Conceptual Design of Dual Cam Mechanisms Using Graph Theory

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by

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ABSTRACT

CONCEPTUAL DESIGN OF CAM MECHANISMS USING GRAPH THEORY

This paper presents the theory of graphs and how they can be applied to the conceptual design of dual-cam mechanisms. The graph theory was used to validate the results of previous authors (Hain and Pryor). Several applications of dual cam mechanisms are reviewed.

Algorithms using a process of bit mapping were used to developed all possible non-isomorphic graphs for various classes of mechanisms. A classification method was developed using common occurring substructures found in the graphs. This classification scheme was used for conversion of graphs into schematics. Once the results of Hain were verified, the investigation turned to mechanisms with two cams. The generated schematics represent the basic configurations for all the non-isomorphic mechanisms studied.

The graph theory was able to generate seven colored graphs for five-bar, 1 D.O.F. mechanisms with one cam. For six-bar, 1 D.O.F. dual cam mechanisms, over 110 colored

graphs were generated. Finally, 12 colored graphs were generated for five-bar, 2 D.O.F. dual cam mechanisms. Two designs of mechanisms using dual cams were examined. The graphs of the mechanisms were checked against the tables of graphs developed during this investigation.

The method presented in this paper has shown to be both exhaustive and complete. The graphs developed are non-isomorphic (unique). The method is not limited to single degree of freedom mechanisms nor is it limited to the number of desired links. This was a problem encountered in previous investigations. The application of graph theory to the generation of mechanisms did not exhibit these limitations.

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Chapter 1

1. Introduction

The purpose of this investigation is to analyze and synthesize mechanisms with one or more cam-contact pairs. A method for generating and enumerating mechanisms is developed and applied to particular classes of cam mechanisms. The major emphasis in this work is the development of an algorithm for the enumerating and modeling mechanisms with one or more cam-pairs using graph theory. This algorithm is then used during the generation of dual-cam mechanisms.

Much work has already been conducted in the study of structural classification of mechanisms. Hain [1] investigated mechanisms with one cam-contact pair. Hain used a well-known method of replacing a single link in a multi-link mechanism with a cam-pair. He applied this method and developed 21 different cam-pair mechanisms. He also noted the limited applications of these mechanisms and issued a challenge for engineers and designers to design better cams.

Pryor et al. [2] expanded on Hain's work and proposed an alternative method for classifying and enumerating mechanisms with cams which they termed, cam-modulated mechanisms. His method was similar to that used by Hain in that it involved the conversion of more complicated mechanisms into cam-modulated mechanisms by replacing some of the links in the mechanisms. Pryor's method may have been an improvement on Hain's method but it too, had several limitations. One such limitation lies in the difficulty of determining non-isomorphism. It is possible that when one proceeds to remove links and their associated joints and replace them with cam-contact pairs, several mechanisms may be generated that are structurally identical. This method therefore, does not lead to unique mechanisms or with a ready means to determine if a converted mechanism is non-isomorphic. Chapter 4 provides further detail on the method used by Hain and Pryor and the problem with isomorphism. This author is proposing a different approach to enumerating conceptual designs of cam-modulated mechanisms, by using graph theory and algorithms developed for this study. The proposed method does not rely on the existence of structurally complicated mechanisms.

This thesis is divided into seven chapters. Chapter 1 gives an introduction and brief summary of the remaining chapters. Chapters 2 and 3 provide the background material on the algorithms used to develop the graphs presented in this thesis. Chapters 4, 5 and 6 present the classification and enumeration process of Five-bar and Six-bar cam-modulated mechanisms. Chapter 7 discusses a few applications of dual-cam-modulated mechanisms.

Chapter 2 provides a discussion on the theory of graphs as well as the key concepts in the combinatorial enumeration of all possible concepts. The concept of bit mapping is developed into a method of performing the permutations required to generate all possible unique mechanisms. The degree-of-freedom equation and matrix representation are also discussed in this chapter.

Chapter 3 gives an overview of the key algorithms written for this thesis. This chapter documents the key elements used by the permutation algorithms. Each of the major algorithms that make up the source code are documented in this chapter, to provide a means to reference the graphs that were generated in Chapters 4, 5 and 6.

Chapter 4 describes five-bar mechanisms with a single cam and a single degree-of-freedom. The analysis of Hain's work as it relates to graph theory is then compared to the results found through using the permutation algorithms. This chapter also describes the method of conversion of mechanisms into cam-modulated mechanisms as used by Hain.

Chapter 5 details the enumeration of six-bar mechanisms with dual-cams and one degree-of-freedom. This chapter describes the method of classifying graphs based on various types of subgraphs and how the subgraphs are converted into schematics of mechanisms.

Chapter 6 describes the structural enumeration of five- link dual-cam mechanisms with two degrees-of-freedom. The method of structural classification based on sub-graphs is discussed along with several tables that tabulate the classifications of all the colored-graphs generated by structural synthesis. The fundamental graphs along with their respective non-isomorphic colored graphs are also illustrated in this chapter.

Chapter 7 contains a survey of several designs for variable valve-timing mechanisms which use dual-cam mechanisms. A detailed analysis of a six-bar dual-cam mechanism used for automobile sunroofs is also found in this chapter.

1.2 Discussion of Earlier Work

Hain [1] systematically derived 21 different five-link cam mechanisms from the six-link Watt and Stephenson linkage mechanisms. This was accomplished by taking each of the fundamental six-link mechanisms mentioned and replacing a single link and two connecting joints with a single cam-contact pair. This method reduces the number of links by one but maintains the degrees-of-freedom of mechanism. Each of these mechanisms contained only one cam-contact pair with one degree-of-freedom. Hain [1] stated that the mechanisms shown, represented all single degree-of-freedom single-cam mechanisms that converted the rotating cam input into an oscillating output. The purpose of this investigation is to apply a different method of enumeration than that used by Hain and Pryor and determine if indeed the 21 cam-contact pairs are the only possible mechanisms. This investigation will also apply kinematic synthesis to this class of mechanisms and higher link cam-modulated mechanisms.

1.3 Taking the Challenge

The first step in taking up the challenge issued by Hain [1] was to analyze what Hain and Pryor had already accomplished. One goal was to verify Hain's work [1] and determine if indeed the 21 mechanisms that are shown in his paper are the only possible five-bar single degree-of-freedom single-cam mechanisms.

The next step was to determine if an alternative method of enumeration existed that could be applied to this problem. Based on past investigations [9,10,11,12], Graph theory has shown to be a useful tool for the generation of new kinematic structures. Algorithms based on graph theory were then developed for this research. The purpose of these algorithms was to enumerate all possible non-isomorphic graphs for five- and six-link mechanisms with two cam-contact pairs. From this list of mechanisms, certain criteria can then be applied to each mechanism, in order to reduce the total number of mechanisms. The final list of mechanisms can then be compared to the list of mechanisms generated from previous research.

The final step was to continue the search for more complicated mechanisms using the graph theory algorithms. In particular, both five-bar and six-bar mechanisms with two cam-contact mechanisms were investigated.

CHAPTER 2

2. Background Theory

In this chapter, a background is presented on combinatorial mathematics and the theory of graphs. Matrix representation of graphs is discussed along with the degree-of-freedom equation. Finally, important algorithms such as bit mapping and matrix permutations are discussed.

2.1 Combinatorial Mathematics

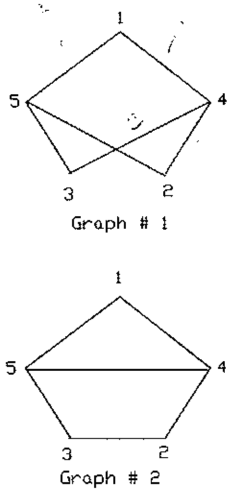
Combinatorial mathematics is a branch of mathematics with many applications in engineering and has become more useful with the wide use of computers. Various applications and manipulations of sets can be carried out that several years ago were either too lengthy to do by hand or required only the most expensive computers. With the availability of computers and the cost of computing time constantly dropping, engineers are finding numerous computer applications for combinatorial mathematics. One particular branch of combinatorial mathematics used by engineers is graph theory. Graph theory will be used to describe the relationship among sets or elements in the mechanical system so that it can be used to enumerate mechanisms.

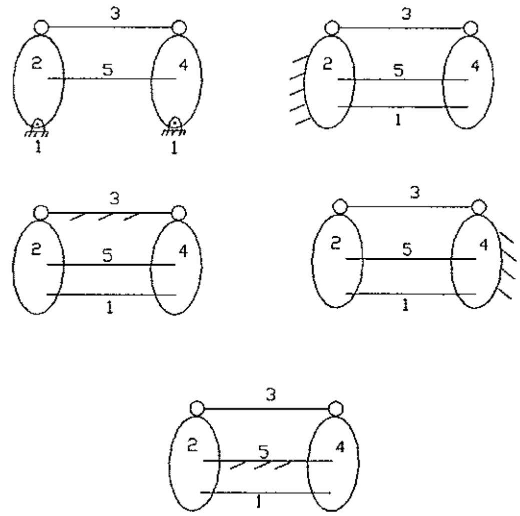
2.2 Graph Theory

A graph is defined in reference [5] to be the ordered pair (A,B), where A is a set and B is a binary relation on A. The elements of A are called vertices and the ordered pairs in B are called edges of the graph. An edge is said to be incident with the vertices it joins. Figure 2.1(a) shows a simple four-bar mechanism and the graph that is used to describe it. The links (vertices of the graph) are labeled A, B, C and D. In this example, edge (A,B) of the graph is incident to vertices A and B. Two vertices are said to be adjacent if they are joined by an edge. There are four edges shown in the figure. Edge #1 joins or connects vertices A and B, Edge #2 joins vertices B and C, Edge #3 joins vertices C to D, and Edge #4 joins vertices D to A.

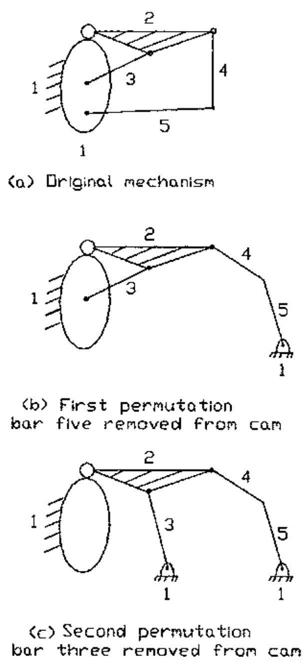
2.3 Isomorphism

Two graphs are said to be isomorphic if there is a one-to-one correspondence between their vertices and edges such that incidence is preserved. This means that for any given edge between two vertices in one graph, there is a corresponding edge between the corresponding vertices in the second graph. Non-isomorphic Graphs are unique in that they describe a relationship that is not found in another graph, no matter how the graph is redrawn or rearranged.





(a) Hain's Fundamental Graphs (b) Kinematic Inversion



(c) Hain's Permutation Method

Figure 4.1 Hain's Graphs, Methods of Permutation

Two structurally equivalent mechanisms are said to be isomorphic if they meet the following criteria:

1 - number of vertices are the same

2 - number of edges are equal

3 - number of vertices with the same degree

4 - topologically the same for each resulting graph

Figure 2.1(b) illustrates two isomorphic graphs. Even though the vertices are labeled the same, the graphs appear somewhat differently, by close comparison between the edge vertex relationships of the first graph, with the second graph shows that both graphs are indeed isomorphic. Obviously, there must exist analytical means by which one can determine isomorphism.

Uicker and Raicu [7] proposed a method for solving the problem of isomorphism. This method is discussed in greater detail in section 2.4.3.

2.4 Matrix Representation of Graphs

The binary relationships between the two sets allows one to describe the relationship in matrix form. The adjacency matrix "A" of a graph with "N" vertices is a NxN matrix. In this matrix, the (i,j)th entry is <u>one</u> if there is an edge joining the ith and jth vertices, and is <u>zero</u> otherwise.

2.4.1 Relationship Between the Mechanism and Its Graph

Since graph theory is used to describe the relationship of sets in a graphical manner, graphs can then be used to describe the relationships between various members of a linkage. Each link in the mechanism may be represented by an edge in the graph and each joint in the mechanism is represented by a vertex in the graph.

A matrix is said to be symmetric if every element A[i,j] is equal to element A[j,i]. In the case of mechanisms, if link "A" connects to link "B", then it can also be said that "B" is connected to "A". The adjacency matrix used to describe a mechanism is therefore symmetric. This property of the adjacency matrix of a graph is useful because in the process of permuting the matrix, only the upper half of the matrix need be considered.

2.4.2 Graph to Matrix Conversion

The process of converting a graph to a matrix form is quite simple. Each vertex in the graph represents a link in the mechanism. These links (vertices) are then represented in the matrix as a row and column of the matrix. For instance, link #3 is denoted by row 3 and column 3 of the matrix. A joint between links #3 and #4 is shown by the elements [3,4] or [4,3]. The equivalent adjacency matrix for the Four-bar mechanism shown in Fig. 2.1(a) is shown elements [3,4] or [4,3]. The equivalent adjacency matrix for the Four-bar mechanism shown in Fig. 2.1(a) is shown below:

$$\begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

The edge A-B of the graph is represented in the matrix as row 1 and column 1. The vertex A is equivalent to the element (1,1) in the matrix and is assigned the value of zero. This is due to the fact that the vertex is not allowed to connect to itself. Vertex A is joined to vertex B and D. Vertex B is row 2 column 2 of the matrix. We set elements (1,2) and (2,1) in the matrix equal to one to show that vertex "A" connects to vertex "B". Likewise, the elements (1,4) and (4,1) are also equal to one.

2.4.3 Matrices and Isomorphism

One advantage to representing graphs of mechanisms with matrices, is the readily available algorithms used for the analysis of matrices and the applications of computers in manipulating matrices. Computers can be used to perform permutations and generate various graphs. A method of determining isomorphism is necessary if one is interested in generating unique graphs.

Uicker and Raicu [7] proposed a method for solving the problem of isomorphism. They showed that the problem of isomorphism reduced to that of assuming a eigenvalue solution to the permutation matrix and solving for the determinant. This determinant was unique for each non-isomorphic graph.

