements for pitch and yaw angles, as well as lateral, vertical, and axial displacements.
The currently proposed sensor framework consists of three rings of similar dimensions as in the 1 DOF case. Two of the rings will hold four sensor pairs and the third one will hold only one sensor pair. Two sensor pairs will be used to measure pitch, two to measure yaw, and one to measure axial displacement of the model. Vertical displacement and lateral displacement will be extracted from the first four sensor pairs. Figure 4.4 shows the side view of the sensor arrangement mounted on the rings; the third ring and the test section pipe are removed for clarity. Since the laser beam width is 15 mm, with this configuration, it is possible to measure $\pm 7.5$ mm displacements in each direction. The sensors will be placed to allow about $\pm 5$ degrees angular variation pitch and yaw measurements. Model size dictates the maximum possible dynamic range of motion.
If the sensors are placed 11 inches from the ends of a 35 inches test model, it will be possible to provide a full 5-degree of freedom control. In order to incorporate roll control of the model, other types of magnetizations could be included and/or the permanent magnet inside the test model should be made non-axisymmetric. The sensor scheme will also need to be augmented.

4.1.3 Real-time Control Software

The dSpace ControlDesk tool was used to develop a real-time control graphical user interface (GUI) environment. It is comprised of the following screens/layouts:

- Title
- Magnetic Suspension and Balance System - Control System
- Sensors, Power Supplies, and Cooling Subsystem
- Bias Currents
- Lift/Sink Model
- Curves and Trends
- Interlocking Diagrams

The "Title" is the opening screen of the GUI, it has no dynamic element. The second screen, "Magnetic Suspension and Balance System – Control System" layout is supposed to be the main window for normal control operation of the HRTF. The "Sensors, Power Supplies, and Cooling Subsystem" screen gives the details of the I/O status, bypass operation for maintenance mode can also be selected in this screen. The "Bias Current" screen allows the operator to set the bias current. Alternatively, bias positions can also be set here. The lift and sink status can be viewed in the "Lift/Sink" screen. Curves of different variables can be configured in the "Curves and Trends" screen. The position measurement together with the model of the MSBS are used to derive curves for the forces acting on the test model. The "Interlocking Diagrams" screen gives the static logic diagrams of digital subsystem. A detailed description of the GUI, I/O assignments, and software interlocking is given in Appendix A.

4.2 Description of Plant Model

For control purposes, a mathematical model of the plant, including the actuators and sensors, needs to be developed. Results of the necessary derivations are presented in this section. For a detailed development of the model, see references [16,18 – 24]. A functional block diagram illustrating the main subsystems in the HRTF MSBS is given in Figure 4.5. This diagram shows that the forces and torques generated by the MSBS
balance the gravity and aerodynamic forces and torques to position the test model according to the reference input. This is accomplished by a feedback compensator and prefilter that determine the necessary change in the coils' currents based on the laser sensors' measurements. The desired coil currents are formed by adding the computed change in currents to the equilibrium coil currents for the specified test model position and orientation. The power amplifiers that drive the coils produce these currents. The coils then generate a magnetic field, which applies forces and torques on the test model.

![Functional block diagram of the MSBS.](image)

Figure 4.5 Functional block diagram of the MSBS.

For computer simulations, detailed nonlinear and linear Simulink block diagrams of the plant were created in [16]. The suspended test model is assumed to be quasi-elliptical, upto 35" long, length to diameter ratio of 12, and weighing about 202.8 N. The test model contains inside a cylindrically shaped permanent magnet core. The test model is suspended inside the test section, which is surrounded by the ten coils. The derivations of the model in this section are based on [18-24]. Note that a change in coordinate axis from [18 – 24] is adopted. The coordinate conversion is given in Table 4.1.
To derive the analytical model two coordinate frames are needed. One frame is fixed to the permanent magnet core and the other one is fixed to the test section. The set of orthogonal ($\bar{x}, \bar{y}, \bar{z}$) body-fixed or core axes is initially aligned with an orthogonal $x, y, z$ system fixed at the center of the test section. Figure 4.6 shows the modified coordinate system.

A nonlinear state-space model was derived in [16] for the plant. It is of the form:

$$\dot{X} = f \left( X, u \right)$$
where $X$ is the state vector and $u$ is the vector of inputs. The state vector is given by

$$X^T = [\bar{\Omega}_x \quad \bar{\Omega}_y \quad \bar{\Omega}_z \quad \bar{\nu}_x \quad \bar{\nu}_y \quad \bar{\nu}_z \quad \theta_x \quad \theta_y \quad x \quad y \quad z]$$

and the input $u$ is given by

$$u^T = \begin{bmatrix} I_1 & I_2 & I_3 & I_4 & I_5 \end{bmatrix}.$$

The state vector $X$ consists of the position coordinates and their velocities and the two angular orientations and their angular velocities. The bar over a variable denotes that it is referenced to the core coordinates; otherwise, it is referenced to the test section coordinates. The states $\theta_x$ and $\theta_y$ are the pitch and yaw angles of the core, respectively. The states $x$, $y$, and $z$ correspond to the lateral, vertical, and axial displacements, respectively. To explicitly write the nonlinear state equation, it is necessary to write the equations for the rotational and translational motions.

First, the rotational motion for the angular displacements requires computation of the torques applied on the core and the moments of inertia. The angular acceleration of the core, in core coordinates, can be written as [22]:

$$\dot{\bar{\Omega}} = \left( \frac{1}{I_c} \right) \nu \left( \bar{M} \times (T_m B) + \bar{T}_d \right)_{\text{net torque}}$$

where $I_c$ (kg-m$^2$) is the core moment of inertia about the axes of symmetry (x and y), $\nu$ is the volume of the core, $\bar{M}$ (A/m) is the magnetization of the core that is nonzero only in the principal $\bar{z}$ axis direction, $T_m$ is the vector transformation matrix from test-section to core coordinates, $B$ (Telsa) is the flux density produced by the electromagnets, and $\bar{T}_d$ (N-m) represents external disturbance torques. Second, the translational acceleration of the core, in core coordinates, can be written as:
\[
\dot{\tilde{V}} = \left(1/m_e\right) \left(v\left(T_{m} \partial B T_{m}^{-1} \tilde{M}\right) + \tilde{F}_d\right)
\]

where \(m_e\) is the mass of the core, \(\partial B\) is a matrix of the gradients of \(B\), and \(\tilde{F}_d\) represents the external disturbance forces in core coordinates. Under the assumptions of reference [18], \(B\) as well as its first and second gradients are linearly related to the coil currents as follows:

\[
\tilde{B} = \left(1/I_{\text{max}}\right) [K_B] I
\]

where \(\tilde{B}\) is the total \(B\) field up to second order gradients at the operating point:

\[
\tilde{B} = [B_x \ B_y \ B_z \ B_{xy} \ B_{xz} \ B_{yz} \ B_{xx} \ B_{yy} \ B_{zz} \ .......
\]

\[
\ ....... \ B_{xxy} \ B_{xzz} \ B_{xyy} \ B_{xyz} \ B_{yyz} \ B_{yyy} \ B_{yzz} \ B_{zzz} \]

\(I_{\text{max}}\) is the maximum coil current, \(K_B\) is a table of \(B\) field coefficients containing the \(B\) field contribution of each coil at the operating point when maximum current is applied to each coil, and \(I\) is the input current. \(K_B\) is a 5×19 matrix, \(B\) is a 1×19 vector, and \(I\) is 1×5 vector.

4.3 Linearization

The linearized state equations when the equilibrium position is \(x_o\) and equilibrium current is \(I_o\), are

\[
\delta \dot{x} = A \delta x + B \delta u
\]

\[
y = C \delta x + D \delta u
\]

where

\[
A = \left. \frac{\delta \dot{x}}{\delta x} \right|_{x_o, I_o}
\]
and

\[ B = \frac{\delta \dot{x}}{\delta I} \bigg|_{x_0, I_0} \]

The suspended model is assumed to be initially at the origin of inertial coordinates. In equilibrium, \( F_x = F_z = 0 \) and the only force on the model is along the y-axis and is equal to the weight of the model.

\[ F_y = m_v g. \]

From equations (18) – (22) of [18],

\[ B_x = B_y = B_{xz} = B_{yz} = 0 \]

and

\[ B_{yz} = \frac{m_v g}{v M_z}. \]

The equilibrium current can be calculated by

\[ \left( \frac{1}{I_{max}} \right) [K] I_o = [\hat{B}] \]

where

\[ [K] = \begin{bmatrix} K_x & K_y & K_{xz} \\ K_y & K_y & K_{yz} \\ K_{xz} & K_{yz} & K_{zz} \end{bmatrix} \quad \text{and} \quad [\hat{B}] = \begin{bmatrix} B_x \\ B_y \\ B_{xz} \\ B_{yz} \\ B_{zz} \end{bmatrix} \]

\( \{ I_o \} \) can be found out by direct inversion of the \([K]\) matrix. \( [\hat{B}] \) and \([K]\) are calculated by OPERA and are found to be
\[
[K] = \begin{bmatrix}
K_x & 0.0065 & -0.0065 & 0.0065 & -0.0065 & 0 \\
K_y & 0.0065 & 0.0056 & 0.0065 & 0.0065 & 0 \\
K_{zz} & 0 & 0 & 0 & 0 & 0.0701 \\
K_{xz} & 0.0468 & -0.0468 & -0.0468 & 0.0468 & 0 \\
K_{yz} & 0.0468 & 0.0468 & -0.0468 & -0.0468 & 0 
\end{bmatrix}
\]

\[
\hat{\mathbf{B}} = \begin{bmatrix}
B_x \\
B_y \\
B_{zz} \\
B_{xz} \\
B_{yz} \\
B_{y2}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0.16
\end{bmatrix}
\]

with these values of \( \hat{\mathbf{B}} \) and \([K]\), the currents required to provide equilibrium suspension are found to be

\[
[I_o] = \begin{bmatrix}
I_1 \\
I_2 \\
I_3 \\
I_4 \\
I_5
\end{bmatrix} = \begin{bmatrix}
102.5641 \\
102.5641 \\
-102.5641 \\
-102.5641 \\
0
\end{bmatrix}
\]

These equilibrium current values are high because preliminary air-cored coil design is used for the calculations of B-fields and gradients. The actual coils will be iron-cored and the resulting equilibrium current values will be significantly lower than these values.

