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A Simulation Study of Convergence Speed for Distributed Codeword Adaptation Algorithms in CDMA Wireless Systems

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A SIMULATION STUDY OF CONVERGENCE SPEED FOR DISTRIBUTED CODEWORD ADAPTATION ALGORITHMS IN CDMA WIRELESS SYSTEMS

by

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ABSTRACT

A SIMULATION STUDY OF CONVERGENCE SPEED FOR DISTRIBUTED CODEWORD ADAPTATION ALGORITHMS IN CDMA WIRELESS SYSTEMS

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In this thesis we present a side-by-side comparison of interference avoidance (IA) algorithms for distributed codeword adaptation in Code Division Multiple Access (CDMA) systems. In CDMA systems, the interference is determined by the values of the cross-correlation of codewords assigned to users, and various algorithms can be used for codeword optimization. The IA algorithms for codeword adaptation considered are the eigen-algorithm, the Minimum Mean Square Error (MMSE) update, and the adaptive IA algorithm, for which we investigate convergence speed using the extensive simulations of several uplink CDMA system scenarios. The results of this thesis were presented at the Fourth IEEE Radio and Wireless Symposium (RWS), San Diego, CA in January 2009.

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CHAPTER I

INTRODUCTION

Interference from both natural sources and other users of the medium has always been the problem to be solved by wireless communication designers [5]. In wireless systems, the transmission medium is shared by all users. Thus, the amount of interference cannot be limited by restricting physical access to the medium and/or reducing relative noise energy. Such restrictions can be applied for wired or optical systems, but in radio communication systems the restrictions on use are legislative in nature and imposed by government regulating agencies such as the Federal Communications Commission (FCC) [5] in the United States and its counterparts in other countries. Hence, mutual interference is a fact of wireless communication systems.

Code Division Multiple Access (CDMA) has been adopted as the major multiple access technique in the third generation (3G) wireless systems and is also proposed for use in future generation wireless communication systems. CDMA enables multiuser communications along with efficient utilization of available spectrum and transmitter power in wireless systems [2, 17, 25]. In a CDMA system, the transmitters can use all available spectrum all of the time unlike Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA). In TDMA, the transmitters are restricted to using the available spectrum part of the time and in FDMA, the transmitters are restricted to using part of the spectrum all of the time [2]. Access to air interface in CDMA systems is controlled by distinct codewords or spreading sequences that are assigned to active users to the system [2, 23]. A major advantage of CDMA is that a large number of users can be

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This thesis follows the IEEE style.

accommodated if each transmits messages for a short period of time. In such a multiple access system, it is relatively easy either to add new users or to decrease the number of users without disrupting the system. Spread spectrum is always used to accomplish CDMA. All user data and most implementations of the controlled channel and signaling information with CDMA are transmitted at the same frequency at the same time. The CDMA system operates asynchronously, which means that the transition times of a user's data symbols do not have to coincide with those of other users.

In general, CDMA system users transmit signature waveforms that are linear superposition of the signal space basis functions. The basis function might be non-overlapping pulses (or "chips") in a typical CDMA system [2] and in another system, it might be "tones" of different frequencies as in multicarrier and Orthogonal Frequency Division Multiplexing (OFDM) systems or even spatial signal distributions. However, the signals coming from different users are, in general, not orthogonal, and the CDMA system is an interference-limited system. In addition, fading and dispersive radio channels further distort transmitted signals by introducing multipath interference.

In order to minimize both the multiple access interference (MAI) and multipath interference (MI), and ensure that the specified quality of service (QoS) requirement is met, the transmitter in a CDMA system may adjust its codewords. IA is a class of methods proposed recently in the literature for adaptation of CDMA codewords through various distributed algorithms such as the eigen-algorithm [5], the MMSE algorithm [10,26], and the adaptive IA algorithm [8]. We note that, while it has been shown that all these algorithms converge to an optimal ensemble of codewords where desired QoS requirements are satisfied with minimum transmitter power, a side by side comparison of these algorithms in terms of convergence speed for similar operating scenarios has not been performed yet.

The relationship between codeword assignment in a CDMA system and performance metrics such as sum capacity (C_s) and general square correlation (GSC) have been studied in several papers [5, 28–30]. These performance metrics are used in conjunction with IA algorithms to obtain optimal codeword ensembles in a finite number of steps for codeword adaptation. The main idea behind the interference avoidance algorithms is to maximize the SINR (Signal-to-Interference plus Noise-Ratio) at the receiver through adaptation of codewords. All of the interference avoidance algorithms replace the current codeword to reduce or minimize interference from other sources, including other users, and is performed at the base station. These codeword updates monotonically decrease GSC and increase C_s and coupled with the fact that these measures are bounded, this implies the convergence of the algorithms [15, 19]. The goal of this thesis is to compare the convergence speed of three algorithms (the eigen-algorithm, the MMSE update, and the adaptive IA algorithm) proposed for distributed codeword adaptation in CDMA wireless systems.

The rest of the thesis is organized as follows: Chapter II presents the system model and formally states the problem. In Chapter III, we discuss different distributed interference avoidance algorithms used in codeword adaptation for the system under consideration. We illustrate the algorithm with numerical examples obtained from the extensive simulations in Chapter IV, and conclude with final remarks and future work in Chapter V.

The work included in this thesis was presented at the Fourth IEEE Radio and Wireless Symposium (RWS) Conference, San Diego, CA in January 2009 [11].

CHAPTER II

SYSTEM MODEL AND PROBLEM STATEMENT

We consider the uplink of a single cell synchronous CDMA wireless system with L users in a signal space of dimension N communicating with common base station [5] and ideal channels. This type of study has never been performed before. The IA-based transmitter adaptation uses information about the interference corrupting the desired signal at the receiver, which is acquired over a feedback channel, and the transmitter is updated in response to changing patterns of interference. The amount of interference information that the receiver may feed back to the transmitter is limited by the capacity of the feedback channel. The N dimensional received signal at the base station is

$$\mathbf{r} = \sum_{\ell=1}^{L} b_\ell \sqrt{p_\ell} \mathbf{s}_\ell + \mathbf{n} = \mathbf{S}\mathbf{P}^{1/2}\mathbf{b} + \mathbf{n} \tag{1}$$

where :

- $\mathbf{S}$ is $N \times L$ codeword matrix having the column $\mathbf{s}_\ell$ as unit norm user codeword corresponding to user ℓ,

$$\mathbf{S} = \begin{bmatrix} | & & | & & | \\ \mathbf{s}_1 & \ldots & \mathbf{s}_\ell & \ldots & \mathbf{s}_L \\ | & & | & & | \end{bmatrix} \tag{2}$$

- $\mathbf{b} = [b_1, \ldots, b_L]^\top$ is the vector containing the information symbols sent by users.
- $\mathbf{P} = \text{diag}\{p_1, \ldots, p_L\}$ is the power matrix containing the received power at the base station.