Another variation of this method for checking for isomorphism is to map the adjacency matrix (represented by a matrix of integer numbers) of a graph, into a matrix of real numbers. If one assumes an arbitrary eigenvalue and determines the determinant of the resulting matrix, non-isomorphic graphs in general will generate unique determinants. While this method lends itself well to a quick and efficient computer solution, the method does have some limitations. There does exist the possibility that this method may eliminate from consideration a mechanism which is actually non-isomorphic. This situation may arise from some non-isomorphic graph that yields the same determinant as that of another non-isomorphic graph.

2.5 Degrees-of-Freedom

In covering the background information on degrees-of-freedom, this section describes the theory and how it applies to mechanisms.

2.5.1 Degrees of Freedom of a Rigid Body

The degree of freedom of a moving object in free space is the number of coordinates used to fully describe the position and orientation of that object at any instant in time. If one uses the cartesian coordinate systems a basis for a three dimensional space, the location of an object requires three coordinates to describe its location and three more coordinates to describe the orientation/rotation of the object. The total number of degrees-of-freedom for this spatial object in three dimensional spaces is six. If we constrain that object to move in a planar motion only, the number of degrees-of-freedom for the mechanism is then reduced to three.

2.5.2 Mechanisms and Degrees-of-Freedom

Often mechanisms are classified by the number of inputs required to generate a particular output. By this, we mean the degrees of freedom for the entire mechanism.

If for example, a link is constrained to move in a plane, the motion consists only of translation and rotation, resulting in three degrees of freedom. Two degrees of freedom arise from the two coordinates required to describe the translation of the link. The third degree of freedom comes from the rotation of the link about the axis normal to the plane of motion. Therefore, the constrained rigid link is said to have three degrees of freedom. An assembly of N links posses a total of N degrees of freedom when they are joined together to form a linkage system. As links are joined, there is a resultant loss of degrees of freedom in the total system.

A pin joint between two links removes two degrees of freedom of relative motion. From this observation, one can write an equation which governs the degrees-of-freedom of an N-link chain connected by N number of pin joints. Ground is considered as one of the links being fixed. The equation then becomes

$$F = 3(N - 1) - 2F1 \qquad [2.1]$$

Where F is the total degrees-of-freedom of the mechanism, (N-1) is the number of mobile links and F1 is the number of single degree-of-freedom joints in the system. Equation [2.1] is known as Greubler's equation.

Several joints provide one degree of freedom. The slider joint for example, provides one degree of freedom in translation only. The revolute or pin joint also allows only one relative degree-of-freedom but in rotation. In this text, all single degree-of-freedom joints are shown as pin (or revolute) joints. It is assumed that any pin joint may be replaced with any joint with one degree of freedom.

2.6 Algorithms of Bit Mapping

This section describes binary numbers and the algorithms which use them.

2.6.1 The Binary Number

Binary numbers are often used when describing how computers store numbers internally. Our counting system has a base of ten. The right-most digit of any number has the weight of one. The next digit to the left has a weight of ten. For example, the number 21 consists of two tens and one. For binary numbers, the base is two. The right hand position weight is one, just as in the decimal system. The value of any number in the right hand position is always one for any system because any number raised to the power of zero is one. The next position to the left has the weight of two (two raised to the power of one). The next position moving in the left direction has the weight of four (two raised to the power of two). Each position to the left increases in the power of two. Each binary digit is called a bit, which is a contraction for "BInary digiT".

Any number in the decimal system can be converted into a binary representation. The number 50 for instance is equivalent to 110010 in the binary system. To understand this, let us take a look at the six binary digits used for the number 50. The second digit from the right is in the ones position. This means the number two is raised to the power of one. This represents the number two. The fifth digit is one raised to the power of four or 16. Finally, the sixth digit is two raised to the power of five or 32. Add each of the values and we get 32 + 16 + 2 = 50.

By making use of the binary system and examining each bit representation, it is possible to use an algorithm of bit mapping in enumeration. Starting with the integer 0 and incrementing by 1, one can examine each bit and enumerate every possible combination of zero and one. For example, if one is looking for every possible combination of 1 & 0 using two digits the combinations are 00, 01, 10, and 11. These combinations could have been found by incrementing an integer from 0 to 3 and examining each bit. An algorithm based on bit mapping was used to develop graphs for kinematic synthesis. The bits are mapped into a matrix which represents a graph. Each graph is stored in the computer if the graph is new and is non-isomorphic.

2.6.2 Bit Mapping and Matrices

There are several methods used for the enumeration of graphs. The process used throughout this study is one of bit mapping and decoding of bits for the purpose of permuting matrices that can be used to represent graphs. The process involves the incremental step increase in an integer number and the decoding of the bits used internally to represent that number. By considering only the upper portion of the matrix of a graph, the task of enumeration becomes one of generating all possible combinations of ones and zeros. Each matrix generated by this process must then be evaluated on certain criteria as follows:

1 - Each row must have at least two elements equal to one

2 - Each column must have at least two elements equal to one

The process of enumerating mechanisms begins with the generation of all fundamental non-isomorphic graphs. This is accomplished by incrementing a number, performing a bit mapping operation, decoding the bits in order to load a matrix and then determining if the resultant matrix is non-isomorphic. Once the fundamental matrices have been determined using the above mentioned algorithm, the same process is applied again to the fundamental graphs in order to generate all non-isomorphic colored-graphs. Color is used to distinguish the edges of a graph that are different from the other edges. In the case of cam mechanisms, color is used to denote the cam-contact pair.

CHAPTER 3

3. Overview of Algorithms

This chapter provides an overview of the algorithms developed during this research. Section 3.1 describes the main algorithm. Key algorithms are documented in sections 3.2 through 3.4. of this chapter.

3.1 General Steps

The main algorithm "SYNTH" is responsible for controlling the other algorithms and maintaining central storage in the computer memory for intermittent and final results. SYNTH makes use of two important algorithms "PERMUT" and "COLGRA". The first performs permutations on a matrix, the second finds all possible combinations of colors for a matrix (graph). SYNTH invokes PERMUT first to generate all possible fundamental non-isomorphic graphs and then passes the results to COLGRA. The results from COLGRA represent the final results of the permutation process.

3.2 Algorithm PERMUT

PERMUT is responsible for determining all possible non-isomorphic matrices (graphs). This is accomplished by looking at the order of an NxN matrix and determining the number of bits required to represent the upper-half of a symmetrical square-matrix. This number is used as a looping index. For example, a 5x5 matrix requires ten (4+3+2+1) bits to represent the upper half of the matrix. Each element of the matrix is represented by a bit. All elements along the diagonal are equal to zero because the vertices are not allowed to connect to themselves.

PERMUT also makes use of the algorithm DECODE to examine the bits of the loop index and map the 1 bits into a symmetrical matrix. After the permuted matrix is generated, PERMUT examines the number of incident edges for each row looking for matrices with two or more edges per row. Those matrices which pass this initial screening process are then passed to another algorithm FDETER which calculates the determinant of the matrix. The results from this process are passed to the algorithm ISOMOR to determine if the matrix represents a non-isomorphic matrix. If the matrix is non-isomorphic, it is added to the array of saved matrices.

3.3 Algorithm COLGRA, For Coloring Graphs

COLGRA receives an adjacency matrix of order N and determines all possible permutations of that matrix using the number of colors passed. The method of bit mapping is once again used for generating all permutations.

COLGRA takes the adjacency matrix passed in and copies the upper half of the matrix into a one dimensional integer array called BIT1. The matrix is then colored by using the bit mapping of an integer number that is initialized to one and incremented by one. Each time the integer is incremented, the bits are tested for the value of one. The number of one bits are summed. If the number of bits turned on is equal to the number of elements that have a value of one in the matrix, the algorithm SETBIT is invoked to copy the bit map into a second one dimensional integer array called BIT2. Arrays BIT1 and BIT2 are then compared element by element. If the arrays have at least two elements that match (the number of edges to color) the matrix is passed to the algorithm COLMAT for coloring. Colored maps differ from fundamental matrices (graphs) in that the integer number three is used for color.

3.4 Checking for Isomorphism (ISOMOR)

The purpose of the algorithm ISOMOR is to determine if a given matrix is non-isomorphic when compared to a list of matrices that are stored in memory. If the matrix is non-isomorphic, it is added to the list of non-isomorphic matrices. The determinant of the matrix is determined by first mapping the integer matrix into a matrix of double precision real numbers and calculating the determinant of the second matrix. Once the determinant has been calculated, this value is compared against an array of saved determinants using the following equation.

$$X = (\ newdeter - savedeter\ ) \ /\ newdeter \qquad [3.1]$$

If X is less than or equal to some very small number, say 1.0E-8, then the determinant is considered a match. Once a match is found, ISOMOR invokes the algorithm COPMAT to copy the matrix and it's determinant into an array of saved non-isomorphic matrices and determinants.

3.5 Coloring the Graph (COLGRA)

The algorithm for coloring a graph is called COLGRA. COLGRA receives the one dimensional array "BIT2" and examines the upper half of graph which is to be colored. Each element of the upper half of the matrix is compared with a corresponding element in the array BIT2. If both elements are equal to one, the element of the matrix is colored (indicated by setting the element equal to three). The colored graph is then passed to the invoking algorithm.

CHAPTER 4

4. Analysis of Hain's Work

In order to understand the work that Hain conducted in structural synthesis, it is necessary to understand the methods he used in obtaining various permutations of mechanisms. We begin with the examination of the degree of freedom equation. This equation will be used to determine the number of one and two degree-of-freedom joints required for a single degree-of-freedom mechanism. Once these values are found, the graph theory approach can then be used to enumerate all possible permutations of mechanisms meeting the same requirements as those investigated by Hain.

The primary difference between the graph theory approach and the approach used by Hain, is that Hain's method of permutation relied on existing linkages that could be converted into mechanisms with higher ordered pairs (cam-pairs in particular) and fewer links (ie. six links with no cam-contact-pairs converted to five-link, one cam-contact-pair mechanisms). The graph theory method is an exhaustive method which generates a complete list of graphs from which mechanisms may be developed.

This approach removes the requirement that one must apply a conversion method to kinematic linkages in order to generate the desired linkages.

4.1 Degree-of-Freedom Equation

The degree-of-freedom equation may be written as

$$F = 3(L - (J1 + J2) - 1) + J1 + 2*J2 \quad [4.1]$$

Where F is the total number of degrees-of-freedom for the mechanism, J1 is the number of single degree-of-freedom joints and J2 is the number of two degree-of-freedom joints. For a five-bar mechanism, one can set the number of links L equal to five. Substitute this value into equation [4.1] and solve for the number of single degree-of-freedom joints required for a single degree-of-freedom mechanism. Equation [4.1] then becomes

$$1 = 3 * (5 - (J1 + J2) - 1) J1 + 2J2 \quad [4.2]$$

If the number of two degree-of-freedom joints J2 = 1, then from Eq. [4.2]

$$J1 = 5 \quad [4.3]$$

Therefore, in order for the linkage towave a single degree of freedom, the number of single degree-of-freedom joints is equal to five and the number of two degree-of-freedom joints is equal to one. The total number of joints for the mechanism is equal to six.

4.2 Hain's Method of Enumeration

As mentioned earlier in section 1.2, Hain used a well-known method of replacing a binary link and its two single degree-of-freedom joints with a higher cam-contact pair in order to enumerate all five-bar single cam-contact pair mechanisms. In doing so, Hain developed 158 five-bar cam-modulated-linkages from four types of six-link mechanisms. These linkages were Watt's mechanism, Stephenson's mechanism, a six-link with one double joint and a six-link two degree-of-freedom chain with two double joints. From this list of 158 mechanisms, Hain reduced the list down to 21 mechanisms that represented all possible arrangements of mechanisms in which the rotating input was converted into oscillating swinging motion output.

The mechanisms were grouped according to the following functions;

1 - cam rotates on an axis that is fixed to frame

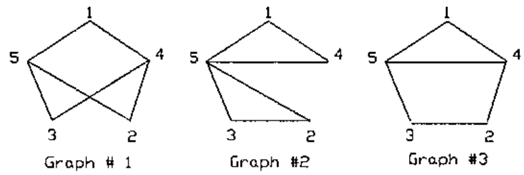
2 - cam is stationary and is fixed to frame

3 - cam moves, follower is fixed to frame

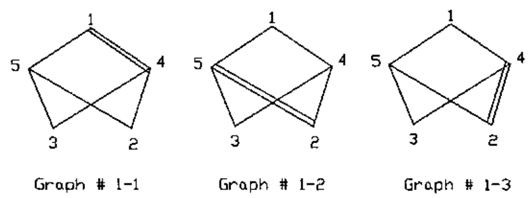
4 - cam is attached (mounted on) to a coupler of a four-bar

5 - basic five link chain with cam & follower used to relate movement of two of these links

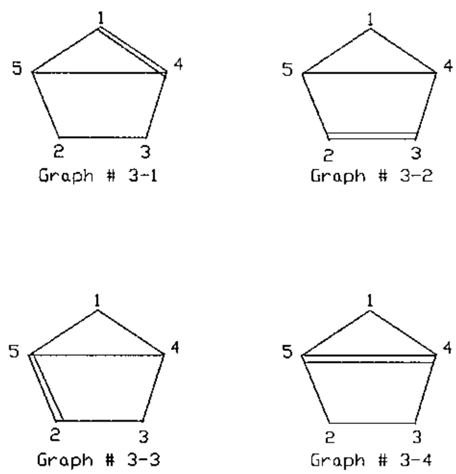
Hain was able to obtain such a large number of mechanisms by performing various permutations on the newly generated five-bar mechanisms. One such method involved the placement of various pin joints when links are joined to the ground link. In the case of the cam link acting as the ground link, the connecting links may be shown to connect to pin joints which are not physically located on the link itself. Figure 4.1(c) shows two permutations of a single schematic generated by applying this method. In Fig. 4.1(c), the cam link is considered as the ground link. This particular link happens to be a ternary link. The first permutation of this schematic involves removing the pin joint that connects Link #5 to the cam. The link is shown to be connected to Link #5 but at a location that is not on the link itself.



(a) Fundamental Graphs



(b) Graph #1 Colored



(c) Graph #3 Colored

Figure 4.2 Fundamental Five-Bar Graphs with Color

The second permutation involves link #3. This schematic which shows the same mechanism with both pin joints removed from the actual cam link. Hain considered the first schematic (Fig 4.1(a)) and it's two permutations as three mechanisms when he developed his 158 mechanisms.