The model parameters are summarized in Table 4.2.
Table 4.2  System variables for 5 DOF plant model

<table>
<thead>
<tr>
<th>Volume of the model $v$</th>
<th>$2.65 \times 10^{-3}$ m$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the model $m$</td>
<td>20.67 Kg</td>
</tr>
<tr>
<td>Weight of the model $W$</td>
<td>202.78 N</td>
</tr>
<tr>
<td>Length of the Model $l$</td>
<td>0.9 m</td>
</tr>
<tr>
<td>Diameter of the Model $a$</td>
<td>0.075 m</td>
</tr>
<tr>
<td>Moment of Inertia $I$</td>
<td>$(\equiv Mh^2/20) 0.837$ Kg m$^2$</td>
</tr>
<tr>
<td>About transverse axes (pitch &amp; yaw)</td>
<td></td>
</tr>
<tr>
<td>Moment of Inertia $I_r$</td>
<td>$(\equiv 2Mr^2/5) 0.0116$ Kg m$^2$</td>
</tr>
<tr>
<td>About longitudinal axes (roll)</td>
<td></td>
</tr>
<tr>
<td>Magnetic core volume</td>
<td>$1.325 \times 10^{-3}$ m$^3$</td>
</tr>
<tr>
<td>Magnetization $M$</td>
<td>$1.2/\mu_0$ A/m</td>
</tr>
<tr>
<td>Field to levitate $B_{c}$</td>
<td>$2g*p/M_z = 0.0160$ T/m</td>
</tr>
</tbody>
</table>

Using the system variables and equilibrium currents, $A$, $B^t$, $C$, and $D$ matrices are calculated from Simulink using Matlab's command *linmod* [30]. State matrices are found to be

$$A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & -4.9e-5 & -4.9e-5 & 0 & 0 & 0 & 0
0 & 0 & 0 & -9.81 & 0 & 0 & 0 & 0 & 0
0 & 0 & -242 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$
Out of ten eigenvalues of a matrix, this model has four non-zero eigenvalues at $\pm 6.9821$ and $\pm 6.9821i$. Figure 4.7 shows the pole map of the plant. There are no transmission zeros.

\[ B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.58e-2 \\
\end{bmatrix} \]

\[ C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\[ D = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} \]

\textsuperscript{4} Note that the letter $B$ is used to denote both flux density and the input matrix of a state-space system.
4.4 **Eddy Current Modeling and Amplifier Dynamics**

The dynamical effects of the eddy currents and the current amplifiers are taken into account for the controller design. This section discusses these effects.

Changing magnetic fields induce electromotive force (EMF), which causes eddy currents. The test section is constructed of stainless steel, so the eddy currents generated by unsteady magnetic fields are a serious concern. The eddy currents are modeled by analyzing the magnetic field exerted by the coils using the software package OPERA-3d™ [31]. The eddy current effects are approximated by a simple first order lag so they only affect the transient response of the magnetic field generated by the coils. Further, the cutoff frequency for the eddy current effects is found to be 27 Hz, which is acceptable from a system-dynamics point of view [15]. For detailed analysis of eddy currents and
modeling of the lag transfer function in the Simulink model, see [16]. Figure 4.8 shows the frequency domain plots of the modeled eddy current effects.

![Bode Diagrams](image)

Figure 4.8  Bode plots of first order lag modeling the eddy current effects on the magnetic field’s dynamical response.

The power amplifiers to be used as current sources, are manufactured by Copley Controls Corporations. They will be used in the project to supply the commanded currents to the coils. It is important to take these effects into account for controller design and tuning [16]. The amplifier dynamics are modeled as a third order transfer function, details can be seen in [17]. The current-voltage transfer function of a representative Copley power amplifier is given by

\[
\frac{I}{V} = K \frac{(s + z_1)}{(s + p_1)(s + p_2)(s + p_3)}
\]
where \( K = -51.585, z_1 = 209.8, p_1 = -499.8, p_2 = -93 + 210i, p_3 = -93 - 210i \). Figure 4.9 shows the frequency domain plots of modeled eddy current effects.

![Bode Diagrams](image)

Figure 4.9 Frequency response of a representative Copley power amplifier.

4.5 Open Loop Analysis

A detailed open loop analysis of the linearized plant is presented in this section in order to understand the model, its characteristics, and the effects of cross-coupling. SISO transfer functions, cross-couplings, and observability and controlability analysis is discussed in this section.

4.5.1 SISO Analysis of MIMO System

The plant model has five inputs and five outputs. The five inputs are coil currents and five outputs are pitch and yaw angles, and lateral, vertical, and axial positions.
Transfer functions from each input to each output are calculated to observe the effects of cross-coupling. It is found that the model is highly coupled, as expected by the use of the 'X' configuration of the electromagnets. Table 4.3 summarises the cross-coupling.

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Effected Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lateral Displacement</td>
</tr>
<tr>
<td>I₁</td>
<td>×</td>
</tr>
<tr>
<td>I₂</td>
<td>×</td>
</tr>
<tr>
<td>I₃</td>
<td>×</td>
</tr>
<tr>
<td>I₄</td>
<td>×</td>
</tr>
<tr>
<td>I₅</td>
<td>×</td>
</tr>
</tbody>
</table>

The SISO transfer functions from input 1, \( I₁ \), to lateral, vertical and axial displacements, and pitch and yaw angles are found to be

\[
TF₁¹₁ = \frac{0.023873}{s^2} \\
TF₁¹₂ = \frac{0.023873(s + 0.01293)(s - 0.01293)}{s^4} \\
TF₁¹₃ = \frac{0.79833}{(s + 6.982)(s - 6.982)(s^2 + 48.75)} \\
TF₁¹₄ = \frac{0.081379s^2}{(s + 6.982)(s - 6.982)(s^3 + 48.75)} \\
TF₁¹₅ = \frac{0.081379}{s^2}
\]

SISO transfer functions from input 2, \( I₂ \), to pitch and yaw angles, and lateral, vertical, and axial displacements, are found to be
$$TF\ 21 = -\frac{0.023873}{s^2}$$

$$TF\ 22 = \frac{0.023873 (s + 0.0002)(s - 0.0002)}{s^4}$$

$$TF\ 23 = \frac{0.79833}{(s + 6.982)(s - 6.982)(s^2 + 48.75)}$$

$$TF\ 24 = \frac{0.081379 s^2}{(s + 6.982)(s - 6.982)(s^2 + 48.75)}$$

$$TF\ 25 = -\frac{0.081379}{s^2}$$

SISO transfer functions from input 3, $I_3$, to pitch and yaw angles, and lateral, vertical, and axial displacements, are found to be

$$TF\ 31 = -\frac{0.023873}{s^2}$$

$$TF\ 32 = -\frac{0.023873 (s + 0.0002)(s - 0.0002)}{s^4}$$

$$TF\ 33 = \frac{0.79833}{(s + 6.982)(s - 6.982)(s^2 + 48.75)}$$

$$TF\ 34 = \frac{0.081379 s^2}{(s + 6.982)(s - 6.982)(s^2 + 48.75)}$$

$$TF\ 35 = \frac{0.081379}{s^2}$$

SISO transfer functions from input 4, $I_4$, to pitch and yaw angles, and lateral, vertical, and axial displacements, are found to be

$$TF\ 41 = \frac{0.023873}{s^2}$$
\[ TF_{42} = -\frac{0.023873}{}(s+0.01293)(s-0.01293) \]

\[ TF_{43} = \frac{0.79833}{(s+6.982)(s-6.982)(s^2+48.75)} \]

\[ TF_{44} = -\frac{0.081379s^2}{(s+6.982)(s-6.982)(s^2+48.75)} \]

\[ TF_{45} = -\frac{0.081379}{s^2} \]

SISO transfer functions from input 5, \( I_s \), to pitch and yaw angles, and lateral, vertical, and axial displacements, are found to be

\[ TF_{51} = 0 \]

\[ TF_{52} = \frac{0.00042492}{s^2(s+6.982)(s-6.982)(s^2+48.75)} \]

\[ TF_{53} = \frac{0.035759s^2}{(s+6.982)(s-6.982)(s^2+48.75)} \]

\[ TF_{54} = -\frac{8.663}{(s+6.982)(s-6.982)(s^2+48.75)} \]

\[ TF_{55} = 0 \]

Since \( I_s \) is the current flowing in the axial coils, it has no effect on lateral displacement and yaw angle. This can also be observed from the SISO transfer functions from \( I_s \).

4.5.2 Controllability Analysis

The 5 DOF MSBS plant model is found to be completely controllable.

Controllability from each input is also calculated and is summarised in Table 4.4.

Eigenvalues of the A matrix of the system are also analysed for controllability from each
input. It is found out that all four non-zero eigenvalues are controllable from all five input currents. Some of the zero eigenvalues are non-controllable from one input but controllable from other inputs, making the overall system controllable.

**Table 4.4  Controllability from each input**

<table>
<thead>
<tr>
<th>Description</th>
<th>Rank</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Controllability</td>
<td>10</td>
<td>Completely controllable</td>
</tr>
<tr>
<td>Controllability from Input 1: I1</td>
<td>8</td>
<td>2 states not controllable</td>
</tr>
<tr>
<td>Controllability from Input 2: I2</td>
<td>8</td>
<td>2 states not controllable</td>
</tr>
<tr>
<td>Controllability from Input 3: I3</td>
<td>8</td>
<td>2 states not controllable</td>
</tr>
<tr>
<td>Controllability from Input 4: I4</td>
<td>8</td>
<td>2 states not controllable</td>
</tr>
<tr>
<td>Controllability from Input 5: I5</td>
<td>6</td>
<td>4 states not controllable</td>
</tr>
</tbody>
</table>

**4.5.3 Observability Analysis**

The 5 DOF MSBS plant model is found to be completely observable. Observability from each output is also calculated and is summarized in Table 4.5.

Eigenvalues of the A matrix of the system are also analysed for observability from each output. It is found out that all four non-zero eigenvalues are observable from all five output positions. Some of the zero eigenvalues are not observable from one output but observable from the other output, thus making the overall system observable.

**Table 4.5  Observability from each output**

<table>
<thead>
<tr>
<th>Description</th>
<th>Rank</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall Observability</td>
<td>10</td>
<td>Completely observable</td>
</tr>
<tr>
<td>Observability from pitch angle</td>
<td>4</td>
<td>6 states not observable</td>
</tr>
<tr>
<td>Observability from yaw angle</td>
<td>2</td>
<td>8 states not observable</td>
</tr>
<tr>
<td>Observability from lateral displacement, x</td>
<td>2</td>
<td>8 states not observable</td>
</tr>
<tr>
<td>Observability from vertical displacement, y</td>
<td>8</td>
<td>2 states not observable</td>
</tr>
<tr>
<td>Observability from axial displacement, z</td>
<td>4</td>
<td>6 states not observable</td>
</tr>
</tbody>
</table>
In this chapter, the 5 DOF plant model is discussed. Open loop analysis of this multivariable system shows that it is highly coupled. Controllability and observability analysis shows that the system is completely controllable and observable. MIMO controller design concepts are developed in the next chapter.
CHAPTER V

STABILIZATION OF 5 DOF MSBS

In this chapter, stabilization and regulation of the 5 DOF magnetic suspension and balance system are discussed. The chapter begins with a brief overview of linear quadratic optimal control theory, necessary for providing a basic understanding of linear quadratic regulator (LQR). Kalman filtering is then explained. Linear quadratic Gaussian/loop transfer recovery (LQG/LTR) design technique is also presented in detail followed by a comprehensive controller design outline. This chapter is concluded with frequency domain analysis.