- $\mathbf{n}$ is an Additive White Gaussian Noise (AWGN) that corrupts the signal at the receiver.

The correlation matrix of the received signal is

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \mathbf{S}\mathbf{P}\mathbf{S}^\top + \mathbf{W}, \tag{3}$$

where $\mathbf{W} = E[\mathbf{n}\mathbf{n}^\top]$ is the noise covariance matrix.

The multiple-access interference (MAI) is a major limitation for this system's performance, which is caused by the correlation among the users of signatures, $\mathbf{s}_\ell$, $k = 1, 2, ..., L$.

The amount of MAI depends on the number of users, L, relative to length N of the user signature sequences [21, 30]. If $L \leq N$ then orthogonal waveforms can be chosen, eliminating the MAI completely and thereby achieving the performance of a single-user system. The more interesting situation is when $L > N$, when the system is overloaded or oversaturated and when one inevitably encounters MAI.

A unit norm receiver filter [5], $\mathbf{c}_k$, is used to estimate the symbol transmitted by a given user k. This estimate is computed as

$$\hat{\mathbf{b}}_k = \mathbf{c}_k^\top \mathbf{r} = \underbrace{b_k \sqrt{p_k} \mathbf{c}_k^\top \mathbf{s}_k}_{\text{desired signal}} + \underbrace{\mathbf{c}_k^\top \left(\sum_{\ell=1, \ell \neq k}^{L} b_\ell \sqrt{p_\ell} \mathbf{s}_\ell + \mathbf{n} \right)}_{\text{interference + noise}} \tag{4}$$

In this case, the SINR for a given user k can be defined as the ratio of the desired signal corresponding to user k at the receiver to the power of interference and noise that affects user k's signal at the receiver. The expression for the signal-to-interference plus noise-ratio (SINR) for user k can be written as

$$\gamma_k = \frac{p_k (\mathbf{c}_k^\top \mathbf{s}_k)^2}{\sum_{\ell=1, \ell \neq k}^{L} p_\ell (\mathbf{c}_k^\top \mathbf{s}_\ell)^2 + E[(\mathbf{c}_k^\top \mathbf{n})^2]} = \frac{p_k (\mathbf{c}_k^\top \mathbf{s}_k)^2}{\mathbf{c}_k^\top \mathbf{R}_k \mathbf{c}_k}, \tag{5}$$

where $\mathbf{c}_k$ is the unit norm receiver filter used to decode the symbol transmitted by user k and $\mathbf{R}_k$ is the correlation matrix of the interference plus noise seen by user k having the expression

$$\mathbf{R}_k = \sum_{\ell=1,\ell\neq k}^{L} p_\ell \mathbf{s}_\ell \mathbf{s}_\ell^\top + \mathbf{W} = \mathbf{R} - p_k \mathbf{s}_k \mathbf{s}_k^\top \tag{6}$$

When codewords are orthogonal, ($\mathbf{s}_k^\top \mathbf{s}_k = 0$) for all the values of k and ℓ and matched filter receivers are used $\mathbf{c}_k = \mathbf{s}_k$ for all the values of k, equation (5) then shows that interference equivalent to $(\mathbf{c}_k^\top \mathbf{s}_\ell) = 0$ for all the values of ℓ.

Using signal to interference ratio as a metric, we present a class of distributed algorithms for synchronous systems that result in an ensemble of optimal waveforms meeting the Welch Bound with equality and therefore achieve minimum average interference over the ensemble of signature waveforms.

The performance metric known as General Square Correlation,(GSC) is defined as

$$GSC = \mathrm{T}race[\mathbf{R}^2] = \mathrm{T}race[(\mathbf{SPS}^\top + \mathbf{W})^2] \tag{7}$$

Since the SINR is inversely proportional to R, and from equations (3), (5) and (6), it can be seen clearly that the small value of GSC leads to larger SINR, it is to be noted that GSC is lower bounded by $\frac{L^2}{N}$ when $L \geq N$ and by L when $L \leq N$. The result obtained was first derived by Welch in [31]. Hence, the codeword ensembles satisfying these bounds are therefore called Welch Bound Equality sequences, or WBE sequences for short [12, 31].By refering to equation (7), we can say that the resulting codeword ensembles satisfying the Welch Bound Equality (WBE) sequences are "tuned" to the particular noise structure specified by covariance matrix $\mathbf{W}$. When the noise matrix is white with covariance matrix $\mathbf{W} = \sigma^2 \mathbf{I}_N$ then the Welch Bound Equality sequences satisfy [31]

$$\mathbf{SPS}^\top = \frac{L}{N}\mathbf{I}_N \tag{8}$$

An alternative performance metric known as sum capacity is defined as the maximum sum of reliable error free rates over all users while another performance metric named user capacity of a CDMA system is defined as the maximum number of admissible users at a given common target SINR γ^* [5]. L users are said to be admissible if there exists powers $p_\ell > 0$ and signature sequences $\mathbf{s}_\ell$ such that each user has an SINR at least as large as γ^*. It can be expressed [16, 19, 20] as

$$\mathrm{C}_s = \frac{1}{2}\log|\mathbf{R}| - \frac{1}{2}\log|\mathbf{W}| \tag{9}$$

Clearly, user capacity depends on the performance criterion γ^* [5]. Therefore, it is necessary to examine the codeword ensembles in terms of objective measures such as information theoretic capacity. The sum capacity is achieved by an ensemble $\mathbf{S}$ when codewords are chosen in such a way that they meet the Welch Bound with equality. Hence, we can say that GSC is a measure of user capacity, but it can be used as a measure of sum capacity in many cases.

Both the $\mathrm{T}race[\mathbf{R}^2]$ and sum capacity depend on the eigenvalues of the received signal covariance matrix $\mathbf{R}$. It is observed that $\mathrm{T}race[\mathbf{R}^2]$ is a convex function in these eigenvalues and sum capacity is a concave function [1, 3, 5]. It can also be shown that both $\mathrm{T}race[\mathbf{R}^2]$ and sum capacity are optimized when identical bounds on these eigenvalues are met with equality. The optimal points that maximize sum capacity and minimize $\mathrm{T}race[\mathbf{R}^2]$ are identical using the constrained optimization methods or the results from the majorization theory. Hence, the minimization of $\mathrm{T}race[\mathbf{R}^2]$ is equivalent to the maximization of sum capacity.