Another method of performing permutations involves the grounding different links. This process is known as kinematic inversion. Figure 4.1(b) shows five different permutations of the same linkage obtained by varying the link that is considered as the ground link.

4.3 Hain's Kinematic Structures

In determining the various structures used by Hain, each of the 21 mechanisms are converted into graph form. Results from this process are shown in Fig. 4.1(a). It is interesting to note that all of the 21 mechanisms are permutations of the two basic graphs shown in Fig. 4.1(a). From the graphs of the mechanisms, the mechanisms can be classified according to properties found in the graphs. Classification of the mechanisms can then be used by the designer of mechanisms when selecting mechanisms to perform specific functions based on structural classification.

4.4 Graph Theory for Enumeration

There are several advantages in using the graph theory approach for the enumeration and permutation process. The first reason is that this method is independent of results obtained from other processes and methods. It provides a check for other enumeration methods. By using the algorithms to generate all possible combinations and then apply criteria to reduce the list of possible mechanisms, one is forced to consider all cases generated. This method is thorough in that it is exhaustive in its enumeration of mechanisms, leaving none unfound. The graph theory method is independent of function, and motion and is not limited to either the number of links or the degrees of freedom desired in the linkages under study.

The graph theory method of permutation was used to find all possible non-isomorphic fundamental graphs. A second application of graph theory was then applied to the fundamental graphs in order to find all possible non-isomorphic colored graphs. For the permutation process, the following input parameters were used by the various algorithms were as follows;

degrees-of-freedom = 2

number of linkages = 5

number of joints = 6

The graph theory method generated a total of eleven fundamental graphs whereby each of the graphs were numbered from one to eleven. However, not all of the graphs represented single degree-of-freedom mechanisms. This was due to the varying number of joints found in each of the graphs. From list of eleven fundamental graphs, only three graphs met the required number of joints (six) for a single degree-of-freedom mechanism. These graphs are shown in Fig. 4.2(a) and are numbered #1, #2 and #3. Examination of Graph #2 reveals that although the graph has the required number of joints (six), this graph does not represent a working mechanism because there are two totally independent loops found in the graph. Graph #2 is shown in Fig. 4.2(a) for reference only because it was generated from the permutation algorithms. From the remaining two fundamental non-isomorphic graphs, seven non-isomorphic colored graphs were generated. Each graph has only one color, denoted with a double line, representing the cam-contact pair. Figures 4.2(b) and 4.2(c) show the non-isomorphic colored-graphs generated from two of fundamental graphs shown in Fig. 4.2(a).

4.5 Classification of Graphs

From the eleven colored-graphs generated by the graphical method, there were several substructures that were found to be common in the graphs. These substructures were used as means of classifying the graphs generated from the algorithms and aided in the process of converting the graphs into their respective schematics.

4.5.1 Type I Substructure

The first substructure found in the graphs is shown in Fig. 4.3(a). This substructure shows a cam-contact-pair connected to two other links. The cam-contact is labeled 1 and 2. Two basic permutations arise from this substructure. Either one of the connecting links may be considered as the ground link. Figure 4.3(a) shows these permutations along with the substructure.

4.5.2 Type II Substructure

The second substructure is shown in Fig. 4.3(b). This substructure represents a rigid structure. As each link is added to the linkage, one degree of freedom is removed until the total degrees of freedom for this substructure is zero. One possible arrangement of links for this structure is shown. As drawn, from the Fig. 4.3(b) one can see that the links in this substructure cannot move.

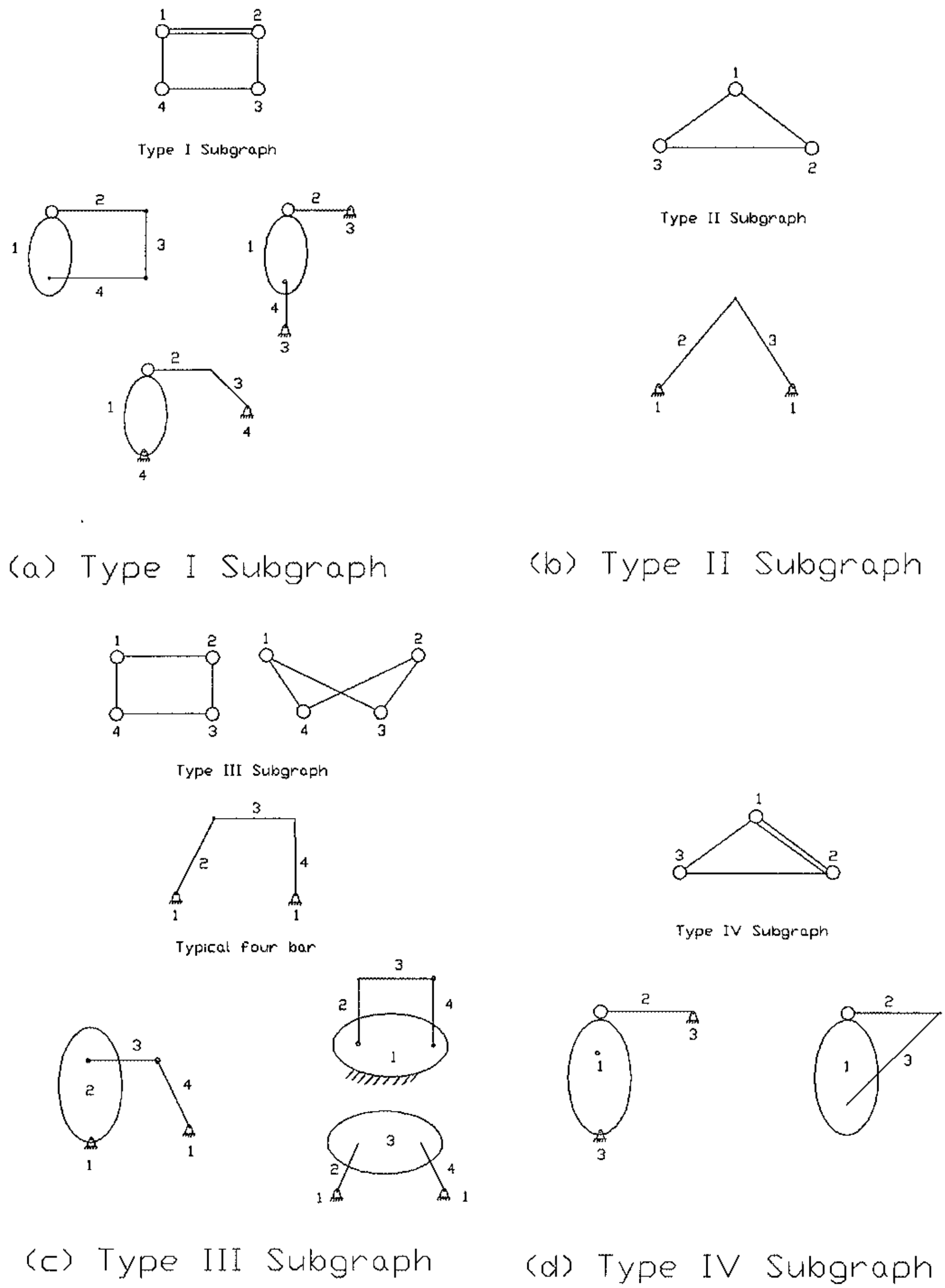


Figure 4.3 Subgraphs and Resultant Linkages

4.5.3 Type III Substructure

The third substructure consists of a four-bar linkage and is shown in Fig. 4.3(c). There are three possible permutations for this structure. As shown, the cam and follower makeup two of the links in the four-bar linkage. Three permutations are shown along with the substructure.

4.5.4 Type IV Substructure

The fourth substructure found in the graphs is shown in Fig. 4.3(d). It consists of the two links in the cam-contact pair connecting to a common third link. Two permutations are possible from this configuration and are shown with their respective subgraphs.

4.6 Tables of Mechanisms

Using the three types of subgraphs described in the previous sections, the colored graphs were classified and listed in Tables 4.1 and 4.2. Table 4.1 lists the first set of colored graphs corresponding to Graph #1. The substructures found in these graphs are listed with each respective graph in the table. Table 4.2 lists all the colored graphs generated from Graph #3. As mentioned in section 4.4 Graph #2 was not colored.

Table 4.1 Classifications of Fundamental Five-Bar Mechanisms Generated from Graph #1

Graph #	Type I	Type II	Type III	Type IV
1-1	x		x	x
1-2	x		x	
1-3	x		x	

Table 4.2 Classifications of Fundamental Five-bar Mechanisms Generated from Graph #3

Graph #	Type I	Type II	Type III	Type IV
3-1			x	x
3-2	x	x		
3-3		x		
3-4	x			

4.7 Promising kinematic Concepts

After generating all the non-isomorphic colored-graphs, the graphs were classified according to substructures and expanded into schematics according to the permutations shown with each of the classes of substructures. These schematics do not represent every possible permutation, but rather the most general permutations.

There are several methods which can be used to generate additional permutations from the fundamental schematics but the intent of this investigation is to determine if there are fundamental mechanisms that were overlooked or not considered by Hain due to the particular method he chose for his enumeration. While this method uses the computer for the enumeration process, the graph theory method of enumeration is exhaustive. One must apply particular criteria in order to reduce the total number of valid non-isomorphic mechanisms.

4.8 Analysis of Results

Comparisons between Hain's schematics and the schematics developed for this study using structural kinematic synthesis can now be made. Of the 21 mechanisms that Hain developed, the graphical method was unable to generate any additional graphs or configurations of mechanisms which were not permutations of one of the 21 mechanisms. However, this comparison does provide a check for the validity and completeness of the graph theory approach. With this in mind, the graph theory method was used to examine other types of more complicated cam-modulated mechanisms.

In particular, this study includes the following types of mechanisms:

1. Six-bar mechanisms with two cam-contact pairs and one degree-of-freedom (see Chapter 5).
2. Five-bar mechanisms with two cam-contact pairs and two degrees-of-freedom (see Chapter 6).

CHAPTER 5

5. Dual-Cam Mechanisms With Six Links and One D.O.F.

The analysis of six-link dual-cam mechanisms with one degree of freedom begins with the degree-of-freedom equation. The equation as shown in Chapter 4 may be written as

$$F = 3(L - (J1 + J2) - 1) + J1 + J2 \qquad [5.1]$$

In this case, we set the number of links L equal to 6 and the number of two degree-of-freedom joints, J2, equal to 2. Equation [5.1] may now be written as

$$1 = 3(6) - 3J1 - 3(2) - 3 + J1 + 2(2) \qquad [5.2]$$

so that

$$J1 = 6 \qquad [5.3]$$

From Eq. [5.3], the number of single degree-of-freedom joints is equal to six. The total number of joints for the mechanisms is equal to eight (six one degree-of-freedom joints and two two degree-of-freedom joints). The results from the degree-of-freedom equation are used as input parameters for the structural synthesis algorithms. To summarize, the input parameters are as follows:

degrees-of-freedom = 1

total # of links = 8

# of joints with 1 degree-of-freedom = 6

# of joints with 2 degrees-of-freedom = 2

5.1 Graphs

The graph theory algorithms generated nine fundamental non-isomorphic graphs that met the criteria for six-link one degree-of-freedom dual-cam-contact mechanisms. Each of these graphs have been expanded into all the possible non-isomorphic colored graphs by the graph theory approach to synthesis.

The results of the algorithms are shown below with each fundamental graph and the total number of non-isomorphic colored graphs generated by the algorithms shown to the right.

graph #1 - 2 permutations

graph #2 - 10 permutations

graph #3 - 17 permutations

graph #4 - 15 permutations

graph #5 - 26 permutations

graph #6 - 15 permutations

graph #7 - 5 permutations

graph #8 - 10 permutations

graph #9 - 10 permutations

Figures 5.1 through 5.5 show each of the fundamental graphs and each colored graph that was developed from the respective fundamental graph. A total of 110 non-isomorphic colored graphs were generated. Section 5.2 discusses the methods used to classify these graphs and how the graphs are converted into schematics of mechanisms.