It should be noted that the linear quadratic design techniques are chosen to design a controller for the 5 DOF MSBS for the HRTF because these techniques have already been tested successfully on real-time magnetic levitation systems (see, for example, [22]).

5.1 Introduction to Optimal Control

Optimal control is a branch of modern control theory that deals with designing controls for dynamical systems by minimizing a performance index (PI) that depends on the system variables. The performance index may include a measure of operating error and/or control effort. Under some mild assumptions, making the performance index small also guarantees that the system variables will be small, thus insuring closed-loop stability. Classical design is concerned with directly selecting the feedback gains \( K \) in the inner real-time control loops [9]. On the other hand, modern control design offers standard algorithms for implementing an outer design loop that automatically selects the inner loop feedback gains in such a fashion that closed-loop stability and performance of
MIMO systems is guaranteed. In contrast to classical sequential closed loop design, in modern control, all of the feedback loops are closed simultaneously by computing the feedback gains by solving standard matrix design equations [25].

5.2 Linear Quadratic Regulator (LQR)

The LQR is a cornerstone of modern optimal control design consisting of explicit matrix design equations easily solved on a digital computer. It has a wide range of relevance, because most nonlinear systems are designed to operate near an equilibrium point. The LQ techniques given in [26] are followed here.

The nonlinear 5 DOF MSBS model, introduced in Section 4.2, is of the form

\[ \dot{X} = f(X, u) \]

where \( X \) is given by

\[ X^T = [\bar{\Omega}_x \bar{\Omega}_y \bar{V}_x \bar{V}_y \bar{V}_z \theta_x \theta_y \theta_z x y z] \]

and the input \( u \) is given by

\[ u^T = [I_1 I_2 I_3 I_4 I_5] \]

The bar over a variable denotes that it is referenced to the core coordinates; otherwise, it is referenced to the test section coordinates. The states \( \theta_x \) and \( \theta_y \) are the pitch and yaw angles of the magnetic core respectively. The states \( x, y, \) and \( z \) are lateral, vertical, and axial displacements respectively. The remaining states correspond to the velocities of the displacement and rotational variables.

The linearized model, developed in Section 4.3, in the neighborhood of the origin of the test section coordinate system can be written as

\[ \dot{X} = AX + Bu \]
\[
y = CX + Du
\]
where for simplicity \( X = \delta X \). The continuous-time performance index (PI) is given by
\[
J(t_0) = \frac{1}{2} X^T(T) S(T) X(T) + \frac{1}{2} \int_{t_0}^{T} \left( X^T Q X + u^T R u \right) dt
\]
with \( S(T) \geq 0, Q \geq 0, R > 0 \).

The design problem is to convert the design specifications into appropriate symmetric control weighting \( R \), state weighting \( Q \), and final state weighting \( S(T) \) matrices. The solution of the minimization problem is made possible by making \( J \) greater than equal to zero. This requires that \( Q \) and \( S(T) \) be positive semi-definite \(( S(T) \geq 0, Q \geq 0 \) and \( R \) be positive definite \(( R > 0 \)). Thus \( Q \) and \( S(T) \) have non-negative eigenvalues so that \( X^T(T)QX \) and \( X^T(T)S(T)X(T) \) are non-negative for all \( X(t), \) and \( R \) has positive eigenvalues, so that \( u^T Qu > 0 \) for all \( u(t) \). The optimal control is given in terms of the auxiliary matrix \( S(T) \) which satisfies the bilinear matrix Riccati equation, given by
\[
-\dot{S} = A^T S + S A - SBR^{-1}B^T S + Q \quad t \leq T
\]
In terms of the Riccati equation solution \( S(T) \), the optimal control is given by
\[
u(t) = -R^{-1}B^T S(t) X(t) .
\]
Thus, the optimal feedback gain is time-varying and can be written as follows
\[
K = R^{-1} B^T S(t)
\]
and the optimal feedback control law can be written as
\[
u = -K(t) X(t)
\]
The block diagram of the LQR is shown in Figure 5.1. It is a feedback control system with time varying feedback gains $K(t)$, and a formal design outer loop. Even if the system is time-invariant, the optimal control $u(t)$ is time varying since the state variable feedback is time varying. Such feedbacks are inconvenient and difficult to implement, and experience shows that a constant gain can work just as good. The constant gain matrix is found using steady state suboptimal control techniques [26].

Steady state suboptimal control is now selected to minimize the quadratic performance index

$$J(t_o) = \frac{1}{2} \int_0^\infty (X^T Q X + u^T R u) dt$$

with $Q \geq 0$, $R > 0$. Since the integration interval is infinite, this is called the infinite horizon performance index. The bilinear matrix Riccati equation is replaced by the algebraic Riccati equation (ARE), which is given by

$$0 = A^T S + S A - S B R^{-1}B^T S + Q$$

This is a symmetric matrix quadratic equation. The ARE can have multiple solutions. However, if certain mild assumptions on the system and PI matrices hold, then there is a single positive definite solution $S_\infty$. If the solution of the ARE exists, then, the optimal infinite horizon gain is the constant matrix given by

$$K_\infty = R^{-1}B^T S_\infty,$$

and the optimal steady state control is

$$u(t) = -K_\infty X(t)$$

or simply

$$u(t) = -K X(t)$$
Even if the control interval $[0, T]$ is not infinite, the steady state gain $K_\infty$ can be used instead of the optimal time varying gain $K(t)$, yielding a suboptimal control strategy. In the limit, as $T$ becomes large, the optimal gain $K(t)$ tends to $K_\infty$, so that the decision to use the steady state gain makes more and more sense. In addition to the ease of implementation of constant feedback gains, this suboptimal controller has other important advantages [26]:

1. It can guarantees stability even for complex multi-loop systems
2. There are efficient numerical routines available for the solution of ARE

![Figure 5.1 Linear state variable feedback controller.](image)

### 5.3 Kalman Estimator

Since all of the states are not available by direct measurements, a Kalman estimator is used to estimate the states. These state estimates, $\hat{X}(t)$, are then fed back to the controller, so the modified control law is of the form $u = -K \hat{X}(t)$. This results in the so called linear quadratic Gaussian (LQG) problem since the noise sources are
typically modeled as zero mean Gaussian stochastic processes. The performance of the Kalman estimator requires that the noise sources be accurately modeled since poorly modeled noise can have catastrophic consequences on observer performance [26]. Since the controller will be implemented digitally, the primary noise sources are assumed to be quantization noise and sensor noise. Quantization of the sensor signal constitutes the system’s process noise while sensor noise provides the measurement noise. The linear state space equations can now be written as
\[
\dot{x} = Ax + Bu + Gw \\
y = Cx + Du + v,
\]
where \(w\) and \(v\) represent the process and measurement noise, respectively. The design of an optimal estimator involves an associated cost function. In this case, the cost function to be minimized is a function of the state error signal, \(\bar{X}\), defined as
\[
\bar{X} = X - \hat{X}
\]
where \(x\) is the true state and \(\hat{x}\) is the estimated state. The cost function is given by
\[
J = \int_0^T \| \bar{X}(t) \|^2 \, dt
\]
which describes the energy in the error signal or equivalently the mean square error.

The closed loop estimator equations are given by
\[
\dot{\hat{X}} = A\hat{X} + Bu + L(y - \hat{y}), \quad \hat{X}(0) = 0
\]
where \(L\) denotes the observer gain and the initial condition of the estimator is arbitrarily chosen to be zero. The optimal observer gain guaranteeing the minimization of cost function is given by
\[ L = P \ C^T \ R_f^{-1} \]

where \( R \) represents the autocorrelation of the measurement noise, \( P \) is the stabilizing solution of the algebraic Riccati equation

\[
0 = PA^T + A^T P - P \ C^T \ R_f^{-1} \ C^T \ P + G \ Q_f \ G^T
\]

The block diagram representation of the observer is shown in Figure 5.2.

![Observer block diagram](image)

**Figure 5.2** Observer block diagram.

It should be noted that the noise statistics \( G \) is used to compute the noise spectral density matrices \( Q_f \) and \( R_f^{-1} \). The transfer function from the measurements \( y(t) \) to the state estimate \( \hat{x}(t) \) is given by

\[
H(s) = [sI - (A - LC)]^{-1} L
\]

so the Kalman filter is a low pass filter that attenuates the effect of sensor noise on the state estimates.
5.4 LQG/LTR Design Technique

In the LQG approach, the feedback gain $K$ is selected by solving the LQR algebraic Riccati equation and the compensator dynamics are given in terms of the observer gain, $L$, designed using the Kalman filter algebraic Riccati equation [26]. Since the late 1980's it is known that the desirable robustness properties of the LQR design are, in general, lost when it's combined with the Kalman filter. Techniques to recover the LQR robustness properties have been developed and are called LQG/Loop Transfer Recovery (LTR) design techniques. This section presents the LQG/LTR procedure from [26] and [29].

In Section 5.2, the matrices $Q$ and $R$ were introduced as the LQR design parameters used to make the closed-loop system meet the design objectives. In Section 5.3, the parameters $Q_f$ and $R_f$ are the noise covariance matrices needed to design the Kalman gain. In this section, it will be shown that the four matrices can be modified to recover the LQR robustness properties.

Consider the plant
\[ \dot{x} = Ax + Bu + Gw \]
\[ y = Cx + v \]
and suppose that the LQR full state feedback law has been designed and is given by
\[ u = -KX. \]

Define the loop gain at the input of the plant that has leads to the nice robustness properties to be
\[ L_i(s) = K(sI - A)^{-1}B. \]

Now supposed that the LQG regulator is designed. The LQG control law is given by
\[ U(s) = -K[sI-(A-LC)]^{-1}BU(s) - K[sI-(A-LC)]^{-1}LY(s) \]

where \( L \) is the Kalman filter gain. After simple matrix manipulation, it can be shown that the equivalent output feedback compensator transfer function, \( K(s) \), can be written as

\[ K(s) = K[sI-(A-BK-LC)]^{-1}L \]

The corresponding LQG loop gain at the plant input is now given by

\[ L_r(s) = (K[sI-(A-BK-LC)]^{-1}L)[C(sI-A)^{-1}B]. \]

To design a Kalman filter so that the regulator loop gain, \( L_r(s) \), at the plant’s input, is the asymptotically approaches the LQR loop gain, \( L_s(s) \), it is assumed that the plant is minimum phase (because there are no zeros), with \( B \) and \( C \) of full rank and \( \text{dim}(u) = \text{dim}(y) \).