II.1 UNDERLOADED AND OVERLOADED SYSTEM

The ratio $\frac{L}{N}$ in a CDMA system is referred as the load factor of the system. The load factor plays a significant role in the study of convergence speed for the distributed algorithms in CDMA wireless systems. When the load factor is less than or equal to 1, the system is called an underloaded system; when it is larger than 1 the system is called an overloaded system. In the underloaded system, users may be assigned orthogonal codewords such that they will not interfere with each other, and codeword adaptation is not necessary. In the overloaded system, the users will adapt their codewords using IA algorithms until an optimal ensemble is reached where the total interference in the system is minimized, and whenever there is a change in the system configuration the IA algorithms should be reiterated to adjust codewords to a new optimal ensemble for the new system configuration. It is shown that the user capacity is maximized when the codewords are either orthonormal or when there are fewer users than signal space dimensions or when they form WBE sequence sets or when the system is overloaded.

In the case of an overloaded system ($L > N$), a set of WBE user codewords and powers with oversized users for which the sum of allocated powers among all valid power allocations for the given SINRs is minimum [8,28,30]. A user is said to be oversized if the effective bandwidth implied by its target SINR is large relative to the effective bandwidths implied by the other user's target SINR. No user is oversized for every user j if and only if the user follows the equation given below:

$$\frac{1}{N}\sum_{\ell=1}^{L} p_\ell \geq p_j \tag{10}$$

When all the input power constraints are equal, no user is oversized, and if a user ℓ is oversized then every user with input power constraint at least p_ℓ is also oversized. Also, there can be at most N - 1 oversized users in the system.

Various algorithms available for distributed codeword adaptation in uplink CDMA systems do not have the same performance under a similar CDMA setup. Therefore, it is worthwhile to compare the performance of the available algorithms to identify a suitable one for a given system. The assumption made for this thesis is that there is no change in users' codewords during the codeword replacement. The codeword adaptation algorithms replace each user's codeword with a new one satisfying the algorithms' requirement [4, 6]. This replacement is done sequentially after all the users have updated their codewords. The codeword ensembles obtained by the application of the eigen algorithm, MMSE algorithm and adaptive algorithm form WBE sets. The main positive side of using the WBE sequences in a CDMA wireless system for users (codewords) is the use of matched filters as optimal linear receivers that minimize the mean squared error (MSE) for each user. Therefore, the results of the matched filters satisfy the sufficient statistics for joint processing of the users and thus achieve the maximum sum capacity. It is to be noted that when the number of users L is less than or equal to the signal space dimension N, the three algorithms (eigen, MMSE and adaptive) produce a set of orthonormal codewords.

$$\mathbf{S}^{\top}\mathbf{S} = \mathbf{I}_L, \quad \text{if} \quad L \leq N \tag{11}$$

For MMSE and an adaptive algorithm, it may take several iterations whereas the eigen algorithm needs a single pass through all codewords as each user chooses a new codeword (eigenvector) orthogonal to the previous codewords [5, 12]. The most important assumption used throughout this thesis is $L > N$. Otherwise, it is observed that the SINR is equal to the signal-to-noise ratio (SNR) since all the resulting optimal signature sequences are orthogonal; thus, the given user interference from the other user is zero.

II.2 SUMMARY

In this chapter, the system model of the uplink synchronous CDMA system is discussed in brief. The concept of load ratio is also introduced for both the overloaded system and underloaded system, which are the equivalent performance criteria. The IA procedure is discussed in which each user will maximize its own signal-to-interference + noise ratio through the adaptation of user codewords. This procedure also implies maximizing the sum capacity as well as minimizing the GSC.

CHAPTER III

DISTRIBUTED CODEWORD ADAPTATION ALGORITHMS

In this chapter, we introduce the algorithms for distributed codeword adaptation based on repeated application of the interference avoidance procedure [5, 8, 26]. In distributed implementation of the algorithms, users update their corresponding codewords individually using only correlation matrix $\mathbf{R}_k$ of the interference plus noise given in equation (6). It is assumed that the correlation matrix $\mathbf{R}$ of the received signal is made available through a feedback channel to all users and they are not required to have complete knowledge about all the other active users in the system [7, 24].

The IA algorithm is an adaptive modulation technique in which the transmitters in wireless communication systems are optimized through the adaptation of individual user codewords; hence, interference is minimized. Thus, it results in a better environment to operate. The idea behind the implementation of the proposed IA algorithms is to maximize the performance metric SINR through the adaptation of user codewords.

III.1 THE EIGEN-ALGORITHM FOR IA

For the eigen algorithm, assuming simple matched filters at the receiver for all users, that is $\mathbf{c}_k = \mathbf{s}_k$, the SINR for user k can be written as

$$\gamma_k = \frac{p_k}{\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k} \tag{12}$$

The denominator of the SINR is the user interference function [5]. The interference function depends on user k's codeword since the receiver filter $\mathbf{c}_k$ depends usually on $\mathbf{s}_k$ and does not depend on user k's power. In this eigen algorithm, the interference function can be expressed as

$$\mathrm{i}_k = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \tag{13}$$

Thus, equation (12) can be used in order to maximize the SINR through the adaptation of user codewords, which is equivalent to minimizing the inverse SINR. The inverse SINR can be defined as,

$$\beta_k = \frac{1}{\gamma_k} \tag{14}$$

A matched filter performs single user detection and can be used for decoding user $\mathbf{R}_k$. It acts as the optimal filter in the presence of AWGN and maximizes the signal-to-noise ratio. The eigen algorithm yields codeword ensembles that minimize the trace of the square of the received signal covariance matrix $\mathbf{R}$.