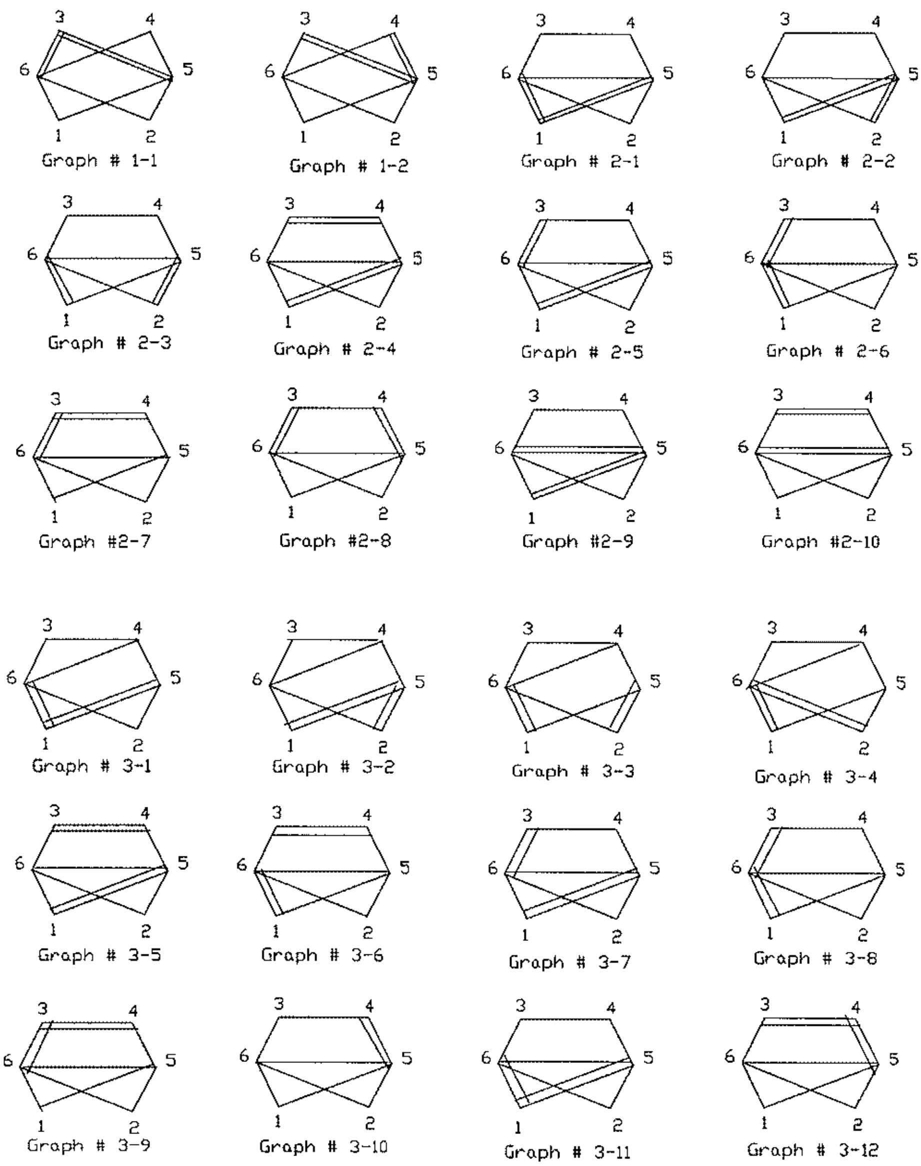


Figure 5.1 Six-Bar Colored-Graphs
#1, #2 and #3

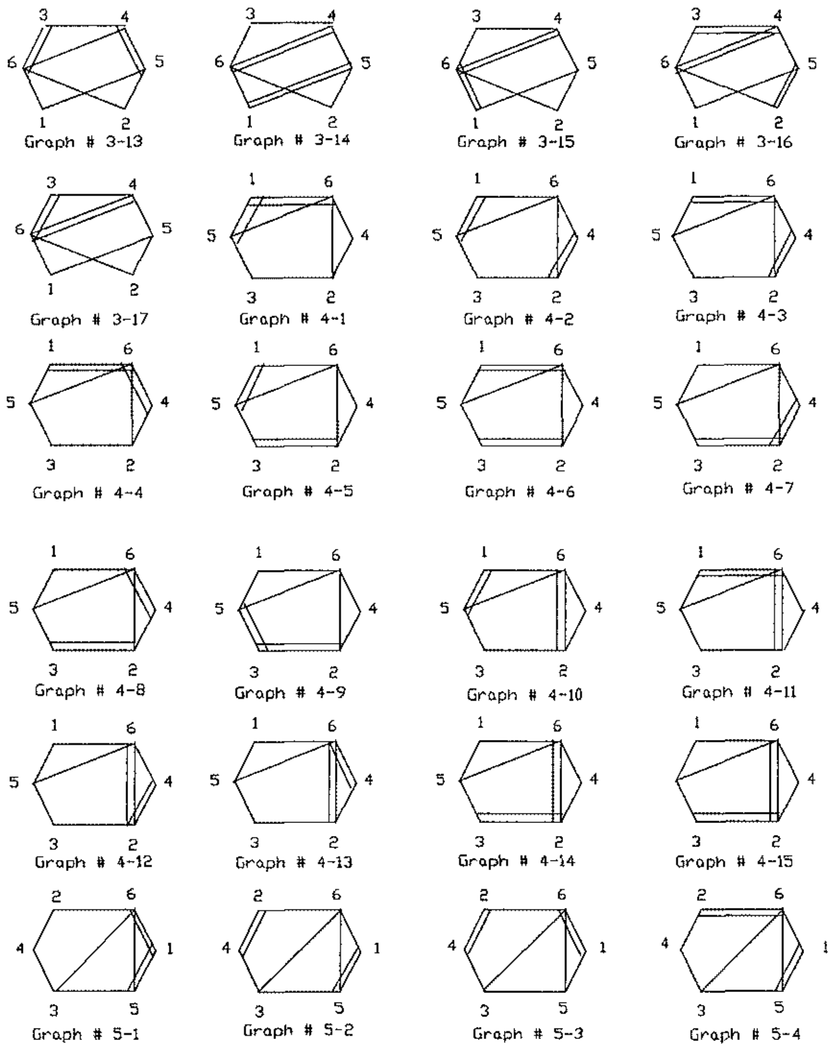


Figure 5.2 Six-Bar Colored-Graphs
#3, #4 and #5

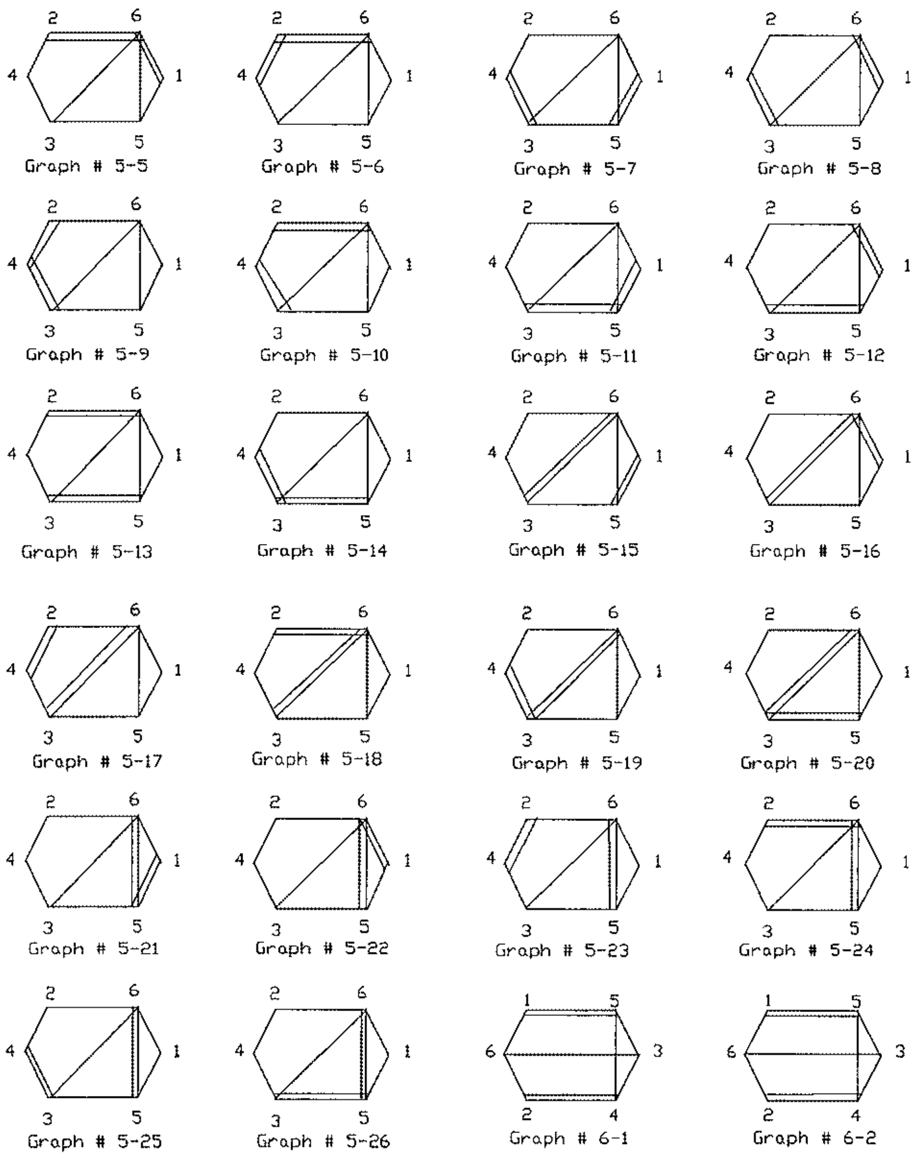


Figure 5.3 Six-Bar Colored Graphs
#5 and #6

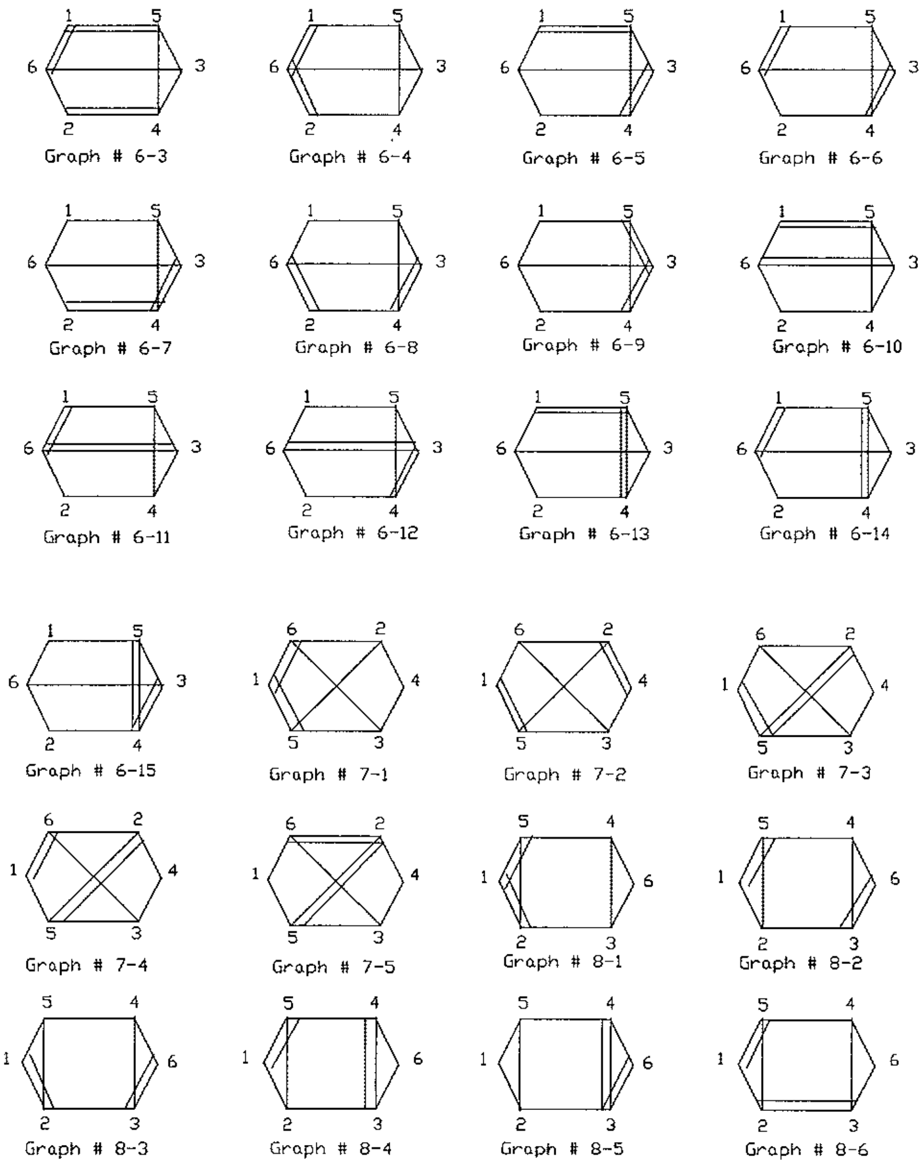


Figure 5.4 Six-Bar Colored Graphs
#6, #7 and #8

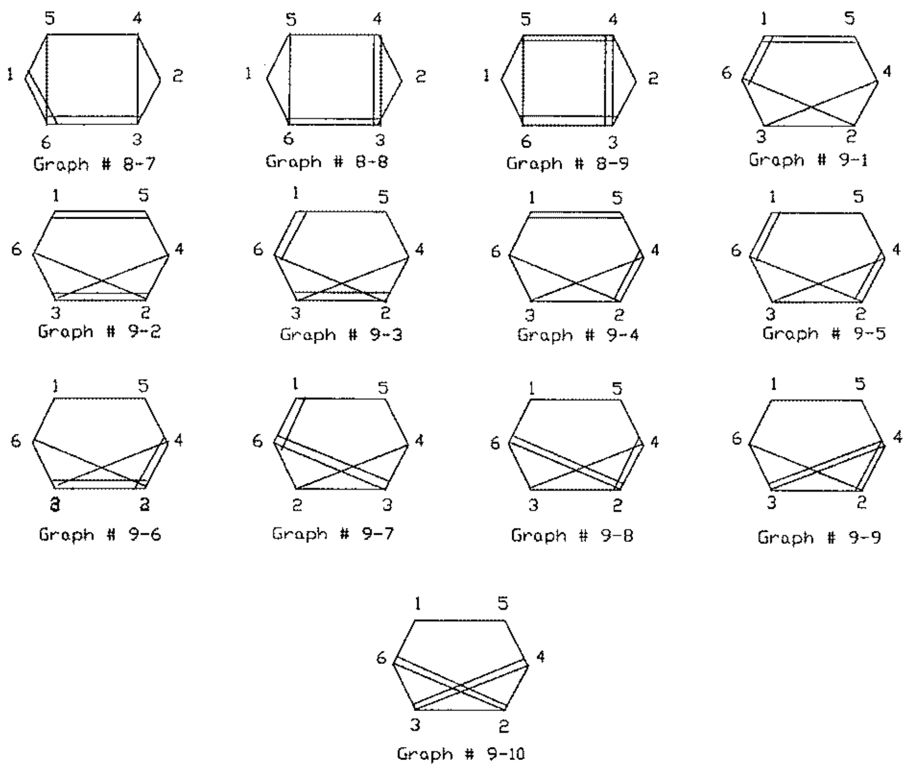


Figure 5.5 Six-Bar Colored Graphs
#8 and #9

5.2 Classification of Graphs

Faced with the task of converting all of the 110 colored graphs into schematics, a method was needed to identify only those graphs that were capable of generating working mechanisms. A method was needed by which the schematics could easily be developed from the colored graphs. The basic arrangements of links was desired when the schematics were being drawn. Once the basic schematics were generated, the methods of permutation discussed in Chapter 4 (ie. kinematic inversion) could be applied by the designer to the basic arrangements developed during this investigation. During the process of examining the graphs prior to conversion into schematics, several substructures appeared over and over again. These substructures were identified and used to classify all 110 colored graphs. Six fundamental substructures were identified. These substructures are shown in Fig. 5.6 and discussed in further detail in the following sections.