To simplify the derivation, let \( G = I \) and the process noise spectral density matrix

\[ Q_f = v^2Q_o + BB^T \]

with \( Q_o > 0 \) and \( v \) a scalar design parameter. Then the Kalman filter gain is given by

\[ L = PC^T \left( v^2 R_f \right)^{-1} \]

where \( P \) is the solution of the Kalman filter ARE

\[ 0 = AP + PA^T + \left( v^2Q_o + BB^T \right) - PC^T \left( v^2 R_f \right)^{-1}CP \]

In [26], it is shown that using \( G = I \) and the process noise given by

\[ Q_f = v^2Q_o + BB^T \]

that as \( v \to 0 \), the dynamic regulator loop gain using a Kalman filter approaches the target feedback loop gain \( K \phi B \) using full state feedback. This means that, as \( v \to 0 \), all the robustness properties of the full state feedback control law are recovered in the
dynamic regulator. It also means that the time response of the LQG regulator should approach the time response of the full state feedback law as $v \to 0$. The compensator block diagram is given in Figure 5.3.

![Compensator block diagram](image)

**Figure 5.3** Compensator block diagram.

### 5.5 Compensator Design Procedure

The design procedure of the LQG/LTR compensator for the magnetic suspension and balance system is outlined in this section. It is an iterative process that requires deep understanding of the behavior of the model in the test section.
A full state feedback controller is designed first for the regulation of the basic nonlinear design-model and then the LQG/LTR techniques are applied to design a Kalman filter. The design procedure is as follows:

1. Linearize the basic non-linear model of the plant, as discussed in Chapter IV.
2. Extract the 5 DOF model, removing the roll states—roll angle and roll velocity, since roll motion is not considered.
3. Design full state feedback LQ static gain feedback controller for the basic model using the linear state space model. The first design object is to stabilize the basic model. Once it is stabilized, it should be tuned for the desired regulation specifications, i.e., reduced rise time and overshoot.
4. Tune the static gain feedback controller by modifying the PI weighting matrices $Q$ and $R$, repeatedly, to achieve the desired time domain and frequency domain characteristics. Analyze the step responses to adjust the time domain specifications, i.e., rise time and overshoot. Analyze maximum and minimum singular values of loop gain, sensitivity transfer function, and complementary transfer function to improve the robustness properties of the controller.
5. Introduce the transient effect of the eddy current into the basic design model.
6. Tune the controller, if necessary, by modifying the PI weighting matrices $Q$ and $R$. Check the time domain and frequency domain characteristics.
7. Introduce the amplifier dynamics into the previously modified basic model.
8. Tune the controller, if necessary, by modifying the PI weighting matrices $Q$ and $R$. Check the time domain and frequency domain characteristics.
9. Introduce the B-coefficient random perturbation in the previously modified model.

10. Tune the controller, if necessary, by modifying the PI weighting matrices $Q$ and $R$. Check the time domain and frequency domain characteristics.

11. Introduce the aerodynamic forces and torques in the previously modified model.

12. Tune the controller, if necessary, by modifying the PI weighting matrices $Q$ and $R$. Check the time domain and frequency domain characteristics.

13. Now remove the eddy current effects, amplifier dynamics, B-coefficient perturbation, and the aerodynamic forces and torques from the model.

14. Design the Kalman full state estimator for the basic model using the LQG/LTR design procedure.

15. Tune the Kalman full state estimator by changing the size of the PI noise covariances matrices $Q_f$ and $R_f$, if necessary.

16. Compare the singular value plots of full state feedback closed loop system with LQG compensator (with the Kalman estimator).

17. Stop tuning the Kalman estimator when the frequency domain plots of the LQG compensator approach the plots of full state feedback controller.

18. Introduce the eddy current effects into the model and check the time domain and the frequency domain characteristics. Tune the static feedback gain controller if necessary.

19. Introduce the amplifier dynamics into the model and check the time domain and the frequency domain characteristics. Tune the static feedback gain controller if necessary.
20. Introduce the B-coefficient uncertainty into the model and check the time domain and the frequency domain characteristics. Tune the static feedback gain controller if necessary.

21. Introduce the aerodynamic forces and torques into the model and check the time domain and the frequency domain characteristics. Tune the static feedback gain controller if necessary.

22. Discretize the linear model.

23. Introduce a zero order hold into the simulation.

24. Use discrete time algorithm to design the LQ static feedback controller and the Kalman estimator with the continuous-time domain performance matrices.

25. Simulate the nonlinear plant with the zero order hold block and the discrete time Kalman estimator and the LQ controller. Check the step responses and tune the controller if necessary.

5.6 LQG/LTR Compensator Simulation

The LQG/LTR compensator is simulated in the Simulink to check the time domain and frequency domain characteristics. The Simulink block diagram for the continuous time system full-state feedback LQ controller is shown in Figure 5.4 and that of the LQ compensator is shown in Figure 5.5. The prefilter is used to adjust the DC gain of the system to unity. The reference position block is used to provide step responses to the model and the graphical subsystem is used for plotting the step responses.
The discretized system is shown in Figure 5.6. After trying a number of different sampling frequencies, a 1 kHz sampling frequency is selected which is the best compromise between the model requirement and the system performance, and sensor bandwidth.
After tuning, the design matrices of the LQ regulator are found to be

\[
Q = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 9990 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 999 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 90000 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 99 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 999 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 99
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
100 & 0 & 0 & 0 & 0 \\
0 & 100 & 0 & 0 & 0 \\
0 & 0 & 100 & 0 & 0 \\
0 & 0 & 0 & 100 & 0 \\
0 & 0 & 0 & 0 & 100
\end{bmatrix}
\]
\[ Q_f = BB^T \]

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[ R_f = 1.0e^{-005} \]

The static feedback gain and the Kalman gain are found to be

\[
K = \begin{bmatrix}
1.3e+3 & 1.3e+3 & 42e+2 & -4.4e+2 & 40e+2 \\
-1.3e+3 & 1.3e+3 & 42e+2 & -4.4e+2 & -40e+2 \\
1.3e+3 & -1.3e+3 & 42e+2 & -4.4e+2 & 40e+2 \\
1.9e-5 & -1.4e-3 & 4.5e+3 & 10e+2 & -2.7e-7
\end{bmatrix}
\]

\[
L = \begin{bmatrix}
4.2178e-1 & 2.3566e-13 & -5.4221e-13 & 5.0604e-16 & 1.5437e-10 \\
2.3566e-13 & 4.2178e-1 & -1.6887e-11 & -1.1244e-10 & -1.4161e-11 \\
-5.4221e-13 & -1.6887e-11 & 3.777e-1 & -6.1650e-5 & 1.3319e-13 \\
5.0604e-16 & -1.1244e-10 & -6.1650e-5 & 6.3359e-1 & 1.0690e-13 \\
1.5437e-10 & -1.4161e-11 & 1.3319e-13 & 1.0690e-13 & 6.3359e-1 \\
1.1481e+2 & 4.2967e-13 & 7.4187e-11 & 1.5727e-12 & 1.4302e-7 \\
-3.4814e-11 & 1.1481e+2 & 2.8671e-9 & -1.2487e-7 & -3.2738e-8 \\
5.7536e-11 & 1.0233e-10 & 8.9199e+1 & -1.1879e-2 & -6.8507e-11 \\
-1.0201e-11 & -7.8966e-8 & -1.1006e-1 & 3.1155e+2 & -2.3097e-11 \\
1.4398e-7 & 1.3833e-8 & 1.1411e-10 & 2.5838e-11 & 3.1155e+2
\end{bmatrix}
\]

Matlab software is used for designing, analyzing, and optimizing the compensator. The related Matlab commands necessary for analysis and design are summarized in Table 5.1.
Table 5.1 Matlab commands used for LQG/LTR controller design and tuning.

<table>
<thead>
<tr>
<th>Matlab Command</th>
<th>Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linmod</td>
<td>For linearization of the design model</td>
</tr>
<tr>
<td>Ssselect</td>
<td>For the extraction of reduced order model</td>
</tr>
<tr>
<td>Lqr</td>
<td>For designing LQ static feedback controller gain</td>
</tr>
<tr>
<td>Lqe</td>
<td>For designing Kalman estimator gain</td>
</tr>
<tr>
<td>C2d</td>
<td>For discretizing continuous time linear model</td>
</tr>
<tr>
<td>Dlqr</td>
<td>For designing discrete LQ static feedback controller gain</td>
</tr>
<tr>
<td>Dlqe</td>
<td>For designing discrete Kalman estimator gain</td>
</tr>
<tr>
<td>Ssdata</td>
<td>For the extraction of discrete time state space model</td>
</tr>
<tr>
<td>Sigma</td>
<td>For the computation of singular values</td>
</tr>
<tr>
<td>Minreal</td>
<td>For the computation of minimal realization</td>
</tr>
<tr>
<td>Bilin</td>
<td>For the conversion of discrete time model back to w-plane</td>
</tr>
</tbody>
</table>

5.7 Frequency Domain Analysis

Unmodeled high-frequency dynamics and structured parameter variations can act to destabilize the closed loop system. Therefore, it is important to design the controller that gives stability robustness, which is the ability to guarantee closed-loop stability in spite of parameter variations at low frequencies and unmodeled dynamics at high frequencies. Robustness issues are conveniently examined in the frequency domain. Matrix transfer relations are developed and the maximum and minimum singular values are reviewed.
Let the plant be denoted by $G(s)$ and the feedback compensator by $K(s)$. The basic control structure is presented in Figure 5.7. Return difference or loop gain, sensitivity transfer function, and complementary sensitivity transfer function are given by

$$L(s) = G(s)K(s)$$

$$S(s) = (I + L(s))^{-1}$$

$$T(s) = I - S(s)$$

It can be shown that the I/O transfer function from command position to actual position, and from aerodynamic disturbances to actual position are given by

$$Y(s) = S(s)*G(s)*ref$$

$$Y(s) = K(s)*S(s)*noise$$

![Figure 5.7](image)

**Figure 5.7** Basic control structure for frequency domain analysis.

The maximum and minimum singular value plots of the plant and of the controller are given in Figure 5.8 and Figure 5.9.
Figure 5.8  Maximum and minimum singular values of the plant transfer function.