The main idea behind the implementation of IA algorithms is to maximize the SINR (considering SINR as a metric) through the adaptation of user codewords. It is to be noted that the eigen algorithm can be used to observe the equivalence between the minimization of $Trace[\mathbf{R}^2]$ and maximization of the sum capacity. As discussed in Chapter II, both the $Trace[\mathbf{R}^2]$ and sum capacity depend on the eigenvalues of the received signal covariance matrix $\mathbf{R}$. While $Trace[\mathbf{R}^2]$ is a convex function in these eigenvalues and sum capacity is a concave function, the equation (6) can be rewritten as

$$\mathbf{R} = \mathbf{R}_k + p_k \mathbf{s}_k \mathbf{s}_k^\top \tag{15}$$

The eigen interference avoidance algorithm consists of changing a user codeword by replacing codeword $\mathbf{s}_k$ (in a distributed manner) with the minimum eigenvector of the correlation matrix $\mathbf{R}_k$ [20] such that the user SINR, γ_k is maximized. The SINR of user k can be greedily maximized by replacing the current codeword $\mathbf{s}_k$ with the minimum eigenvector of the correlation matrix $\mathbf{R}_k$ of the interference + noise seen by user k. Replacing the current codeword with the minimum eigenvector of the interference + noise correlation matrix seen by the individual user avoids interference, placing its transmitted energy in that region of signal space with minimum interference + noise energy and greedily maximizes SINR without any negative effects on the other users in the system [5, 18, 22]. Thus, the sequential application of this greedy procedure by all users defines the minimum eigenvector algorithm for interference avoidance.

The Eigen Algorithm

1. Initialize the user codeword and power matrices $\mathbf{S}$, $\mathbf{P}$, and the noise covariance matrix $\mathbf{W}$.
2. For $k = 1, \ldots, L$
   (a) Compute the correlation matrix $\mathbf{R}_k$ of the interference-plus-noise seen by user k using equation (6).
   (b) Replace the current user codeword by the minimum eigenvector of $\mathbf{R}_k$.
3. Repeat Step 2 until a fixed point is reached

When user k replaces its codeword $\mathbf{s}_k$ with a new codeword $\mathbf{x}$, the difference in GSC can be written as

$$\triangle = Trace[(\mathbf{R}_k + \mathbf{s}_k p_k \mathbf{s}_k^\top)^2] - Trace[(\mathbf{R}_k + \mathbf{x}_k p_k \mathbf{x}_k^\top)^2] \tag{16}$$

After cancelling the like terms and replacing the traces by the corresponding quadratic form, we have

$$\triangle = 2(\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k - \mathbf{x}_k^\top \mathbf{R}_k \mathbf{x}_k) \tag{17}$$

Choosing $\mathbf{x}$ as the minimum eigenvector of $\mathbf{R}_k$, we have

$$\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \geq \mathbf{x}_k^\top \mathbf{R}_k \mathbf{x}_k \tag{18}$$

This equation concludes that $\triangle \geq 0$. Therefore, the eigen algorithm monotonically decreases GSC.

We can emphasize from equation (18) that it is an inequality, and the same logic applies with regard to the eigen algorithm. If $\mathbf{x}_k$ is chosen as the minimum eigenvector of $\mathbf{R}_k$ then $\mathbf{x}_k$ is the maximum eigenvector of $\mathbf{R}_k^{-1}$.

In summary, the eigen algorithm decreases the GSC and increases the sum capacity at each step. As the sum capacity and GSC are bounded from above and below the eigen algorithm must converge to some value. This convergence does not refer to the convergence of codewords, but the convergence only show the iterative procedure must converge GSC and sum capacity.

Numerically, the fixed point is reached when the Euclidian distance between a given codeword and its corresponding replacement is within the tolerance ϵ. The convergence of the eigen algorithm is discussed in [5, 19] where it is shown that it converges to generalized Welch Bound Equality (WBE) sequences maximizing the sum capacity.

When $L \leq N$, the convergence of the eigen algorithm to a set of orthogonal codewords in a single ensemble iteration can be proved in a simple way as follows: The correlation matrix of the interference plus noise seen by user k, $\mathbf{R}_k$, cannot have full rank when $L \leq N$ as the number of available vectors $\mathbf{s}_k$ is at most $N-1$ which means that $\mathbf{R}_k$ is singular and positive semi definite, hence its minimum eigenvalue is zero for all values of $k = 1, ..., L$. The new $\mathbf{s}_k$ is chosen as the eigenvector corresponding to the zero eigenvalue of

$\mathbf{R}_k$ resulting in $\mathbf{R}_k \mathrm{s}_k = 0$. This implies that s_k is orthogonal to s_ℓ for $\ell \neq k$. Thus, we can conclude that if $L \leq N$ then the minimum eigenvalue produces an orthonormal set of vectors after a single pass through all the vectors.

III.2 THE MMSE ALGORITHM FOR IA

This algorithm is proposed in order to mitigate the interference where the current codeword is replaced with the MMSE receiver filter, and the expression of the unit norm MMSE receiver filter for user k is [10, 26]

$$\mathrm{c}_k = \frac{\mathbf{R}_k^{-1} s_k}{(\mathrm{s}_k^\top \mathbf{R}_k^{-2} \mathrm{s}_k)^{1/2}} \tag{19}$$

The SINR for user k in the MMSE receiver filter is expressed as

$$\gamma_k = p_k \mathrm{s}_k^\top \mathbf{R}_k^{-1} \mathrm{s}_k \tag{20}$$

The minimum mean squared error (MMSE) receiver filter is the linear receiver that minimizes the mean squared error (MSE) between the transmitted symbol and the corresponding decision variable. It is also defined as the optimal linear multiuser detector that maximizes the SINR [5].

The interference function for the MMSE receiver can be written as

$$\mathrm{i}_k = \frac{\mathrm{s}_k^\top \mathbf{R}_k^{-1} \mathrm{s}_k}{\mathrm{s}_k^\top \mathbf{R}_k^{-2} \mathrm{s}_k} \tag{21}$$

The SINR for user k can be maximized by replacing the current codeword s_k (in a distributed manner) with the normalized MMSE receiver filter c_k [26].

The MMSE Algorithm

1. Initialize user codeword and power matrices $\mathbf{S}$ and $\mathbf{P}$ and the noise covariance matrix $\mathbf{W}$.

2. For each user $k = 1, \ldots, L$

   (a) Determine its MMSE receiver $\mathbf{c}_k$ using equation (19).

   (b) Replace the current user codeword by $\mathbf{c}_k$.