5.2.1 Type "I" Substructure

The first basic substructure shown in Fig. 5.6(a) involves the sharing of a common element between the two cam-contact pairs. This may be either a shared cam or follower.

There are four possible combinations of cams and followers that arise for this particular sub-structure. They are listed as follows:

1. cam - follower - cam
2. follower - cam - follower
3. follower - cam - cam
4. cam - cam - follower

Note that the cam-to-cam contact is a permissible cam-contact pair. It is also apparent that arrangements #3 and #4 are the same. This reduces the number of possible arrangements to three. These permutations are shown along with the substructure in Fig. 5.6(a).

This arrangement reduces the number of links which must be considered from four links to just three links. This allows for the remaining links to participate in other arrangements such as four-bar linkages with the overall mechanism.

For the arrangement of the three links, if one link is desired as the input link and a second link as the output link, the shared link between the cam contact-pairs can be used to transmit the input from the input link to the output link. Altering the location of the common link can alter or vary the movement of the output link.

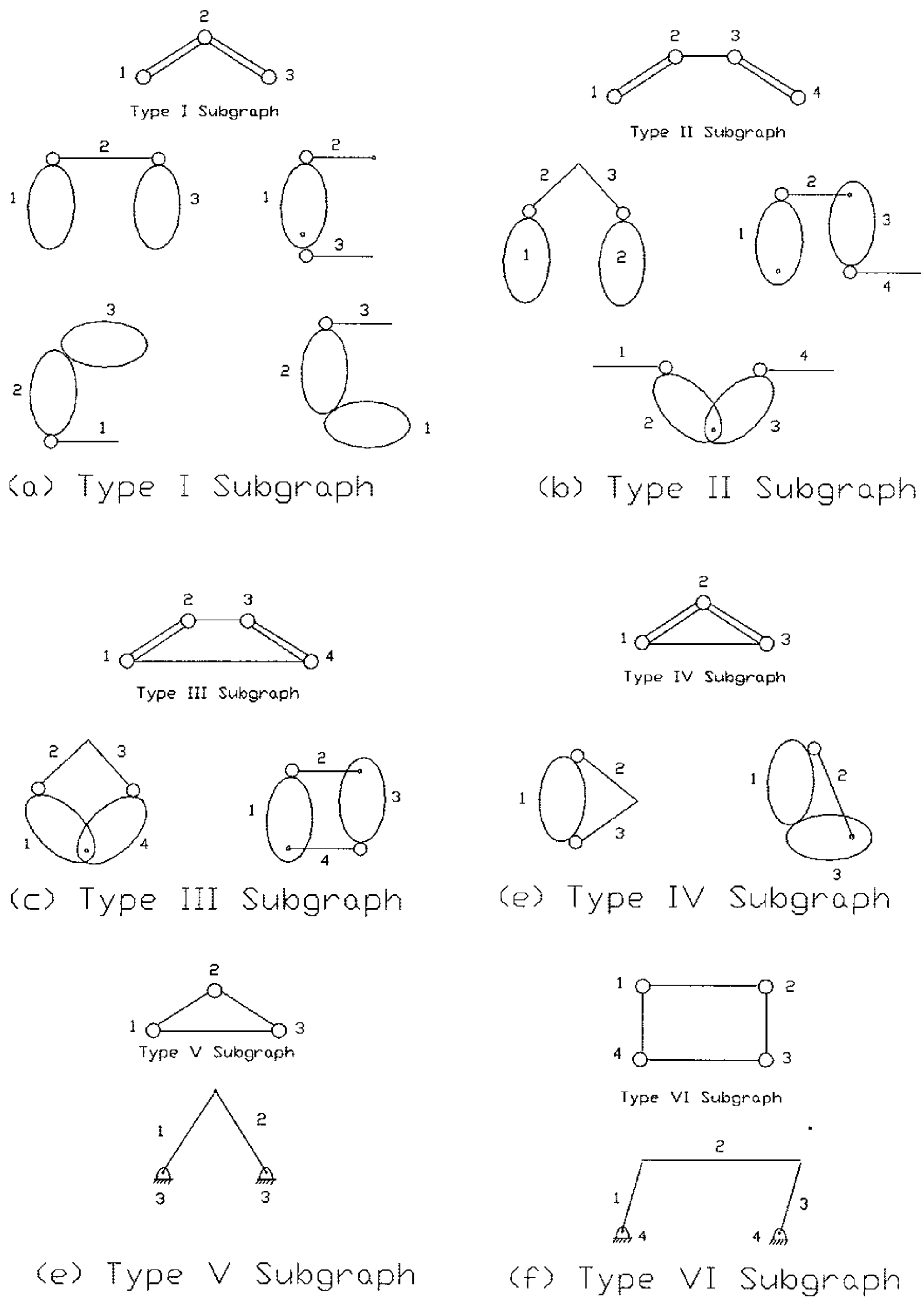


Figure 5.6 Subgraphs for Classifying Linkages

5.2.2 Type "II" Substructure

Figure 5.6(b) depicts the second class of substructures found. In this structure, one link from each cam-pair is pinned to a link from the other cam-pair. This may be a cam-to-cam or a cam-to-follower pinning. The possible permutations of links for this substructure are shown below

1. cam - follower - follower - cam
2. cam - follower - cam - follower
3. follower - cam - cam - follower

This arrangement allows one of pinned links to move if the other pinned link is fixed. The arrangement of links along with the associated substructure are shown in the figure.

5.2.3 Type "III" Substructure

The next configuration found is the double pinning of links from each of the cam-contact pairs. Figure 5.6(c) shows this substructure. Each member of the pair is pinned to a member of the opposite pair. There are only two possible combinations for this configuration. They are listed below and shown in Fig. 5.6(c) along with the substructure.

1. cam - follower - follower - cam
2. cam - follower - cam - follower

In the Fig. 5.6(c), the links labeled #1 and #4 are pinned together. Links #2 and #3 are also pinned. This arrangement tightly couples the movement in one cam pair to the movement of the other pair.

5.2.4 Type "IV" Substructure

Figure 5.6(d) shows the fourth substructure found. It is essentially the same substructure as a Type I, with the exception that the remaining links from each of the cam-pairs are pinned together. This may be a cam-to-cam/follower-to-follower, or a cam-to-follower/cam-to-follower pinning arrangement. This substructure reduces the number of links involved in the two cam-pairs to three.

5.2.5 Type "V" Substructure

Figure 5.6(e) illustrates a substructure that appears quite often in the various graphs. This substructure consists of three links joined together by three joints. All of the joints have one degree-of-freedom. For this reason, the total degree-of-freedom for the substructure is zero. The links cannot move and therefore, represent a rigid structure. Any graphs having this substructure can be eliminated from any further study. This single substructure eliminated 51 percent of the 110 colored graphs from further investigation.

5.2.6 Type "VI" Substructure

The final type of substructure considered for classification is shown in Fig. 5.6(f). This substructure consists of a four-bar linkage. Any link that participates in the cam-contact pair and appears in this substructure is a part of a four-bar mechanism. There are several permutations possible if a cam or follower is a link in the four-bar substructure. The permutations are shown in Fig. 5.6(f).

5.3 Tables of Mechanisms

Using the classifications detailed in the previous section, all non-isomorphic colored graphs generated by the algorithms were examined and classified. In most cases, graphs were found to contain more than one sub-class of structure. Some of the classes are subsets of other classes. Tables 5.1 through 5.8 list each of the fundamental graphs with the numbers assigned to each graph by the algorithms and all non-isomorphic colored graphs along with the substructures found in each graph.

Table 5.1 Classification of Six-Bar Graphs #1 and #2

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
1-1	x					x
1-2	x					x
2-1	x			x	x	x
2-2	x					x
2-3						x
2-4		x			x	
2-5		x			x	
2-6	x				x	x
2-7	x				x	x
2-8		x	x		x	x
2-9	x			x		
2-10		x	x			x

Table 5.2 Classification of Six-Bar Graph #3

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
3-1	x				x	
3-2	x				x	
3-3					x	
3-4	x				x	
3-5		x				
3-6		x				
3-7		x				
3-8	x					
3-9	x			x		x
3-10	x				x	
3-11			x		x	
3-12	x					x
3-13		x				x
3-14			x			
3-15	x					
3-16	x			X		X
3-17	x			x		X

Table 5.3 Classifications for Graph #4

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
4-1	x			x	x	x
4-2						x
4-3						x
4-4	x					x
4-5		x			x	
4-6		x			x	
4-7	x				x	
4-8		x			x	
4-9	x				x	
4-10		x				
4-11	x					
4-12	x			x	x	
4-13	x			x	x	
4-14	x				x	
4-15		x			x	

Table 5.4 Classifications for Graph #5

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
5-1	x			x	x	x
5-2					x	
5-3		x			x	
5-4		x			x	
5-5	x				x	
5-6	x				x	
5-7		x			x	
5-8	x				x	
5-9		x	x		x	
5-10	x				x	x
5-11	x					x
5-12		x	x			
5-13		x			x	
5-14	x				x	
5-15			x			
5-16	x					
5-17			x		x	
5-18	x				x	
5-19	x				x	
5-20	x			x	x	
5-21	x			x		x
5-22	x			x		x
5-23		x				
5-24	x					
5-25		x				
5-26	x			x		x

Table 5.5 Classifications for Graph # 6

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
6-1	x				x	
6-2					x	
6-3					x	
6-4	x				x	
6-5		x				
6-6						
6-7	x					
6-8		x				
6-9	x			x		
6-10			x		x	
6-11	x				x	
6-12	x					
6-13	x					
6-14						x
6-15	x			x		

Table 5.6 Classifications for Graph # 7

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
7-1	x					
7-2						
7-3	x					
7-4			x			
7-5	x					

Table 5.7 Classifications for Graph #8

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
8-1	x			x	x	
8-2					x	
8-3						
8-4						
8-5	x			x	x	
8-6		x			x	
8-7	x				x	
8-8	x				x	
8-9			x		x	

Table 5.8 Classifications for Graph #9

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
9-1	x				x	x
9-2						x
9-3						x
9-4		x			x	x
9-5					x	x
9-6	x			x		x
9-7	x				x	
9-8	x					
9-9	x			x	x	
9-10			x			

5.3 Possible Mechanisms

Using the tables of mechanisms from section 5.2 and the possible fundamental arrangements of links shown for each type of subgraph, Figs. 5.7 through 5.11 depict the fundamental schematics for each of the colored graphs that were expanded from Tables 5.1 through 5.8. Only 50 graphs were expanded into schematics because 60 of the colored graphs contained Type V substructures (rigid structures). From the Tables 5.1 through 5.8 only four graphs (Graph #6-6, Graph #7-2, Graph #8-3 and Graph #8-4) did not contain any of the substructures used to classify the colored graphs.

The substructures can be easily identified when examining each colored graph. The designer can use the tables in this chapter to identify linkages with similar characteristics when consideration is being made during the design of particular mechanisms. When manufacturing is taken into consideration during the design process, the designer may use the tables to identify linkages which possess the same characteristics.