Figure 5.9  Maximum and minimum singular values of the controller.
Figure 5.10  Maximum and minimum singular values of the loop gain of the full-state feedback controller and the LQG/LTR compensator.

Figure 5.11  Maximum and minimum singular values of the sensitivity transfer function of the full-state feedback controller and the LQG/LTR compensator.
Figure 5.12 Maximum and minimum singular values of complementary sensitivity transfer function of the full-state feedback controller and the LQG/LTR compensator.

Figure 5.13 Maximum and minimum singular values of the I/O transfer function from the reference position to the actual position.
Frequency domain closed-loop objectives can be summarized as:

1. For disturbance rejection, the maximum singular value of $S(s)$ should be small.
2. For noise attenuation, the maximum singular value of $T(s)$ should be small.
3. For reference tracking, the maximum singular value and the minimum singular value of $T(s)$ should approximately be equal to 1.
4. For control energy reduction, the maximum singular value of $L(s)$ should be small.
5. For robust stability in the presence of an additive perturbation, the maximum singular value of $L(s)$ should be small.
6. For robust stability in the presence of a multiplicative output perturbation, the maximum singular value of $T(s)$ should be small.

These closed-loop requirements cannot be satisfied simultaneously. Feedback design is therefore a trade-off over frequency of conflicting objectives [28]. This is possible because the frequency range over which the objectives are important are quite different. For example, disturbance rejection is typically a low frequency requirement, while noise mitigation is only relevant at higher frequencies.

Figures 5.10, 5.11, and 5.12 show the minimum and maximum singular values of the loop gain, the sensitivity transfer function, and the complementary sensitivity transfer functions of the full state feedback controller and that of the LQG/LTR compensator. It should be noted that the singular values of the LQG/LTR compensator are approaching
the singular values of the full state feedback controller for almost all frequencies of interest. This means that with the LQG/LTR design technique maximum frequency-domain performance is achieved. Figure 5.13 shows maximum and minimum singular values of the I/O transfer function from the reference positions to the actual positions. It can be observed that there is almost no steady state error for the complete system bandwidth.

It should be noted that all of the frequency domain plots are generated in the w-plane. The continuous time plant model is discretized and then converted to the w-plane in order to include the discretization effects into the plots. Figure 5.14 shows the conversion process.

![Diagram showing frequency conversion](image)

**Figure 5.14** Frequency conversion.

In this chapter, a comprehensive theoretical background of LQG/LTR compensator technique is developed. This technique is then applied to the 5 DOF HRTF
magnetic suspension and balance system. The iterative process of controller design is discussed in detail. Frequency domain analysis shows that acceptable frequency domain objectives have been achieved by the iterative design procedure. Simulation results are given in the next chapter.
CHAPTER VI

SIMULATIONS AND RESULTS

This Chapter presents the simulation results of the 1 DOF controllers designed in Chapter III, and the 5 DOF controller designed in Chapter V. The main objective for the 1 DOF controllers was to find the maximum dynamic range of the nonlinear plant model for tracking reference pulses with minimum overshoot and minimum rise time. Step responses of the maximum dynamic range for the 1 DOF system are presented. Simulation plots of the 5 DOF MSBS are then presented followed by a detailed time domain analysis. Step responses from all five outputs, i.e., pitch and yaw angle, vertical, lateral, and axial displacements, are shown separately. All of the simulations described in this chapter are performed using Matlab 5.3 release 11.1 on a PC platform with an Intel Pentium II 500 MHz processor.

6.1 1 DOF Simulations

As discussed in Chapter II and Chapter III, a classical lead-lag controller and a sliding mode controller were designed to test the real time controller. In this section, simulation plots for the 1 DOF system are presented. These plots are generated from Matlab/Simulink simulations. The main objective was to find the maximum dynamic range of the classical and nonlinear controller with the nonlinear plant model. With the classical controller, step responses of nonlinear simulation model are compared with that of the linear design model. The nonlinear model is linearized at 10 mm and the equilibrium current is found to be 0.56 A. The maximum dynamic range for the classical linear controller is found to be 27 mm, 18 mm away from the magnet and 9 mm towards
the magnet. Figure 6.1 compares the step responses of the linear design model and the nonlinear simulation model. Coil current is also shown. It is observed that the behavior of the nonlinear simulation model is not as smooth as that of the linear design model because nonlinear effects come into play as the model moves away from the equilibrium position. Coil currents follow the tracking trends and are in the range of 1.5 A. The closed loop system is critically damped.

![Figure 6.1 Step responses with classical controller.](image)

Step responses using the sliding mode controller are given in Figure 6.2. It is observed that the response is very smooth and there is no overshoot or undershoot. However, at the same time coil current is not smooth, resulting in the chattering effect explained in Chapter III. The maximum dynamic range is found to be 27 mm, the same as that of the classical linear controller, 18 mm away from the magnet and 9 mm towards
the magnet. The tuning parameters, $\beta$, the control gain, $\Phi$, the boundary layer, and $\lambda$, the control bandwidth, are found to be 65, 0.0125, and $5\pi$, respectively.

![Figure 6.2 Step response with sliding mode controller.](image)

6.2 5 DOF Simulations

The 5 DOF MSBS is described in Chapter IV and the LQG/LTR compensator is explained in Chapter V. The LQG/LTR compensator was designed in the continuous time domain and was then discretized. Tuning of the performance matrices, as described in Chapter V was done in the continuous time domain. The simulation plots presented in this chapter are comprised of five step responses in both directions, i.e., positive and negative directions. All of the 5 DOF simulations are based on the digital LQG/LTR compensator with the sampling frequency of 1kHz and the simulation step size is set 10 times less than the sampling time, i.e., 0.0001 seconds. The simulation model for these simulations includes disturbances due to aerodynamic forces and torques, uncertainty in
B-coefficients, eddy current effects, and amplifier dynamics. Each simulation given below is the average of five Monte Carlo runs for random values of the B-coefficient perturbation. The LQ static feedback controller and the Kalman estimator used in these simulations are based on discrete time algorithms using the continuous time design matrices.

![Response of the closed loop system with simulation model for ± 5.8 degrees pitch command.](image)

Figure 6.3    Response of the closed loop system with simulation model for ± 5.8 degrees pitch command.

Figure 6.3 shows the step responses of pitch angle tracking. The coupling of pitch angle and axial displacement, which is a natural phenomenon for wind tunnel applications [32], is clear from the response.
Figure 6.4  Response of the closed loop system with simulation model for ± 5.8 degrees yaw command.

Figure 6.4 shows the step responses of yaw angle tracking. It is clear from the response that there is no overshoot and the response is very smooth.
Figure 6.5  Response of the closed loop system with simulation model for ± 7.5 mm axial displacement.

Figure 6.5 shows the step responses of axial displacement tracking. Note that the coil current is within the range, i.e., 120 amperes. There is absolutely no overshoot or steady state error. The settling time is less than half a second.
Figure 6.6  Response of the closed loop system with simulation model for ± 7.5 mm vertical displacement.

Figures 6.6 shows the step responses of vertical displacement tracking. Note that the coil current is within the range, i.e., 120 amperes. There is absolutely no overshoot or steady state error. The settling time is less than half a second.
Figure 6.7  Response of the closed loop system with simulation model for ± 7.5 mm lateral displacement.

Figure 6.7 shows the step responses of lateral displacement tracking. Note that the coil current is within the range, i.e., 120 amperes. There is absolutely no overshoot or steady state error. The settling time is less than half a second.

The pitch and yaw loop dynamics are approximately the same as the translational ones, i.e., lateral, vertical, and axial. The main difference comes in the angular response of the model to coil current or moment inputs. The model behaves as if held by a torsional spring, and possesses natural frequencies in pitch and yaw corresponding to the magnetic and aerodynamic restoring moments and the moments of inertia of the model about the pitch and yaw axes. This has the effect of separating the model dynamic poles symmetrically from the origin, along the imaginary axis. This effect has also been observed in our linearized model of the plant.
The time domain characteristics, i.e., rise time and settling time, are summarized in Table 6.1.

Table 6.1  Time domain analysis summary for the 5 DOF MSBS

<table>
<thead>
<tr>
<th>Tracking</th>
<th>Rise Time</th>
<th>5% Settling Time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(seconds)</td>
<td>(seconds)</td>
</tr>
<tr>
<td></td>
<td>Positive direction</td>
<td>Negative direction</td>
</tr>
<tr>
<td>Lateral displacement</td>
<td>0.272</td>
<td>0.286</td>
</tr>
<tr>
<td>Vertical displacement</td>
<td>0.275</td>
<td>0.282</td>
</tr>
<tr>
<td>Axial displacement</td>
<td>0.248</td>
<td>0.255</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>0.277</td>
<td>0.273</td>
</tr>
<tr>
<td>Yaw angle</td>
<td>0.289</td>
<td>0.279</td>
</tr>
</tbody>
</table>

It can be observed from the data that the rise time is below half a second for all five cases. Steady state error and maximum error for all five cases are summarized in Table 6.2. Steady state error is calculated by averaging 10 data points in positive direction of response. Maximum error is also calculated from positive direction of each response.
Table 6.2  Steady state and maximum error summary for the 5 DOF MSBS

<table>
<thead>
<tr>
<th>Tracking</th>
<th>Lateral</th>
<th>Vertical</th>
<th>Axial</th>
<th>Pitch angle</th>
<th>Yaw angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\text{Ess}_x)</td>
<td>(\text{Emax}_x)</td>
<td>(\text{Ess}_y)</td>
<td>(\text{Emax}_y)</td>
<td>(\text{Ess}_z)</td>
</tr>
<tr>
<td>Lateral disp. (mm)</td>
<td>0.29</td>
<td>0.60</td>
<td>0.33</td>
<td>0.56</td>
<td>0.32</td>
</tr>
<tr>
<td>Vertical disp. (mm)</td>
<td>0.29</td>
<td>0.60</td>
<td>0.33</td>
<td>0.56</td>
<td>0.33</td>
</tr>
<tr>
<td>Axial disp. (mm)</td>
<td>0.28</td>
<td>0.61</td>
<td>0.49</td>
<td>0.49</td>
<td>0.32</td>
</tr>
<tr>
<td>Pitch Angle (degree)</td>
<td>0.001</td>
<td>0.90</td>
<td>0.59</td>
<td>0.59</td>
<td>2.40</td>
</tr>
<tr>
<td>Yaw Angle (degree)</td>
<td>0.30</td>
<td>0.63</td>
<td>0.73</td>
<td>0.73</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Stiffness is a rough estimate of the quality of stability inside the wind tunnel. Stiffness for each degree of freedom is calculated and summarized in Table 6.3. Maximum aerodynamic force (torque) is divided by the maximum displacement (angle) to calculate the respective stiffness. It is clear from the stiffness table that the stiffness of pitch angle is higher than that of yaw angle while tracking lateral, vertical, and axial displacement. It should be noted that the axial stiffness while tracking pitch angle, and pitch stiffness while tracking yaw angle, are quite low. These phenomena need further investigation.
Table 6.3 Stiffness table for the 5 DOF MSBS