3. Repeat Step 2 until a fixed point is reached.

For the MMSE update from equation (18), we have

$$\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \geq \mathbf{c}_k^\top \mathbf{R}_k \mathbf{c}_k \tag{22}$$

which is equivalent to

$$\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k \geq \left(\frac{\mathbf{R}_k^{-1} \mathbf{s}_k}{(\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k)^{1/2}} \right)^\top \mathbf{R}_k \left(\frac{\mathbf{R}_k^{-1} s_k}{(\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k)^{1/2}} \right) \tag{23}$$

or

$$\mathbf{s}_k^\top \mathbf{R}_k^{-1} \mathbf{s}_k \leq (\mathbf{s}_k^\top \mathbf{R}_k^{-2} \mathbf{s}_k)(\mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k) \tag{24}$$

Clearly $\mathbf{R}_k$ is the covariance matrix of the interference plus noise seen by user k, and it is symmetric and positive definite. Since it is invertible and $\mathbf{s}_k$ is unit norm, we have

$$||\mathbf{s}_k^2|| = (\mathbf{s}_k^\top \mathbf{R}_k^{-1/2} \mathbf{R}_k^{1/2} \mathbf{s}_k) = 1 \tag{25}$$

Applying the Schwarz inequality, we can write

$$1 = (\mathbf{s}_k^\top \mathbf{R}_k^{-1/2} \mathbf{R}_k^{1/2} \mathbf{s}_k)^2 \leq ||\mathbf{R}_k^{-1/2} \mathbf{s}_k||^2 ||\mathbf{R}_k^{1/2} \mathbf{s}_k||^2 \tag{26}$$

Equivalently, we can write,

$$1 \leq (\mathbf{s}_k^{\top}\mathbf{R}_k^{-1}\mathbf{s}_k)(\mathbf{s}_k^{\top}\mathbf{R}_k\mathbf{s}_k) \tag{27}$$

We can further simplify the Schwarz inequality and express it as

$$(\mathbf{s}_k^{\top}\mathbf{R}_k^{-1}\mathbf{s}_k)^2 \leq ||\mathbf{s}_k||^2||\mathbf{R}_k^{-1}\mathbf{s}_k||^2 = \mathbf{s}_k^{\top}\mathbf{R}_k^{-2}\mathbf{s}_k \tag{28}$$

From equations (27) and (28), we have

$$\mathbf{s}_k^{\top}\mathbf{R}_k^{-1}\mathbf{s}_k \leq (\mathbf{s}_k^{\top}\mathbf{R}_k^{-1}\mathbf{s}_k)^2(\mathbf{s}_k^{\top}\mathbf{R}_k\mathbf{s}_k) \leq (\mathbf{s}_k^{\top}\mathbf{R}_k^{-2}\mathbf{s}_k)(\mathbf{s}_k^{\top}\mathbf{R}_k\mathbf{s}_k) \tag{29}$$

Equation (29) proves that the MMSE algorithm monotonically decreases GSC for its codeword update.

As with the eigen algorithm, the convergence of the MMSE algorithm is guaranteed by the increase of sum capacity and decrease of GSC coupled with the upper and lower bounds of C_s and GSC. The fixed point is reached when the condition as in the eigen algorithm is achieved. As with the eigen algorithm, the MMSE algorithm monotonically increases sum capacity [26,27] and decreases GSC. It is to be noted that the MMSE update algorithm converges to the fixed point.

III.3 THE ADAPTIVE ALGORITHM FOR IA

The adaptive interference avoidance algorithm can be applied in a distributed manner by active users in the CDMA system to obtain an optimal codeword and power for the specified admissible target SINR $\gamma_1{}^*...\gamma_L{}^*$ that satisfy the condition

$$\sum_{k=1}^{L} \frac{\gamma_k{}^*}{1+\gamma_k{}^*} < N \tag{30}$$

The adaptive algorithm provides flexibility to the system [8] as the active users individually adapt their codewords and power when they are admitted to the system such that they optimize a local criterion. The criterion is based on the idea of using little of the feedback information from the base station. It is to be noted that it must be performed at the base station after which the codewords and powers are assigned to users.

The local criteria that we mentioned above is the spectral efficiency, which can be expressed in terms of the user SINR as

$$\eta_k = ln(1 + \gamma_k) \quad [nats/s/Hz] \quad k = 1, ..., K \tag{31}$$

This equation represents the spectral efficiency of a single-user bandlimited AWGN channel. The expression is a reasonable optimization criterion for individual users [8] in the system having access only to their corresponding SINR. The fact to be noted here is the corresponding SINR has no knowledge of the other user SINRs.

The spectral efficiency for user k can be rewritten as a function of its codeword and power when replacing the expression of γ_k it can be written as

$$\eta_k = ln\left(1 + \frac{p_k}{\mathbf{s}_k^{\mathsf{T}} \mathbf{R}_k \mathbf{s}_k}\right) \quad [nats/s/Hz] \quad k = 1, ..., K \tag{32}$$

Hence, it can be observed that each user k will perform joint codeword and power adaptation to maximize its corresponding spectral efficiency with target SINR constraint $\gamma_k = {\gamma_k}^*$ and unit norm constraint on codeword

$$\mathbf{s}_k^{\mathsf{T}} \mathbf{s}_k = 1 \tag{33}$$

The algorithm updates the codeword and power using incremental updates [8] to avoid abrupt changes of the user codeword and/or power which are not desirable in practical implementations. Hence, the receiver is capable of following codeword changes and continues to detect the transmitted symbol accurately. It is assumed that there is no latency associated with a change in the number of active users [29, 31]. The adaptive algorithm adds and drops the users to and from the system as soon as changes occur by updating the codeword and power matrices ($\mathbf{S}$ and $\mathbf{P}$ respectively) as per changes.

The codeword and power updates equations for the adaptive algorithm can be derived by solving the constrained optimization problem based on the maximization of user spectral efficiency in equation (32).

$$\max_{\mathbf{s}_k, p_k} \eta_k \quad \text{subject to} \quad \begin{cases} \dfrac{p_k}{\mathbf{s}_k^{\mathsf{T}} \mathbf{R}_k \mathbf{s}_k} = \gamma_k^* \\ \\ \mathbf{s}_k^{\mathsf{T}} \mathbf{s}_k = 1 \end{cases} \tag{34}$$

User k's Lagrangian function can be defined using the Lagrange multipliers method, which can be expressed as

$$\begin{aligned} L_k(\mathbf{s}_k, p_k, \lambda_k, \xi_k) &= \eta_k(\mathbf{s}_k, p_k) + \lambda_k(\gamma_k - \gamma_k^*) + \xi_k(\mathbf{s}_k^{\mathsf{T}} \mathbf{s}_k - 1) \\ &= \ln\left(1 + \frac{p_k}{\mathbf{s}_k^{\mathsf{T}} \mathbf{R}_k \mathbf{s}_k}\right) + \lambda_k \left(\frac{p_k}{\mathbf{s}_k^{\mathsf{T}} \mathbf{R}_k \mathbf{s}_k} - \gamma_k^*\right) + \xi_k(\mathbf{s}_k^{\mathsf{T}} \mathbf{s}_k - 1) \end{aligned} \tag{35}$$

where λ_k and ξ_k are the Lagrange multipliers associated with user k's constraints in equation (34).