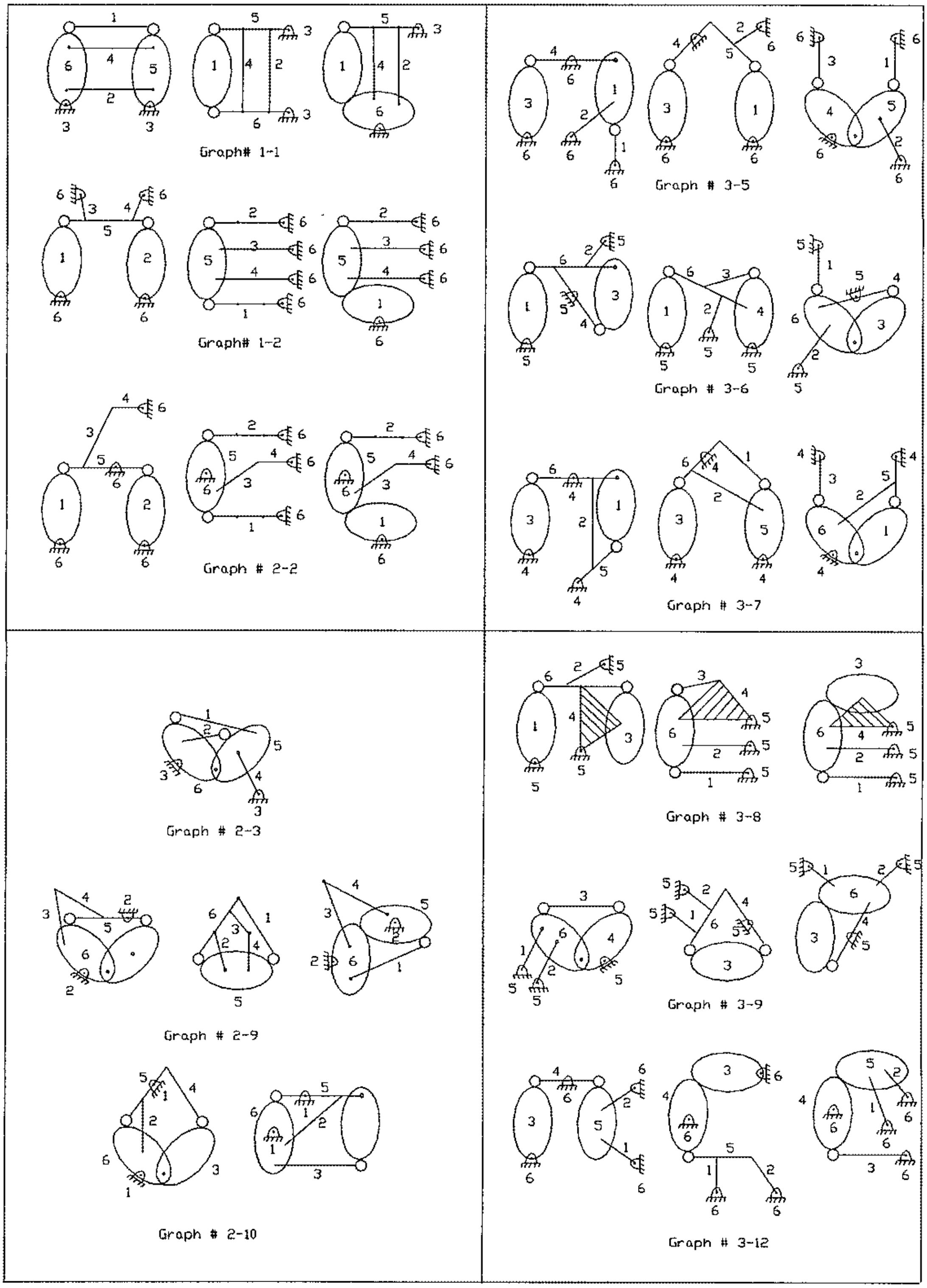


Figure 5.7 Six-Bar Schematics for Graphs #1, #2 and #3

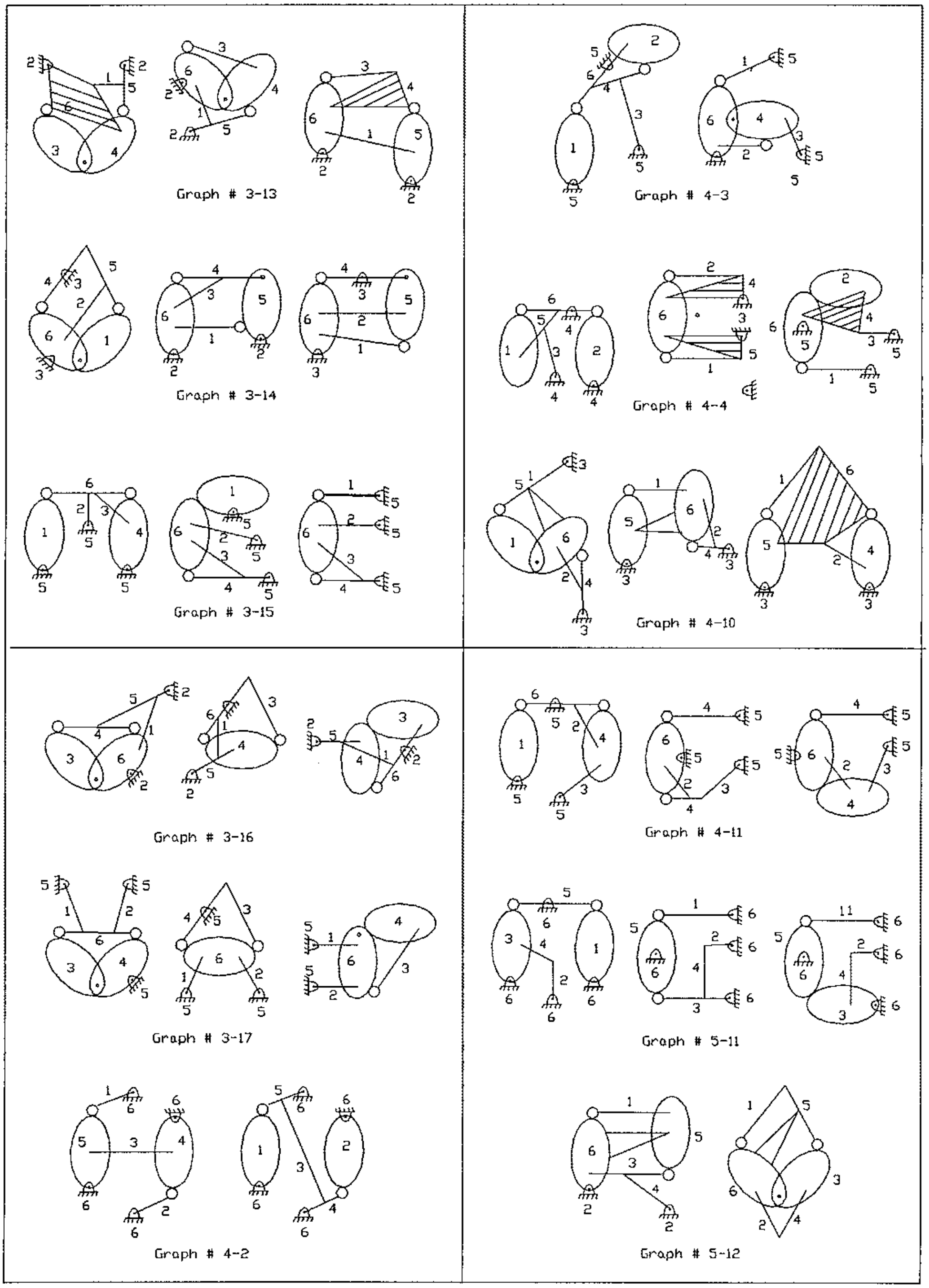


Figure 5.8 Six-Bar Schematics for Graphs #3, #4 and #5

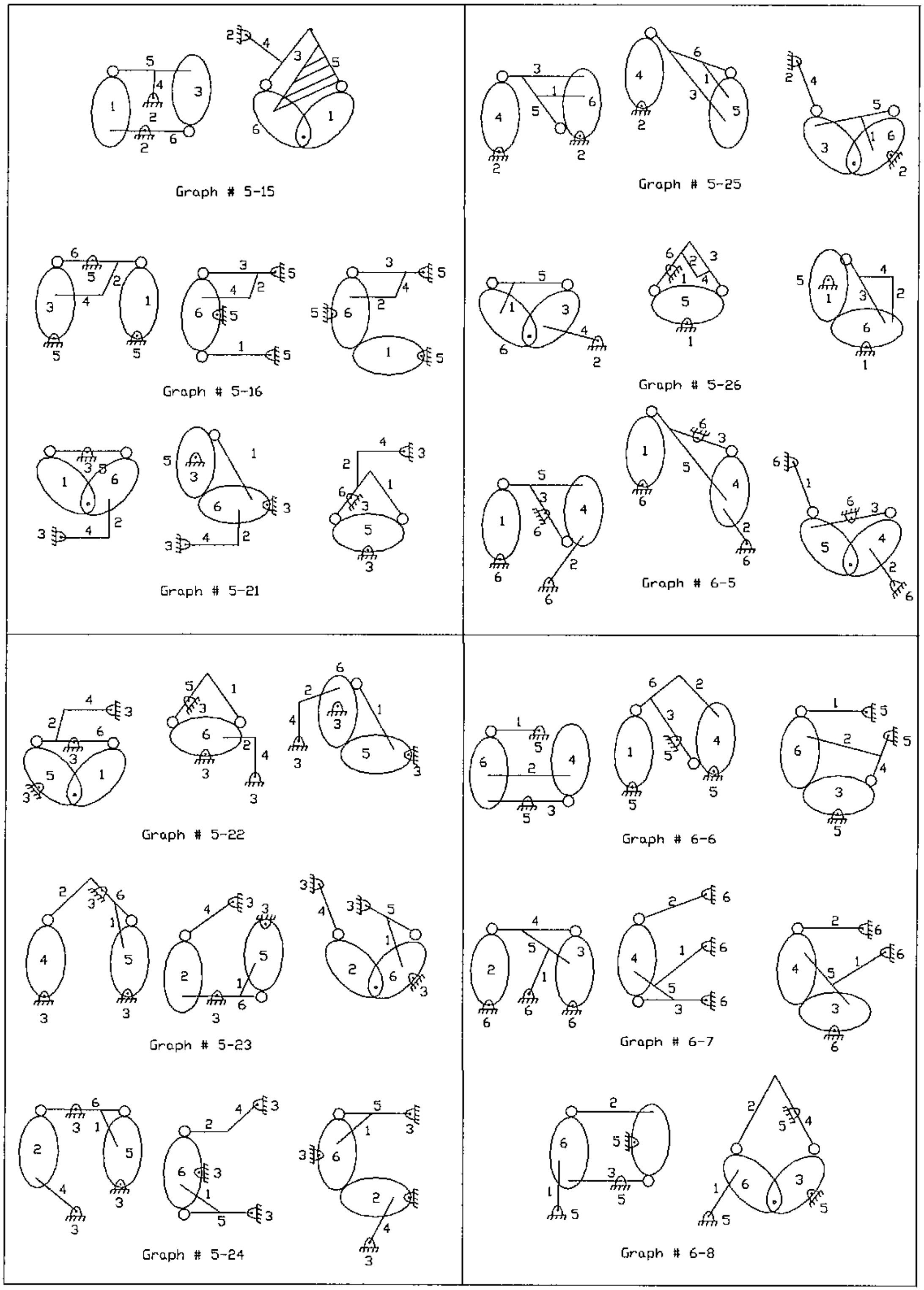


Figure 5.9 Six-Bar Schematics for Graphs #5 and #6

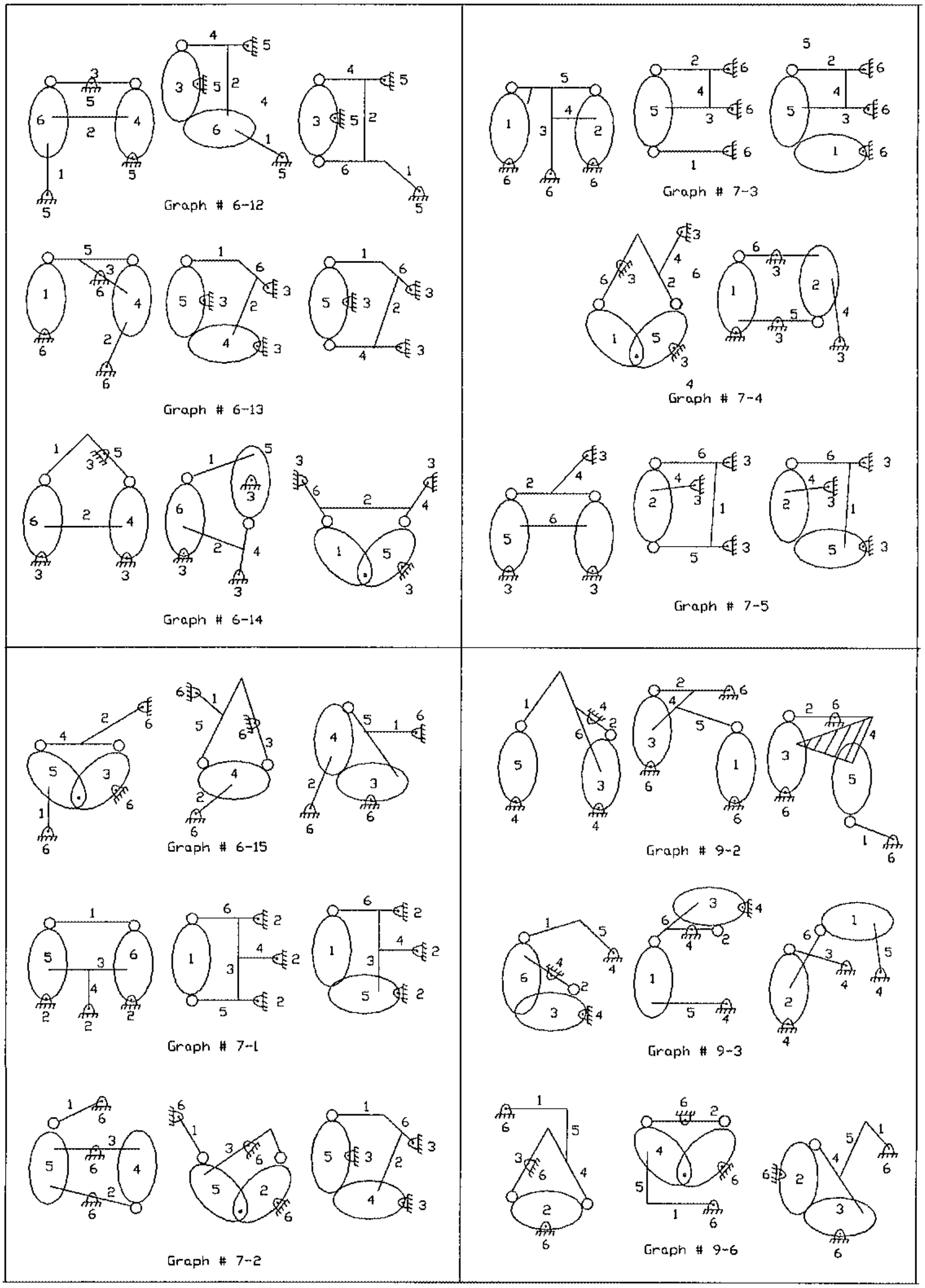
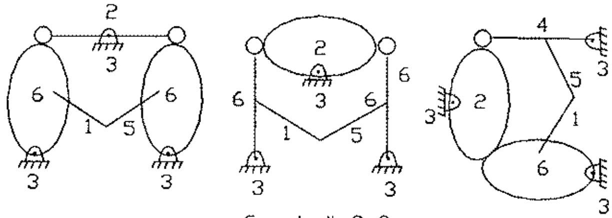
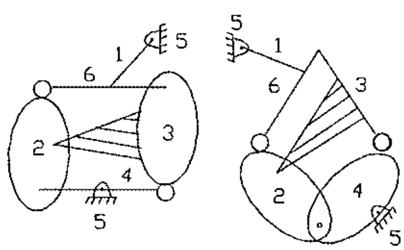


Figure 5.10 Six-Bar Schematics for Graphs #6, #7 and #9



Graph # 9-8



Graph # 9-10

Figure 5.11 Six-Bar Schematics for Graph #9

5.4 Analysis of Results

There are several means of classifying the six-bar dual- cam mechanisms. One method, discussed in chapter 4, used substructures to classify the mechanisms. This was a useful classification for expanding the graphs into mechanisms. Another possible classification of the mechanisms is based on the schematics. This classification is shown below;

1 - single rotating cam
2 - both cams rotating
3 - cams pinned together
4 - 2 cams 1 follower
5 - 1 cam 2 followers
6 - cam fixed to ground
7 - 1 floating cam
8 - four-bar sub structure

If for example, a designer wishes to use a mechanism in which both cams are pinned together, then referring back to section 5.2, we see that a Type II substructure will meet this requirement. Using this information, one may proceed to look through the graphs listed in Tables 5.1 through 5.7. Once the graphs have been identified, Figs. 5.7 through 5.11 illustrate the schematics of the corresponding graphs.