<table>
<thead>
<tr>
<th>Tracking</th>
<th>Lateral Stiffness (N/m)</th>
<th>Vertical Stiffness (N/m)</th>
<th>Axial Stiffness (N/m)</th>
<th>Pitch Stiffness (N-m/m)</th>
<th>Yaw Stiffness (N-m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral displacement</td>
<td>3332</td>
<td>3580</td>
<td>3968</td>
<td>880</td>
<td>121</td>
</tr>
<tr>
<td>Vertical displacement</td>
<td>3332</td>
<td>3580</td>
<td>3978</td>
<td>868</td>
<td>125</td>
</tr>
<tr>
<td>Axial displacement</td>
<td>3282</td>
<td>4058</td>
<td>3978</td>
<td>160</td>
<td>125</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>2231</td>
<td>3369</td>
<td>885</td>
<td>3403</td>
<td>634</td>
</tr>
<tr>
<td>Yaw angle</td>
<td>3181</td>
<td>2729</td>
<td>4257</td>
<td>915</td>
<td>1211</td>
</tr>
</tbody>
</table>

In this chapter, simulation results of the controllers designed for the 1 DOF MSBS and the 5 DOF MSBS are presented. The 5 DOF simulations presented in this chapter includes the most realistic nonlinear plant model and the simulation plots are the average of five Monte Carlo runs. As described in Chapter V, the LQ static feedback controller and the Kalman estimator, used in the simulations, are designed in the continuous time domain and then discretized using discrete time algorithms. The time domain analysis presented in this chapter validates the stability robustness and the performance robustness. Conclusions and final remarks are given in the next chapter.
CHAPTER VII

CONCLUSIONS

Controller design and implementation issues for the HRTF MSBS are investigated in this thesis. A linear quadratic, multiple input multiple output (MIMO), LQ compensator is designed using loop transfer recovery (LTR) techniques to stabilize and regulate the model inside the test section of the HRTF. Since linear control techniques are used to control a nonlinear plant, it was important to determine the region of operation of the controllers. This is done by determining, via simulations, the maximum dynamic range. The maximum dynamic range for displacements and angles from the equilibrium position are tracked and validated via simulations. The simulation plots show an acceptable transient and steady state response, i.e., 5 % settling time less than 2 seconds, rise time less 0.3 seconds, average steady-state error for displacements less than 0.2 mm, and average steady state error for angular positions less than 0.4 degrees. The stability robustness of the LQR/LTR compensator is validated by the time domain analysis of the closed-loop system with random perturbations of the B coefficients, random band limited noise model for aerodynamic forces and torques, and a complete nonlinear model of the MSBS.

An interactive graphical user interface is also developed for operation and maintenance of the HRTF MSBS. Important system components, including hardware interfacing, software workbench, protection interlocking, operational modes, and operational procedures, are defined and implemented. A user’s guide for the system is also composed. The user’s guide is included in Appendix A.
For future work, a few suggestions are given below:

1. It is observed that the model stiffness is higher with the 1 DOF sliding mode control. Though the stability robustness and performance robustness of the LQG/LTR compensator seems to be acceptable for the practical implementation, further research on MIMO sliding mode controller for the 6-DOF HTRF MSBS is recommended for two reasons: sliding mode controllers are inherently robust, and tuning parameters of sliding mode controller is very well defined.

2. Since the model lift/retrieve mechanism is not clear at the moment, the control system software doesn’t have any provision for an automatic lift or retrieve mechanism. It can be included in the control system later; however, I/O’s have been assigned.

3. Sensor calibration routines and procedures should be defined and implemented.

4. Though the interface for model animation in the GUI is not necessary for operation of the MSBS, it would be helpful for the operator to have the model animation.

5. The dSpace’s software component Mlib/Mtrace can be used for the direct access to the hardware through Matlab, this feature can be used for the automatic tuning procedures.

6. To improve the robustness of the HRTF MSBS it might help to investigate a digital LQG/LTR design technique as opposed to the technique developed in this thesis.
REFERENCES


Appendix A

User’s Guide

This user’s guide is composed to describe the integrated environment of the magnetic suspension and balance system designed for the Princeton/ONR High Reynolds Number Testing Facility. Issues of practical concern, i.e., hardware interfacing, controller implementation, system operation, and graphical user interface (GUI) are explained with extensive details.

The magnetic suspension and balance system, discussed in this thesis, is a part of the Princeton/ONR High Reynolds Number Testing Facility (HRTF). The High Reynolds Number Test Facility (HRTF) is a specialized wind tunnel. It will be used for aero/hydrodynamic testing of submersible shapes. The facility will operate at very high pressures, up to 230 atmospheres, and relatively low velocities. The objective of the MSBS for HRTF is to reliably suspend and position in six degrees of freedom a test model with a cylindrical permanent magnet core. In addition to regulating the position in six degrees of freedom, the system should also allow tracking of slow time-varying signals. Since the roll control is not addressed, controllers and graphical user interface are designed with only five degrees of freedom, i.e., pitch and yaw angles, and vertical, axial, and lateral displacements.

Since the magnetic suspension problem is inherently unstable, feedback controller is designed using Matlab and Simulink. Section A.1 deals with the issues of hardware interfacing and I/O description. The implementation details of multivariable controller are given in section A.2. The system operation is described in section A.3, which deals
with startup/shutdown sequences, alarm messages, and software interlocking. A detailed description of GUI is given in section A.5. For the details of GUI development and special features, see dSpace User's Guides.

A 1.1 Controller Description

Optimal control techniques are used. The Linear Quadratic (LQ) compensator is designed using Loop Transfer Recovery (LTR) techniques to stabilize and regulate the model inside the test section of the Princeton/ONR High Reynolds Number Testing Facility. Matlab and Simulink are used to design and optimize the controller. Controller gain matrix and Kalman estimator gain matrices are calculated in offline mode, i.e., without any I/O interfacing. The matrices are then used in Simulink system model for the real time application in online mode, i.e., with I/O interfacing. The Simulink system model is interfaced with the I/Os using dSpace’s Real Time Interface (RTI) libraries. The RTI libraries provide an easy software interface to the dSpace I/Os. The Simulink system model is then compiled using Matlab’s Real Time Workshop (RTW). Five positions, i.e., lateral, vertical, and axial displacements, and pich and yaw angles are input to the controller. Five coil currents are outputs of the controller to regulate the model inside the wind tunnel. Figure A1 shows 5-DOF MSBS model developed in Simulink for real-time application.
Figure A1  RT Simulink model for the real-time controller.

A 1.2 Hardware Interfacing

Hardware interfacing will be done using CP1103 connector panel, which provides easy access to all input and output signals on dSpace controller board DS1103. It is connected to the controller board with three 100-line high-density ribbon cables. Analog signals are accessed via BNC connectors and digital signals are accessed via sub-D connector. Figure A2 shows a simplified diagram of proposed I/O interface of the HRTF magnetic suspension and balance system.

Five analog outputs are commanded coil currents, listed in Table A1. Coil temperatures and sensor measurements are analog inputs to the control system; they are summarized in Table A2.
Figure A2  Simplified I/O interface of the HRTF MSBS.
Table A1  Analog Outputs.

<table>
<thead>
<tr>
<th>Analog Outputs</th>
<th>Symbolic Name</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coil Current 1</td>
<td>CC_1</td>
<td>AO 1</td>
</tr>
<tr>
<td>Coil Current 2</td>
<td>CC_2</td>
<td>AO 2</td>
</tr>
<tr>
<td>Coil Current 3</td>
<td>CC_3</td>
<td>AO 3</td>
</tr>
<tr>
<td>Coil Current 4</td>
<td>CC_4</td>
<td>AO 4</td>
</tr>
<tr>
<td>Coil Current 5</td>
<td>CC_5</td>
<td>AO 5</td>
</tr>
</tbody>
</table>

Table A2  Analog Inputs.

<table>
<thead>
<tr>
<th>Analog Inputs</th>
<th>Symbolic Name</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Position 1</td>
<td>SP_1</td>
<td>AI 1</td>
</tr>
<tr>
<td>Sensor Position 2</td>
<td>SP_2</td>
<td>AI 2</td>
</tr>
<tr>
<td>Sensor Position 3</td>
<td>SP_3</td>
<td>AI 3</td>
</tr>
<tr>
<td>Sensor Position 4</td>
<td>SP_4</td>
<td>AI 4</td>
</tr>
<tr>
<td>Sensor Position 5</td>
<td>SP_5</td>
<td>AI 5</td>
</tr>
<tr>
<td>Coil 1 Temperature</td>
<td>C1_T</td>
<td>AI 6</td>
</tr>
<tr>
<td>Coil 2 Temperature</td>
<td>C2_T</td>
<td>AI 7</td>
</tr>
<tr>
<td>Coil 3 Temperature</td>
<td>C3_T</td>
<td>AI 8</td>
</tr>
<tr>
<td>Coil 4 Temperature</td>
<td>C4_T</td>
<td>AI 9</td>
</tr>
<tr>
<td>Coil 5 Temperature</td>
<td>C5_T</td>
<td>AI 10</td>
</tr>
</tbody>
</table>
Coil temperatures are used only for display and monitoring purpose. As pitch and yaw angles are not measured directly, they are calculated from sensor measurements. “Position conversion” block of Simulink system model is used to extract position signals from the measured analog signals. Eight feedback signals, i.e., 1 from the sensor power supply, 1 from the cooling subsystem, 1 from the launch/retrieve subsystem, and 5 from the main power supplies, are digital inputs to the control system. These feedback signals are used in start up/shut down sequences and operational interlockings. Digital inputs are listed in Table A3 and digital outputs are listed in Table A4. Digital I/O’s are used for the automatic operation of different subsystem in a sequence.