By taking the partial derivatives of equation (34) [8] with respect to the corresponding variables, we get the necessary conditions for maximizing the Lagrangian. We obtain an eigenvalue/eigenvector equation corresponding to matrix $\mathbf{R}_k$ by equating the partial derivative of the Lagrangian with respect to the codeword $\mathbf{s}_k$ to zero.

$$\frac{\partial L_k}{\partial \mathbf{s}_k} = 0 \Rightarrow \mathbf{R}_k \mathbf{s}_k = v_k \mathbf{s}_k \tag{36}$$

where v_k, all user power p_k and codeword $\mathbf{s}_k$ are expressed in terms of Lagrange multipliers. The best choice for user k's codeword satisfying the necessary condition in equation (36) is the eigenvector $\mathbf{x}_k$ corresponding to the minimum eigenvalue/eigenvector $v_k{}^*$ of $\mathbf{R}_k$: for given power p_k it maximizes user k's SINR and implicitly its spectral efficiency by minimizing the effective interference corrupting user k's signal at the receiver.

The incremental update that adapts the codeword in the direction of the minimum eigenvector $\mathbf{x}_k$ can be expressed as

$$\mathbf{s}_k(t+1) = \frac{\mathbf{s}_k(t) + m\beta \mathbf{x}_k(t)}{\| \mathbf{s}_k(t) + m\beta \mathbf{x}_k(t) \|} \tag{37}$$

where :

- $x_k(t)$ is the minimum eigenvector of correlation matrix $\mathbf{R}_k$,
- $m = \text{sgn}(\mathbf{s}_k^\top \mathbf{x}_k)$ and
- β is the parameter that limits the Euclidian distance between the current codeword and new codeword. This is an incremental interference avoidance codeword update, which for given power p_k implies an increase in user k's SINR and implicitly in its spectral efficiency.

Similarly, the power update can be expressed using a gradient-based approach,

$$p_k(t+1) = p_k(t) - \mu[p_k(t) - \gamma^* i_k(t)] \tag{38}$$

with $0 < \mu < 1$, and

$i_k(t) = \mathbf{s}_k^\top \mathbf{R}_k \mathbf{s}_k$ is the interference function.

It is to be noted that the user power will be adapted incrementally to avoid sudden changes in the system. With the increase in spectral efficiency, the adaptation in the direction of the corresponding gradient increases as the Lagrangian is a concave function of user power. Hence, the power update equation can be modified as

$$p_k(t+1) = p_k(t) + \mu_p \left.\frac{\partial L_k}{\partial p_k}\right|_{\mathbf{s}_k=\mathbf{s}_k(t+1)} \tag{39}$$

where $\partial L_k/\partial p_k$ after the user codeword has been updated as specified by equation (37) is

$$\begin{aligned} \left.\frac{\partial L_k}{\partial p_k}\right|_{\mathbf{s}_k=\mathbf{s}_k(t+1)} &= \frac{1}{\mathbf{s}_k(t+1)^\top \mathbf{R}_k(t)\mathbf{s}_k(t+1) + p_k(t)} \\ &+ \frac{\lambda_k(t)}{\mathbf{s}_k(t+1)^\top \mathbf{R}_k(t)\mathbf{s}_k(t+1)} \end{aligned} \tag{40}$$

The Lagrange multiplier $\lambda_k(t)$ is adapted incrementally which can be expressed as

$$\lambda_k(t) = -\mu_\lambda \left.\frac{\partial L_k}{\partial \lambda_k}\right|_{\mathbf{s}_k=\mathbf{s}_k(t+1)} \tag{41}$$

where $\mu_\lambda > 0$ and

$$\left.\frac{\partial L_k}{\partial \lambda_k}\right|_{\mathbf{s}_k=\mathbf{s}_k(t+1)} = \frac{p_k(t)}{\mathbf{s}_k(t+1)^\top \mathbf{R}_k(t)\mathbf{s}_k(t+1)} - \gamma_k^* \tag{42}$$

The Lagrangian L_k is a linear function of λ_k, and the slope is determined by $\partial L_k/\partial \lambda_k$. It is increased by moving λ_k in the corresponding direction indicated by the slope. We note that the update in equation (41) is the steepest ascent gradient update and, it acts as an extra correction factor in the power update equation (39) having more or less influence depending on the closeness between the SINR after codeword adaptation and the target SINR γ_k^*.

$$\gamma_k(t) = \frac{p_k(t)}{\mathbf{s}_k(t+1)^\top \mathbf{R}_k(t)\mathbf{s}_k(t+1)} \tag{43}$$

The Adaptive Algorithm

1. Initialize user codeword and power matrices $\mathbf{S}$ and $\mathbf{P}$, the noise covariance matrix W, the target SINRs $\{\gamma_k^*\}$ and the algorithm constants μ and β.

2. IF the user target SINRs are admissible GO TO step 3. Otherwise STOP.

3. For $k = 1, \ldots, L$

   (a) Compute current correlation matrix $\mathbf{R}_k$ using equation (6) and determine minimum eigenvector $\mathrm{x}_k(t)$ of $\mathbf{R}_k$.

   (b) Update user codeword $\mathrm{s}_k(t)$ using equation (37).

   (c) Update user k's power using equation (38).

4. Repeat Step 3 until a fixed point is reached.

The adaptive IA algorithm converges to a fixed point where Euclidian distance between the codewords of a given user and its corresponding replacements is within some specified tolerance ϵ. The tolerance and speed of convergence of the adaptive algorithm can be adjusted with parameters such as ϵ, β, μ. The algorithm can also be run independently by active users to adapt to changes in the system configuration as reflected by changes in their SINRs and corresponding spectral efficiencies.

The optimal stopping point for the adaptive interference avoidance algorithm is the point where the sum of allocated powers among all valid power allocations for the given target SINRs is minimum and corresponds to generalized Welch Bound Equality (GWBE) sequences. At this fixed point [9], the specified target SINR values are met for all users within the specified tolerance ϵ, and user powers are minimized.

III.4 SUMMARY

The three different distributed codeword algorithms in an adaptive wireless system are clearly described in this chapter. The distributed IA algorithms are presented in which each user will greedily seek to maximize its own signal-to-interference + noise ratio through adaptation of user codewords. This chapter also deals with the distributed algorithm where the users individually adjust codewords and powers to meet a set of specified target SINRs. It is also shown that the algorithm is guaranteed to converge in GSC and/or sum capacity, C_s.