CHAPTER 6

6. Dual-Cam Mechanisms with Five Links and Two Degrees-of-Freedom

Just as we began in Chapters 4 and 5, the analysis of five-link dual-cam mechanisms with two degrees-of-freedom begins with the degree-of-freedom equation. Greubler's equation may be written as:

$$F = 3(\,L - (\,J1 + J2\,) - 1\,) + J1 + J2 \qquad [6.1]$$

If we set the number of links L equal to 5 and the number of two degree-of-freedom joints, J2, equal to 2, then by collecting terms, Eq. [6.1] becomes

$$2 = 3(5) - 3 - 3J1 - 3(2) + J1 + 2(2) \qquad [6.2]$$

Solving for J1,

$$J1 = 4 \qquad [6.3]$$

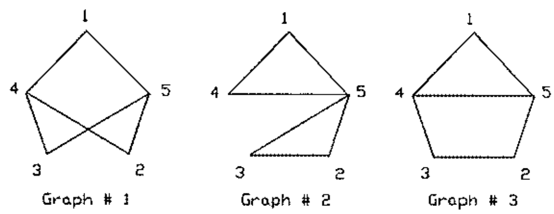
From Eq. [6.3], the number of single degree-of-freedom joints is equal to 4. The total number of joints for the mechanism is equal to 6. The results from the degree-of-freedom equation can be used as input parameters for the algorithm SYNTH.

6.1 Graphs

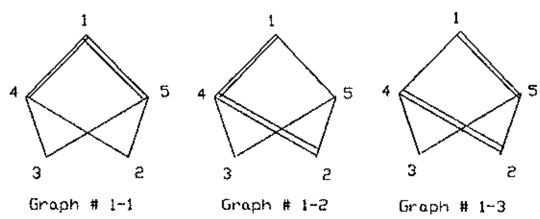
From the main algorithm SYNTH, eleven fundamental non-isomorphic graphs were generated. Of these graphs, only two graphs met the criteria for the number of joints (six). These fundamental graphs are shown in Fig. 6.1(a). The algorithm also generated all non-isomorphic colored-graphs for each of the fundamental graphs representing five-bar mechanisms with two degrees-of-freedom. The graphs contain two colors which represent the two cam-contact pairs. The number of colored-graphs are shown below along with the number of the colored graphs generated for each fundamental graph to the right.

graph # 1 - 3 colored graphs

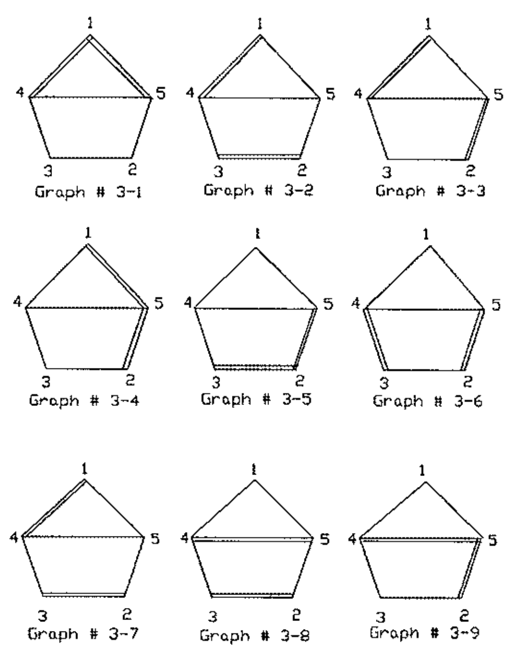
graph # 2 - 9 colored graphs



(a) Fundamental Graphs



(b) Graph #1 Colored



(c) Graph #3 Colored

Figure 6.1 Fundamental Five-Bar Colored Graphs

From the two fundamental graphs, a total of 12 non-isomorphic colored graphs were generated. The colored-graphs are shown in Figs. 6.1(b) and 6.1(c).

6.2 Tables of Mechanisms

Using the classification method developed in Chapter Five, all of the non-isomorphic colored graphs were classified. The results from this classification are tabulated into Tables 6.1 and 6.2. After classifying the graphs, the graphs were then converted into schematics of mechanisms. The results from this process are shown in Figs. 6.6 through 6.9.

and converted into schematics.

Table 6.1 Classifications of Fundamental Graph #1 (Five-Bar, Two Degrees-of-Freedom)

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
1-1	x					x
1-2	x					
1-3		x	x			

Table 6.2 Classifications of Fundamental Graph # 3 (Five-Bar, Two Degrees-of-Freedom)

Graph#	Type I	Type II	Type III	Type IV	Type V	Type VI
3-1	x			x		x
3-2		x				
3-3		x				
3-4	x					
3-5	x				x	
3-6		x	x		x	
3-7		x				
3-8		x	x			
3-9	x					

6.3 Possible Mechanisms

Figure 6.1 shows each type of subgraph and the various permutations that can be generated for that graph. This data is used for expanding the graphs. Figure 6.2 illustrates the schematics for each colored graph.

In addition to the possible schematics shown in the figures, there are at least four other permutations possible for each of the mechanisms shown. These permutations are found by kinematic inversion and by placement of the links joined to the ground link.

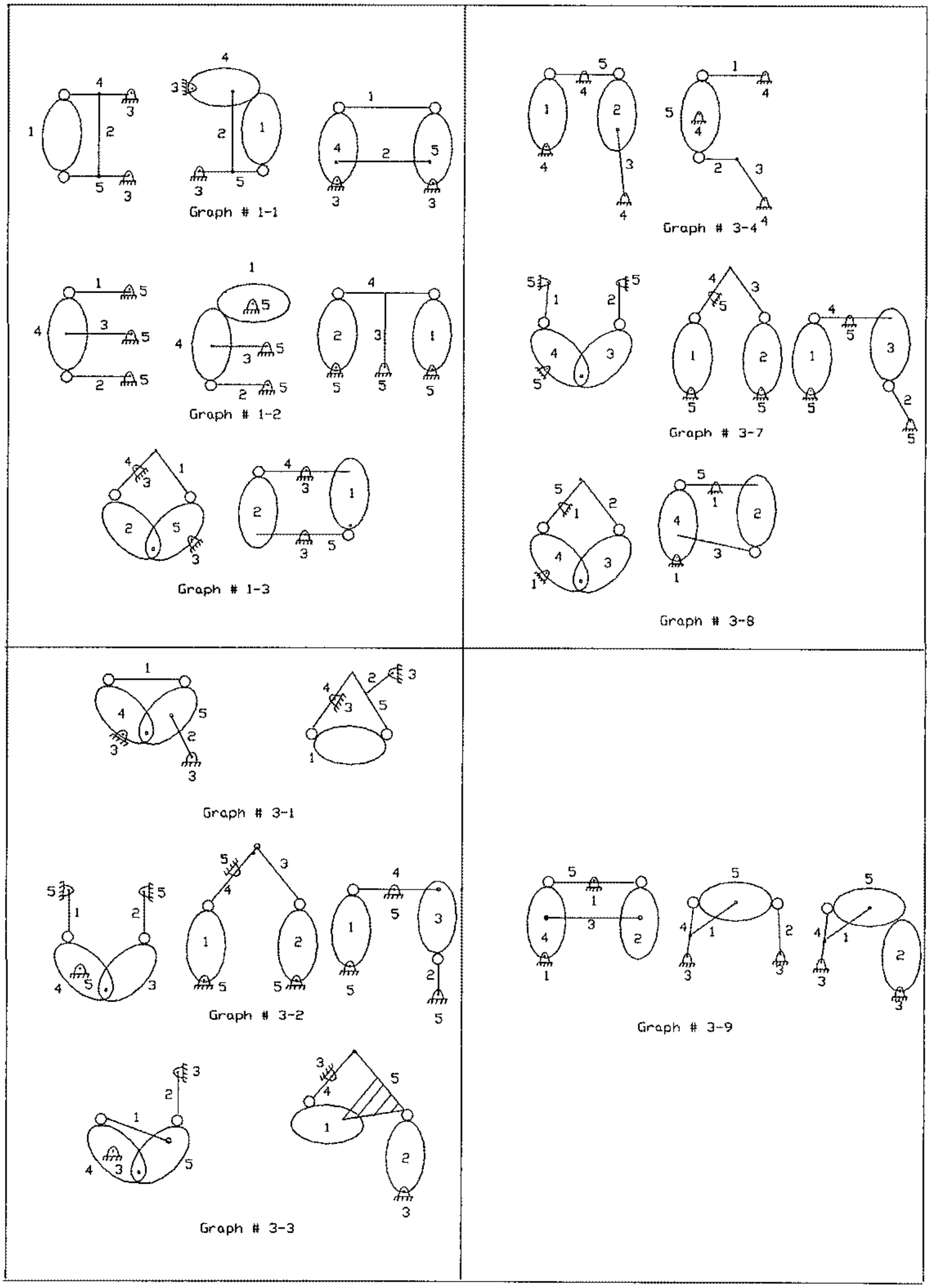


Figure 6.2 Five-Bar Schematics for Graphs #1 and #3

6.4 Analysis of results

The process of enumerating five-bar two degrees-of-freedom mechanisms using Graph theory resulted in two fundamental graphs which generated a total of 12 fundamental colored-graphs. Two of the graphs contained rigid structures and were therefore not converted into schematics of mechanisms. From the remaining 10 graphs, each colored-graph was converted into the fundamental schematics using the same classification technique as found in section 5.2. Of the 10 graphs, eight graphs were converted into scematics in which both cams pinned together. Five of the 10 graphs required the sharing of either a follower or cam between the two cam-contact pairs. Graph #3-1 was the only graph in which the common or shared cam/follower had only two two-degree-of-freedom joints.

CHAPTER 7

7. Applications of Dual Cam Mechanisms

There are many applications of dual-cam-contact mechanisms. This research investigates two such mechanisms and how the tables of classified graphs and their associated schematics found in Chapters 5 and 6 can be used.

7.1 Variable Valve Timing

One application of dual-cam mechanisms is found in mechanisms with variable valve timing. Typically, a cam (rotating input) is used to open an intake or exhaust valve for a particular length of time as the crankshaft of an engine rotates. This allows for the passage of gasses into and out of the cylinder. As the engine revolutions increase to a certain threshold, additional horsepower can be obtained if the dwell on the intake valves can be varied (opened longer). This allows more fuel mixture to enter the cylinder. The problem for the designer is to find a mechanism that will vary the valve timing.

This can be accomplished through several methods. One such method involves a mechanism with two cam-contact pairs.

7.2 Variable Valve Timing with Dual-Cam-Contact Pairs

Several configurations of linkages can be used for variable valve timing. One such linkage makes use of pinned cams which share a common follower. By changing the alignment of the cams with respect to each other, one can alter the dwell of the overall mechanism. Such a mechanism requires two inputs. One input is required to drive the cams while the second input is required to change the relative positions of each cam. In order to reduce the number of moving parts, it is desired to use five-bar mechanisms with two cam-contact-pairs. However, this requires the mechanism to also use two inputs (two degrees-of-freedom).

7.3 The Five-Bar Dual-Cam Mechanism

One such linkage was proposed by CHEW [8] in which the valve timing is varied by changing the position of the common or shared cam-contact. Figure 7.1 shows this particular mechanism. The analysis of this mechanism begins with the detailed description of each link and it's function in the linkage. After the analysis of each link, the information derived in Chapter Six will then be applied to this mechanism.

7.3.1 Analysis of the Five-Bar Dual-Cam Mechanism

The following descriptions detail each of the individual links in the mechanism, as to their purpose and their interaction with each of the other links as shown in Fig. 7.1.

Link #1 - This link is the follower link and produces the output motion of the mechanism. The follower moves along a sliding joint with the ground link. The follower links receives its' input from the movement of link #4.

Link #2 - Link #2 is referred to as the driving cam (input link). This link rotates 360 degrees and provides the primary source of input for the mechanism.

Link #3 - This link is known as the slider link and provides the second source of input. As the slider link moves along the ground link the position of link #4 is changed.

Link #4 - As shown in the Fig 7.1, the position of this link relative to link #2 causes the input from the driving cam (link #2) to vary the position and motion of the follower (link #1). The position of this link changes as the position of the slider link (Link #4) moves back and forth.

Link #5 - This link is the ground link. Link # 1 is joined to this link via a sliding joint. Link #2 is also pinned to the ground link.

The analysis of the Five-Bar variable valve timing mechanism continues with the application of Graph theory to this mechanism. After labeling each of the links, the mechanism was converted into graph form. Figure 7.1(b) illustrates the Graph used to describe this mechanism. Using the colored-graphs of Chapter Six (Fig. 6.1), we note that this mechanism is similar to Graph #1-2.

Using the subgraphs described in Chapter 5, we see that this particular mechanism contains a Type I subgraph. Tables 6.1 through 6.3 can be used to determine what the configuration of the schematic should look like. There are three possible schematics drawn for this mechanism in Fig. 6.2. One of the schematics is shown in Fig. 7.1(c).