<table>
<thead>
<tr>
<th>Digital Inputs</th>
<th>Symbolic Name</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor power supply feedback</td>
<td>SPS_FB</td>
<td>DI 1</td>
</tr>
<tr>
<td>Cooling system feedback</td>
<td>COS_FB</td>
<td>DI 2</td>
</tr>
<tr>
<td>Launch/Retrieve model</td>
<td>LR_MU</td>
<td>DI 3</td>
</tr>
<tr>
<td>Power supply 1 feedback</td>
<td>PS1_FB</td>
<td>DI 4</td>
</tr>
<tr>
<td>Power supply 2 feedback</td>
<td>PS2_FB</td>
<td>DI 5</td>
</tr>
<tr>
<td>Power supply 3 feedback</td>
<td>PS3_FB</td>
<td>DI 6</td>
</tr>
<tr>
<td>Power supply 4 feedback</td>
<td>PS4_FB</td>
<td>DI 7</td>
</tr>
<tr>
<td>Power supply 5 feedback</td>
<td>PS5_FB</td>
<td>DI 8</td>
</tr>
</tbody>
</table>
Table A4   Digital Outputs.

<table>
<thead>
<tr>
<th>Digital Outputs</th>
<th>Symbolic Name</th>
<th>Address</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor power supply command ON</td>
<td>SPS_ON</td>
<td>DO 1</td>
</tr>
<tr>
<td>Cooling system command ON</td>
<td>COS_ON</td>
<td>DO 2</td>
</tr>
<tr>
<td>Launch/Retrieve Launch</td>
<td>LR_L</td>
<td>DO 3</td>
</tr>
<tr>
<td>Launch/Retrieve Retrieve</td>
<td>LR_R</td>
<td>DO 4</td>
</tr>
<tr>
<td>Power Supply 1 command ON</td>
<td>PS1_ON</td>
<td>DO 5</td>
</tr>
<tr>
<td>Power Supply 2 command ON</td>
<td>PS2_ON</td>
<td>DO 6</td>
</tr>
<tr>
<td>Power Supply 3 command ON</td>
<td>PS3_ON</td>
<td>DO 7</td>
</tr>
<tr>
<td>Power Supply 4 command ON</td>
<td>PS4_ON</td>
<td>DO 8</td>
</tr>
<tr>
<td>Power Supply 5 command ON</td>
<td>PS5_ON</td>
<td>DO 9</td>
</tr>
</tbody>
</table>

A 1.3 System Operations

An integrated GUI environment is developed for the system operation. The GUI provides an easy and user-friendly access for operation and maintenance of the HRTF MSBS. Software interlockings are programmed in the Simulink system model. Alarm messages are configured to monitor coil temperatures. Setpoints for the alarm can be set easily from the user interface. Status displays are also configured to monitor current system state.

Basic interlockings are embedded in the control system software for the upstream (downstream) startup (shutdown) of the equipment in a controlled sequence. Instead of
the commanded signals, feedback signals from respective equipment are used in the interlocking to get a fault-tolerant solution. Normal operation requires all the interlocking conditions be satisfied for controller release. For calibration and maintenance, it is sometimes necessary to bypass one or more interlocks. Each interlock can be bypassed from the user interface by respective bypass push button. Table A5 lists 8 bypass variables, 1 from the sensor power supply, 1 from the cooling subsystem, and five from the main power supplies.

<table>
<thead>
<tr>
<th>Internal Flags</th>
<th>Symbolic Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor power supply bypass</td>
<td>SPS_BP</td>
</tr>
<tr>
<td>Cooling System bypass</td>
<td>COS_BP</td>
</tr>
<tr>
<td>Power supply 1 bypass</td>
<td>PS1_BP</td>
</tr>
<tr>
<td>Power supply 2 bypass</td>
<td>PS2_BP</td>
</tr>
<tr>
<td>Power supply 3 bypass</td>
<td>PS3_BP</td>
</tr>
<tr>
<td>Power supply 4 bypass</td>
<td>PS4_BP</td>
</tr>
<tr>
<td>Power supply 5 bypass</td>
<td>PS5_BP</td>
</tr>
</tbody>
</table>

A digital subsystem is modeled in Simulink for startup/shutdown sequence. The software is structured in a highly modular form in order to simplify the task of incorporating any amendments necessary to accommodate different test model, experiment set-ups or run-time features. Startup sequence is as follows:
1. With the startup command from the user interface, sensor power supply will turn ON first, as shown in figure A3.

2. Sensor power supply feedback input will go “high” when the sensors are energized. Control system will then generate an ON command to the cooling subsystem, as shown in figure A4. The cooling subsystem will generate a feedback signal when it is completely “up.”

3. Five ON commands will then be generated by the control system to energize the main power supplies, as shown in figure A5.

Shut down sequence is the reverse of startup sequence. After the startup sequence, when the system has completely started, controller and bias currents can be turned ON from the user interface by pressing “Bias Currents and Controllers ON” push button present in MSBS-CS screen. Feedback signals from the sensor power supply, the cooling subsystem, and the main power supplies are logically ANDed to interlock the controller and the bias currents.
Figure A3  The sensor power supply interlockings.

Figure A4  The cooling subsystem interlockings.
Figure A5  The main power supplies interlockings.

Manual mode of operations is also provided. It gives the operator an opportunity to force digital outputs used to energize the sensor power supply, the cooling subsystem, and the main power supplies, directly from the user interface. This mode can be used for maintenance or testing purpose. Table A6 lists system variables used for manual operation.

Table A6  System variables for manual mode.

<table>
<thead>
<tr>
<th>Internal Flags</th>
<th>Symbolic Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor power supply manual ON</td>
<td>SPS_MO</td>
</tr>
<tr>
<td>Cooling system manual ON</td>
<td>COS_MO</td>
</tr>
<tr>
<td>Power supply 1 manual ON</td>
<td>PS1_MO</td>
</tr>
<tr>
<td>Power supply 2 manual ON</td>
<td>PS2_MO</td>
</tr>
<tr>
<td>Power supply 3 manual ON</td>
<td>PS3_MO</td>
</tr>
<tr>
<td>Power supply 4 manual ON</td>
<td>PS4_MO</td>
</tr>
<tr>
<td>Power supply 5 manual ON</td>
<td>PS5_MO</td>
</tr>
</tbody>
</table>
The MSBS startup sequence of operations is:

1. Turn the computer ON.
2. Launch dSpace ControlDesk from Start -> Programs -> dSpace Tools -> ControlDesk.
3. Load HRTF experiment by File -> Open Experiment, the experiment file HRTF.cdx is stored in c:\dSpace\Work\HRTF_MSBS.
4. Maximize any of the screen file in order to switch the user interface in Work Book view.
5. Initialize the processor by stopping and starting the processor. This can be done by Hardware -> Application -> Stop Real-Time Processor/Start Real-Time Processor.
6. System is now ready to start. Switch to Animation Mode by either Instrumentation -> Animation Mode, or by pressing F5 function key.
7. Now check the bias current settings in the Bias Current screen.
8. System can now be started by pressing “System Startup” push button in the MSBS-CS screen. This will turn on the sensor power supply, the cooling system, and the main power supplies in a sequence.
9. Model should now be lifted by pressing “Lift” push button in the same screen.
10. When the model is at equilibrium position (center of the test section), the bias currents and the controller can be turned on by pressing “Bias Current and Controller ON” push button.
11. When the model is stabilized at equilibrium position, the frame should be retrieved by pressing “Retrieve” push button.
System is now in normal operation. As the commanded positions are initialized to zero, command sequence can be performed by entering respective commands in the "numeric input" blocks.

The shutdown sequence of operations is:

1. Turn the commanded positions to zero so that the model comes back at equilibrium position.
2. Now lift the frame by pressing "Lift" push button.
3. Turn off the bias currents and the controller by pressing "Bias Currents and Controller OFF" push button.
4. The model can now be retrieved by pressing "Retrieve" pushbutton.
5. System should be shut down by pressing "System Shut down" push button. This will turn off the power supplies, the cooling system, and the sensor power supply in a sequence.
6. Now switch the user interface to edit mode either by Instrumentation -> Edit mode, or by pressing shift+F5 key combination.
7. Microprocessor can now be turned off by Hardware -> Application -> Stop Real-Time Processor.
8. Exit dSpace ControlDesk by File -> Exit, without saving the experiment unless any changes has been made.
9. Shut down the computer.
10. Turn off the power supplies of the CPU and the monitor.

Alarms and messages are configured for coil temperatures. The operator can set the temperature limits by entering the temperature values in the "numeric input" blocks.
present in the “Sensors and Power Supplies” screen. An alarm message will appear in the message block present in the “MSBS-CS” screen as soon as the actual temperature is greater than the set point. Message files, *.csv, are configured for each message. Message text can be changed in the corresponding *.csv file.

Four different status of operations, i.e., system started, controller ON, controller OFF, and shutdown, are configured in the control system software for display purposes. Status display appears in the status display block present in the “MSBS-CS” screen. Status display text can be changed in the corresponding *.csv file.

A 1.4 Description of GUI Screens

The graphical user interface (GUI) is developed in dSpace’s software tool ControlDesk. ControlDesk is an integrated tool for the control, monitoring and automation of real-time experiments. Its graphical user interface (GUI) is capable of drag and drop, context menus, floating windows, and online help. ControlDesk includes experiment management and hardware management.

ControlDesk instrumentation is comprised of several virtual and data acquisition instruments. Instrument properties can be set easily. Base Instrumentation kit consists of LED, multistate alert, slider bar, push button, radio button, frame knob, gauge, numeric display, tabular display, text field, plotter, logic analyzer, 2-D curve editor, and 3-D table editor. Any variable from the “variable list” can be assigned to respective instrument by drag and drop. All of the Matlab/Simulink variables are accessible in dSpace environment with READ or READ/WRITE permissions. Parameter sets comprising of any of these variables can be defined for controller tuning, and can be loaded onto the
running real-time application. The parameter sets can also be saved in a file to keep record of system behavior.

The GUI for the HRTF MSBS is comprised of following screens/layouts:

- Title
- Magnetic Suspension and Balance System - Control System
- Sensors, Power Supplies, and Cooling Subsystem
- Bias Currents
- Lift/Sink Model
- Curves and Trends
- Interlocking Diagrams

The “Title” is shown in Figure A6. There is no dynamic element in this screen. This screen is supposed to be the first screen of the user interface.

The “Magnetic Suspension and Balance System - Control System” screen is shown in Figure A7. It is supposed to be the main window for normal operation of the HRTF MSBS. Push buttons are provided for system startup, bias current and controller, system shut down, model lift, and model retrieve. Alarm messages and status displays are present at the top right corner of the screen. “Numeric Displays” are configured for the display of actual positions and commanded positions. The presence of trends of controller current, reference positions, and actual positions in the same screen facilitates the operation.

Manual operation and feedback bypasses can be selected/deselected in “Sensors, Power Supplies, and Cooling Subsystem” screen, as shown in Figure A8. The power supplies, the sensor power supply, and the cooling subsystem can be operated in manual by pressing respective ON pushbutton. Moreover, their feedback signals can be bypassed
by pressing respective ON pushbuttons. The coil temperature alarm setpoint can also be set using “Numeric Input” blocks. LEDs present on the right side of the screen are dynamic displays of feedback signals (DI) and command ON signals (DO).