CHAPTER IV

SIMULATION SETUP AND NUMERICAL RESULTS

In order to study the convergence speed for distributed codeword adaptation algorithms in a CDMA wireless system, we consider a CDMA system with L active users in signal space of dimension N and white noise with covariance matrix $\mathbf{W} = 0.1\mathbf{I}_N$. We performed extensive simulations to evaluate the convergence speed of the IA algorithms discussed in Chapter III. We considered a scenario in which we have initially $L = 6$ active users in $N = 5$ signal dimensions with the load factor, $\frac{L}{N} = 1.2$. In each simulation experiment we recorded the number of ensemble iterations needed for convergence within tolerance $\epsilon = 0.001$ in $1,000$ trials for three scenarios: starting from a random set of L user codewords, starting from an optimal WBE set of L user codewords and adding a random codeword to this set and starting from an optimal WBE set of L user codewords and removing one of the codewords from this set. These scenarios correspond to a dynamic wireless system where the IA algorithms are applied to optimize the user codewords from random initializations as well as from optimal ensembles when one user is added to/removed from the system. We vary the different values of the active users and signal dimension for the same load factor 1.2. Similarly, the scenario was considered for the load factor, $\frac{L}{N} = 1.8$ and 2.4 with the varying numbers of active users and signal dimensions. Thus, the load factor was varied from light system overload $(\frac{L}{N}) = 1.2$, to moderate overload $(\frac{L}{N}) = 1.8$ and heavy overload $(\frac{L}{N}) = 2.4$, and the number of signal dimensions was varied from $N = 5$ to $N = 30$. Similarly, the number of users was varied from 6 to 72. In this thesis, we mainly dealt with eighteen different cases. Tables I, II and III give the value of the average ensemble iterations for eighteen different cases with three sub cases including random initialization, adding one user to the system and dropping one user from the system respectively. For the load factor $\frac{L}{N} = 1.2$, the number of users is 6 and the signal space dimension is 5. The nine different histogram plots for three different algorithms has been considered when the load factor $\frac{L}{N} = 1.8$. The histogram plots for the proposed algorithms when the load factor $\frac{L}{N} =$ 1.2 and 2.4 are not shown in this thesis, but the plots can be plotted in a similar way. The nature and properties of these plots are similar to the histogram plots when the load factor $\frac{L}{N} = 1.8$. In the nine different histogram plots, the z-axis represents the performance index named "Occurences/bin" while the y-axis represents the "Number of iterations" and the x-axis represents the "Scenario index". We observe that the value of the average ensemble iterations can be estimated from the histogram plots, and the plots of the ensemble iterations of all algorithms follow the Gaussian distribution with the mean/average value given in their respective table from Tables I, II and III. We notice that there are six "scenario indexes" each representing its own scenario in terms of different active users L and signal dimension N for random initialization, one user added to the system and one user dropped from the system. The rest can be found in a similar manner. These three cases are repeated for eigen, MMSE and adaptive algorithm. The three different tables for three different distributed codeword adaptation algorithms are shown below in Tables I, II and III respectively.

Since Matlab is widely used due to ease of use, setup, debugging and simplicity due to library standardization, the simulation study has been completed in Matlab 7.5.

TABLE I: Convergence Speed of the Eigen Algorithm

Average Iterations						
	N=5	N=10	N=15	N=20	N=25	N=30
$\frac{L}{N} = 1.2$	12 9 10	14 11 11	13 12 10	13 13 9	13 14 9	13 15 8
$\frac{L}{N} = 1.8$	6 7 5	5 8 5	5 6 5	5 6 4	5 6 4	5 6 4
$\frac{L}{N} = 2.4$	5 6 4	5 7 4	5 8 4	5 8 4	5 9 4	5 9 3

IV.1 SIMULATION SETUP AND RESULTS FOR EIGEN ALGORITHM

In Table I, the first row corresponding to each load factor gives the average ensemble iterations when user codewords and powers were initialized randomly; the second row gives the average ensemble iterations when one user is added without increasing N (by appending randomly generated codeword and power for a new user to the corresponding optimal matrices) to the system when the algorithm reached the fixed point; the third row gives the average ensemble iterations when one user is dropped from the system when the algorithm reached the fixed point. We observed the convergence speed of the algorithms given in Table I.

With the help of the data for $1,000$ trials and the average ensemble iteration, we observe the three different plots for the eigen algorithm shown in Figures 1, 2 and 3. Figure 1 shows the convergence speed for random initialization, while Figure 2 shows the convergence speed when one user was added to the system and Figure 3 represents the scenario when one user was dropped from the system for the eigen algorithm.

The codeword and power for previous users were taken from the fixed point and the codeword and power for a newly added user are initialized randomly and appended to the corresponding matrices. We have observed that the eigen algorithm diagonalizes the correlation matrix of the interference plus noise seen by the given user at each step. Later, the given user's codeword is replaced by the minimum eigenvector. The eigen-decomposition has been performed L times separately on an $N \times N$ matrix for one iteration of the algorithm. The system is considered to have L users and N signal dimensional space.

Figures 4, 5 and 6 show the histogram plots for the eigen algorithm for random initialization, one user added to the system and one user dropped from the system respectively.

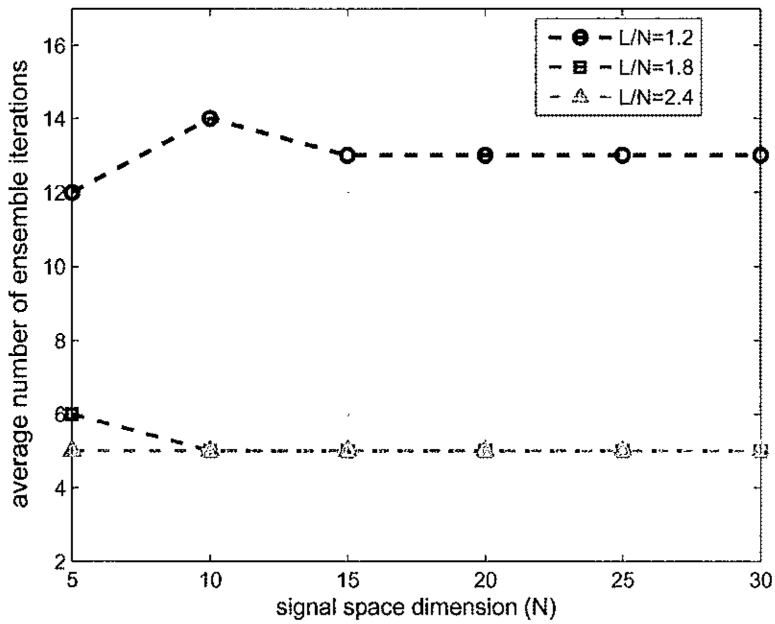

FIG. 1: Convergence speed for the Eigen Algorithm for the random initialization scenario.