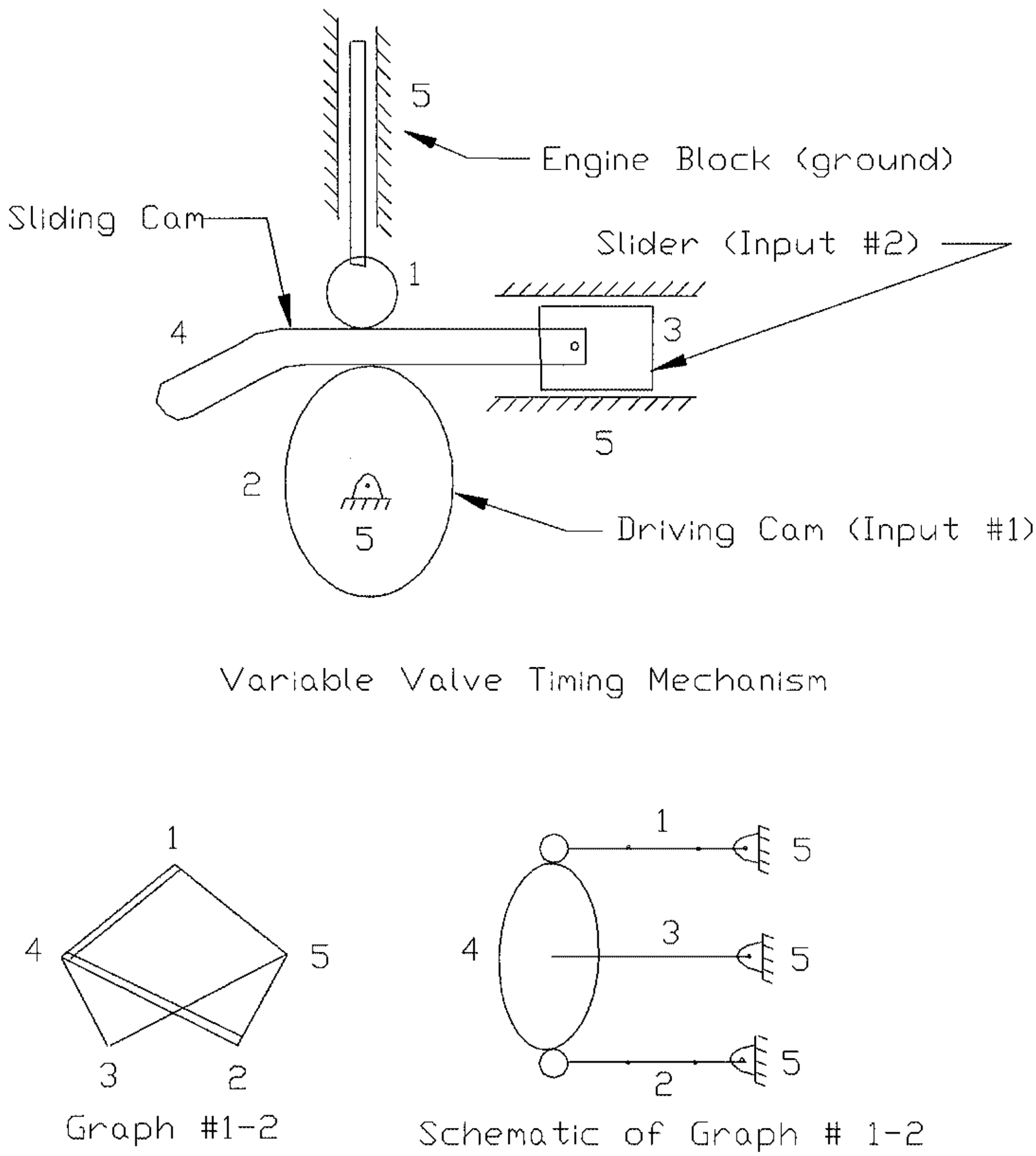


Figure 7.1 Variable Valve Timing Mechanism

7.4 The Sunroof Mechanism

Another application of dual-cam mechanisms is found a particular design of automobile sunroofs. In this design, one input is used to generate two different types of motion. The first type of motion is the retraction of the sunroof. During this motion, the sunroof slides along a track in the roof until the opening in the roof of the vehicle is completely open. The second motion causes the rear end of the sunroof to tilt open approximately one to two inches while the front of the sunroof stay stationary. Both motions are generated through the use of a single motor supplying a single input to the mechanism. The mechanism is illustrated in Fig. 7.2(a). This particular mechanism consists of six links. Two of the links are cams contact pairs. The total degree-of-freedom for the mechanism is one.

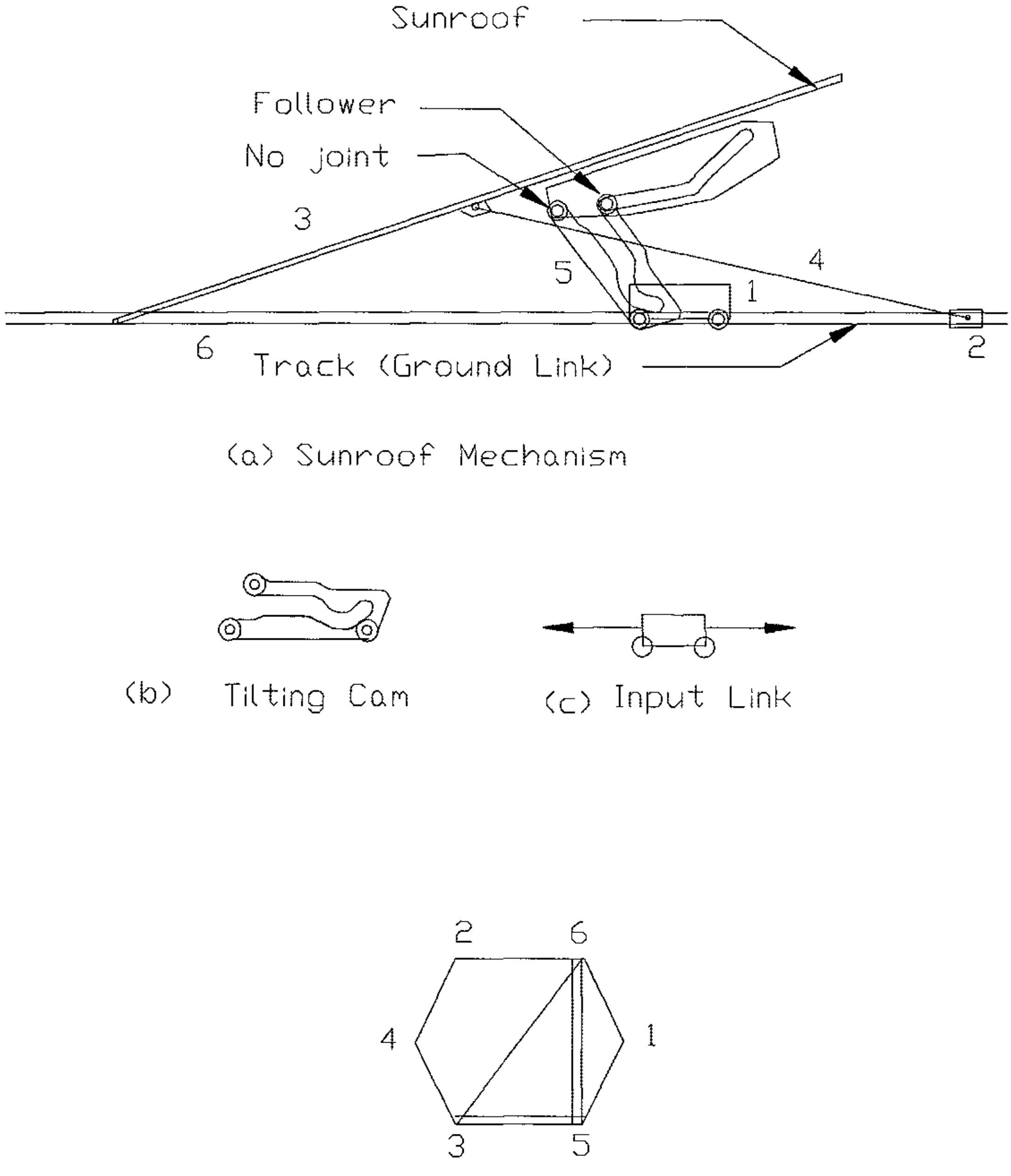


Figure 7.2 Sunroof Mechanism with Graph

In order to describe the mechanism in detail, each link is discussed separately so that one may understand the motion of each link and the interaction with the other links.

Link #1 - Link #1 (Fig.7.2(c)) is the input link who's motion is restricted by the track it slides along (Link #6). A cable is attached from this link to the motor. As the motor winds and unwinds the cable, Link #1 moves along the track and provides the input for the mechanism.

Link #2 - This link is a slider link which moves along the ground link. This link is connected to Link #4 via a pin joint

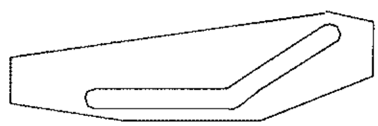
Link #3 - Referring to Fig. 7.2(a), the sunroof lid is shown as link #3. This link is a ternary link with a joints connecting it to the ground link (track), a pin joint connecting it to Link #4, and a cam-contact-pair joint with Link #5.

Link #4 - This link provides stability to the sunroof lid (Link #3) when the sunroof is in the upward tilted position. Otherwise, this link helps the sunroof lid to maintain alignment when the lid is retracting.

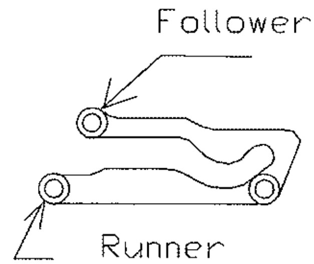
Link #5 - Link #5 is referred to as the tilting cam. This cam is the key to the operation of the entire mechanism and is shown in Fig. 7.2(b). It converts the input provided by link #1 into the tilting motion of the sunroof or pulls the sunroof back into the roof of the vehicle.

Link #6 - This link is the ground link and is the track in which the Links #1, #2 and #3 slide along. There is a stop pin mounted onto the track (ground link) such that the pin acts as a follower when the tilting cam moves into the proper position along the track.

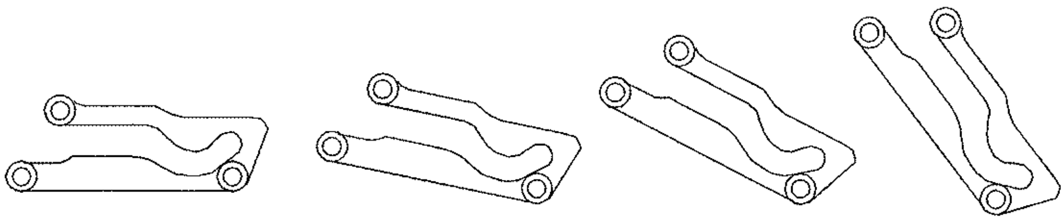
As with the linkage discussed in section 7.3, This mechanism was also converted into graph form. Using the colored-graphs of Chapter 5, The mechanism was not found in the atlas of graphs. The mechanism as described contains a total of nine joints. The mechanisms of Chapter Five considered only eight joints with two cam-contact-pairs. The Sunroof mechanism maintains only eight joints while the sunroof is retracting. However, as the input link drives the tilting cam towards the front of the vehicle, the tilting cam engages with the protruding pin mounted on the track (Link #6). This pin acts as a follower and forces the cam to tilt in an upward position. Figure 7.3 shows the interaction of the tilting cam with the stationary cam as the tilting cam engages with the pin mounted on Link #1. Figure 7.3(a) shows the cam mounted under the sunroof lid. The tilting cam is detailed in Fig 7.3(c). Figures 7.3(c) through Fig. 7.3(f) depict the motion of the cam as it tilts into position and forces the sunroof lid to tilt open.



(a) Cam mounted
in sunroof

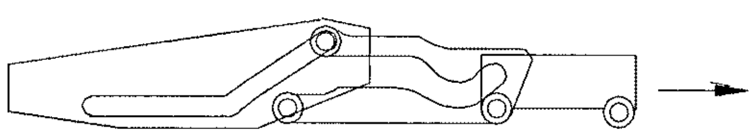


(b) Tilting Cam

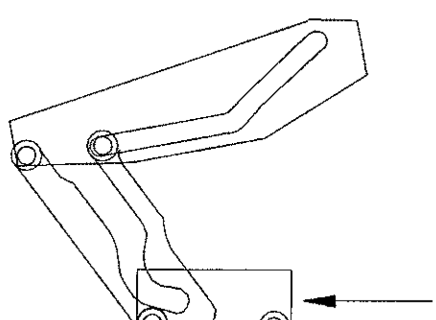


(c) (d) (e) (f)

Movement of Tilting Cam
as Sunroof Tilts to Open Position



(g) Position of Tilting Cam When Sunroof is Retracking



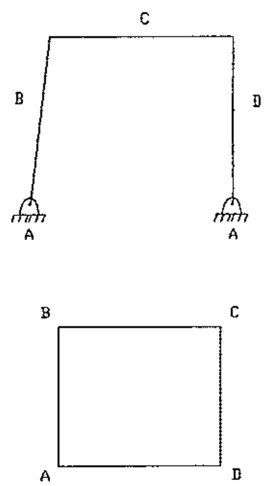
(h) Position of Tilting Cam When Sunroof is Tilted Open

Figure 7.3 Movement and Position of Tilting Cam

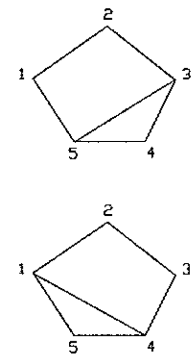
Figure 7.3(g) shows the relative position of the tilting cam as the input link pulls the sunroof along the track. The pin mounted to the track hits the lower surface of the tilting cam and forces cam in a downward motion. This causes the sunroof to tilt downward until it is in the position to slide along the track. While this mechanism may seem complicated, it illustrates the usefulness of dual-cam-modulated mechanisms. The schematics shown in Chapters 5 and 6 should aid the designer when considering alternate configurations of linkages.

7.5 Concluding Remarks

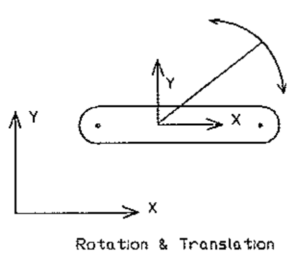
This investigation has proposed an alternative method to enumerating mechanisms structurally using Graph Theory. Through the use of a bit mapping algorithm, all possible non-isomorphic graphs were enumerated. Each of the colored-graphs were then expanded into schematics of mechanisms. Finally several applications of dual-cam-contact mechanisms were shown in order to illustrate that the enumeration scheme and tables of classified graphs (mechanisms) can be used by the designer to either verify the existence of isomorphism or generate mechanism from the tables and schematics. The selection of mechanisms based on the characteristics of the subgraphs shown in Chapter 5 and 6 could be added to an engineering database. Once added to a database, Artificial Intelligence could be used to aid the designer in the selection of mechanisms based on the subgraphs, the number of joints, the number of links and the number of degrees-of-freedom.



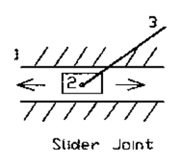
(a) Four-Bar Mechanism with Graph Representation



(b) Isomorphic Graphs



(c) Link with Three Degrees-of-Freedom



(d) Slider joint One Degree-of-Freedom

Figure 2.1 Graphs, Isomorphism and Degrees-of-Freedom

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