Equilibrium currents and positions of the model can be set in “Bias Current and Positions” screen, as shown in Figure A9. “Numeric Displays” and “Numeric Inputs” are configured for the setting and monitoring of equilibrium currents and positions. Default values of the equilibrium currents are calculated equilibrium currents. The default value of the equilibrium positions are zero, i.e. origin. Manual operation can be selected by respective pushbuttons.

Lifting or sinking of the model frame can be done using the “Start” pushbutton of “Lift” or “Sink”. The “Start” pushbutton will turn respective digital output “high” and the “Stop” pushbutton will turn it back to “low” state. The LEDs, present in the screen, display dynamic status of digital outputs and digital inputs. Figure A10 shows “Lift/Sink Model” screen.

Real-time trends for model position, coil currents, forces and torques can be monitored in “Curves and Trends” screen, as shown in Figure A11.

Basic interlocking diagrams of the digital subsystem are included in the “Interlocking Diagrams” screen, as shown in Figure A12. All of the elements of this screen are static. This screen is just for information purpose.
Figure A6  "Title" screen.

Figure A7  "Main Operations" screen.
Figure A8  "Sensors, Power Supplies and Cooling Subsystem" screen.

Figure A9  "Bias Currents and Positions" screen.
Figure A10  "Lift/Sink Model" screen.

Figure A11  "Curves" screen.
All of the screens are grouped in an experiments, more screens can be added if necessary. For detailed information about development of screens, refer to dSpace manuals.
APPENDIX B

SENSOR INTERCONNECTION DIAGRAM OF 1-DOF SYSTEM

Figure B.1   Sensor Interconnection Diagram of 1-DOF MSBS
APPENDIX C

CONTROL SYSTEM SOFTWARE

1. For controller design and tuning:

```matlab
% Load model parameters
load new_model
% Bcoefficients uncertainties
Bpert=diag(.02/.5*(rand(1,5)-.5)+1);
alpha=Bcoeff*Bpert;
% Eddy current effects
num=[2*pi*27];
den=[1 2*pi*27];
% Amplifier Dynamics
num1=[1 2*pi*209.8];
den1=poly([-499.8*2*pi (-93+210*i)*2*pi (-93-210*i)*2*pi])];
% Low pass filter for disturbances due to drag
% forces and torques
num5=[2*pi*200];
den5=[1 2*pi*200];
amp_tf_gain= 4.9609e+006;
% Commanded signals
%max_ref=7.5/1000;
%min_ref=-7.5/1000;
max_ref=.1;
min_ref=-.1;
Ts=.001; % Sampling Time
ts=.8; % Rise Time
xo=[0 0 0 0 0 0 0 0 0 0]'; % Initial States
heading='Q and r are identity';
% LQ controller tuning matrices
q1=i*[1 0 0 0 0 0 0 0 0 0;
0 9990 0 0 0 0 0 0 0 0;
0 0 1 0 0 0 0 0 0 0;
0 0 0 999 0 0 0 0 0 0;
0 0 0 0 90000 0 0 0 0 0;
0 0 0 0 0 1 0 0 0 0;
0 0 0 0 0 0 99 0 0 0;
0 0 0 0 0 0 0 1 0 0;
0 0 0 0 0 0 0 0 999 0;
0 0 0 0 0 0 0 0 0 99];
q2=100*eye(5);
% LQ controller gain
[y,x,t,Klqr]=lqsim(arl,brl,Ts,xo,q1,q2,ts,heading);
% Conversion to discrete domain
sysG=ss(arl,brl,crl,drl);
sysdG=c2d(sysG,Ts);
[darl,dbrl,dcrl,drdl]=ssdata(sysdG);
% Kalman estimator tuning matrices
G1=10000*[1 0 0 0 0 0 0 0 0 0;
0 0 0 0 1 0 0 0 0 0;
0 0 0 0 0 1 0 0 0 0;
0 0 0 0 0 0 1 0 0 0;
0 0 0 0 0 0 0 1 0 0;
0 0 0 0 0 0 0 0 1 0;
0 0 0 0 0 0 0 0 0 1];
```
0 1 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 1 0 0
Q1=dbr1*dbr1';

R1=.00001*[1 0 0 0 0;

Kalman estimator gain
L = dlqe(darl,G1,dcrl,Q1,R1);
sysT_lqr=ss(arl-brl*Klqr,brl,crl-dr1*Klqr,dr1);
sysTf_lqr=minreal(sysT_lqr);
Kalman estimator state space matrices
A_est=darl-L*dcrl;
B_est=[(dbrl+L*ddrl) L];
C_est=eye(10);
D_est=zeros(10);
sysT_est=ss(A_est,B_est,C_est,D_est);
% Simulate model
sim('r_new_lqr_estd_noisel');

2. For frequency domain plots:

load new_model
% Conversion to W-plane using bilin
sysG=ss(arl,brl,crl,dr1);
sysdG=c2d(sysG,Ts);
[dar1,dbr1,dcrl,ddrl]=ssdata(sysdG);
[at,bt,ct,dt]=bilin(dar1,dbr1,dcrl,ddrl,-1,'Tustin',.001);
Ts=.001;
ts=.8;
xo=[0 0 0 0 0 0 0 0 0 0];
% LQ controller and Kalman estimator's tuning matrices
heading='Q and r are identity';
q1=1*[1 0 0 0 0 0 0 0 0 0;

q2=100*eye(5);
G1=10000*[1 0 0 0 0 0 0 0 0 0;
Q1 = bt*bt';
R1 = .00001*[1 0 0 0 0; 0 1 0 0 0; 0 0 1 0 0; 0 0 0 1 0; 0 0 0 0 1];

[y,x,t,Klqr]=lqsim(at,bt,Ts,xo,q1,q2,ts,heading);

w = logspace(-2,3,400);

% singular value plots
% state feedback open loop transfer function
sv_Lsf_d = sigma(at,bt,Klqr,dt,w);

% sensitivity function
[ASsf_d,BSsf_d,CSsf_d,DSsf_d] =
feedback([],[],[],eye(5),at,bt,Klqr,dt);
sv_Ssf_d = sigma(ASsf_d,BSsf_d,CSsf_d,DSsf_d,w);

% complementary sensitivity function
[ATsf_d,BTsf_d,CTsf_d,DTsf_d] =
feedback(at,bt,Klqr,dt,[],[],[],eye(5));
sv_Tsf_d = sigma(ATsf_d,BTsf_d,CTsf_d,DTsf_d,w);

% observer gain
L_d = lqe(at,G1,ct,Q1,R1);

% state space description of the observer based controller
AC_d = at-bt*Klqr-L_d*ct;
BC_d = L_d;
CC_d = Klqr;

% Loop gain
[ACP_d,BCP_d,CCP_d,DCP_d] =
series(at,bt,ct,dt,AC_d,BC_d,CC_d,zeros(5));
sv_Cp_d = sigma(ACP_d,BCP_d,CCP_d,DCP_d,w);

% Sensitivity transfer function
[ASobs_d,BSobs_d,CSobs_d,DSobs_d] =
feedback([],[],[],eye(5),ACP_d,BCP_d,CCP_d,DCP_d);
sv_Sobs_d = sigma(ASobs_d,BSobs_d,CSobs_d,DSobs_d,w);

% Complimentary sensitivity transfer function
[ATobs_d,BTobs_d,CTobs_d,DTobs_d] =
feedback(ACP_d,BCP_d,CCP_d,DCP_d,[],[],[],eye(5));
sv_Tobs_d = sigma(ATobs_d,BTobs_d,CTobs_d,DTobs_d,w);

% I/O transfer function from reference position to actual position
[Aref,Bref,Cref,Dref] =
series(ASobs_d,BSobs_d,CSobs_d,DSobs_d,at,bt,ct,dt);
reftf = ss(Aref,Bref,Cref,Dref);
reftf_f=K_lqr*reftf;
sv_ref=sigma(reftf_f,w);

% Singular values of the plant
sv_plant = sigma(at,bt,ct,dt,w);

% Singular values of the controller
sv_controller = sigma(AC_d,BC_d,CC_d,zeros(5),w);

figure(1);
loglog(w,sv_Lsf_d(1,:),'-',w,sv_Lsf_d(5,:),'-',w,sv_CP_d(1,:),'-',w,sv_CP_d(5,:),'--')
xlabel('frequency, rad/sec')
ylabel('magnitude')
title(['open loop singular values, Kinv(sI-A)B and C(s)P(s), with sampling time = ', num2str(Ts)])
legend('-','smax_L_S_F','--','smin_L_S_F','-','smax_L_o_b_s','--','smin_L_o_b_s')

figure(2);
loglog(w,sv_Ssf_d(1,:),'-',w,sv_Ssf_d(5,:),'-',w,sv_Sobs_d(1,:),'-',w,sv_Sobs_d(5,:),'--')
legend('-','smax_S_S_F','--','smin_S_S_F','-','smax_S_o_b_s','--','smin_S_o_b_s')
title(['singular values of sensitivity function with sampling time = ', num2str(Ts)])
xlabel('frequency, rad/sec')
ylabel('magnitude')

figure(3);
loglog(w,sv_Tsf_d(1,:),'-',w,sv_Tsf_d(5,:),'-',w,sv_Tobs_d(1,:),'-',w,sv_Tobs_d(5,:),'--')
legend('-','smax_T_S_F','--','smin_T_S_F','-','smax_T_o_b_s','--','smin_T_o_b_s')
title(['singular values of complementary sensitivity function with sampling time = ', num2str(Ts)])
xlabel('frequency, rad/sec')
ylabel('magnitude')

figure(4);
loglog(w,sv_ref(1,:),'-',w,sv_ref(5,:),'-')
legend('-','SV_m_a_x','-',SV_m_i_n')
title(['singular values of I/O transfer function from reference position to actual position'])
xlabel('frequency, rad/sec')
ylabel('magnitude')

figure(5);
loglog(w,sv_plant(1,:),'-',w,sv_plant(5,:),'-')
legend('-','SV_m_a_x','-',SV_m_i_n')
title(['singular values of the plant'])
xlabel('frequency, rad/sec')
ylabel('magnitude')

figure(6);
loglog(w,sv_controller(1,:),'-',w,sv_controller(5,:),'-')
legend('-','SV_m_a_x','-',SV_m_i_n')
title(['singular values of the compensator'])
xlabel('frequency, rad/sec')
ylabel('magnitude')
CURRICULUM VITA
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