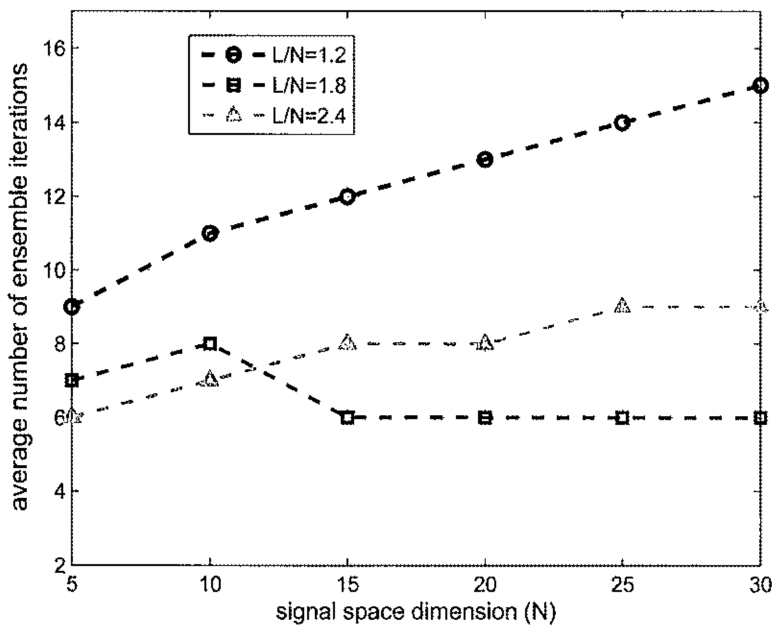

FIG. 2: Convergence speed for the Eigen Algorithm for the one user added to the system scenario.

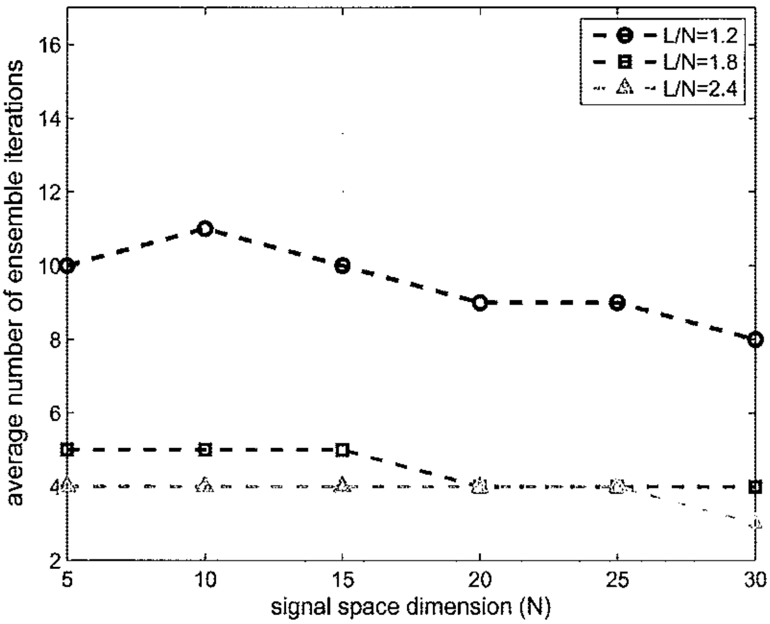

FIG. 3: Convergence speed for the Eigen Algorithm for the one user dropped from the system scenario.

IV.2 SIMULATION SETUP AND RESULTS FOR MMSE ALGORITHM

The first row corresponding to each load factor in Table II gives the average ensemble iterations when user codewords and powers were initialized randomly, and the second row gives the average ensemble iterations when one user is added without increasing N to the system when the algorithm reached the fixed point. Moreover, the third row gives the average ensemble iterations when one user is dropped from the system when the algorithm reached the fixed point. From Table II, we can observe that the average ensemble iterations for random initialization when signal dimension $N = 5$ and active users $L = 6$ is 14. Table II can be referred to the rest of the data with different values of L and N for the load factor

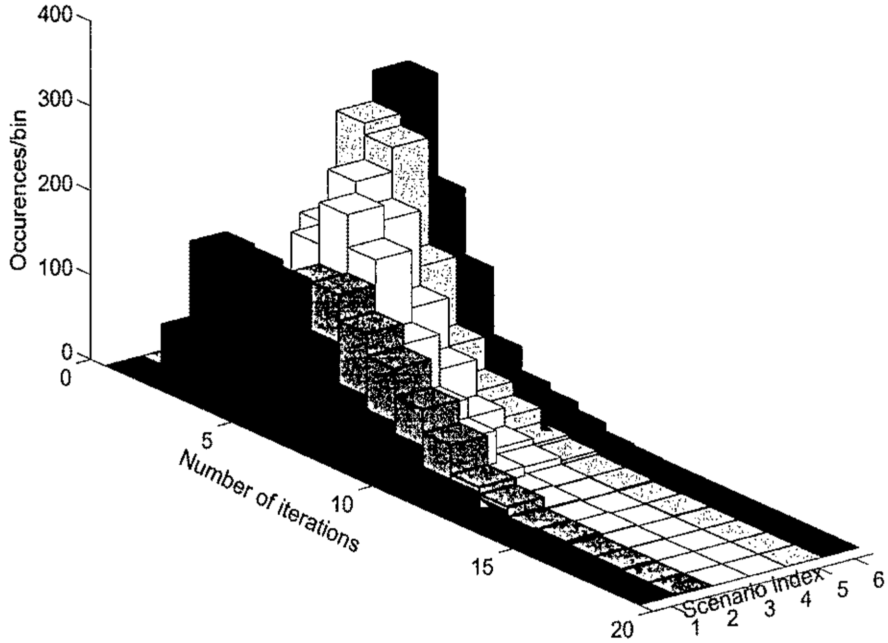

FIG. 4: Histogram Plot for the Eigen Algorithm for random initialization with $\frac{L}{N} = 1.8$.

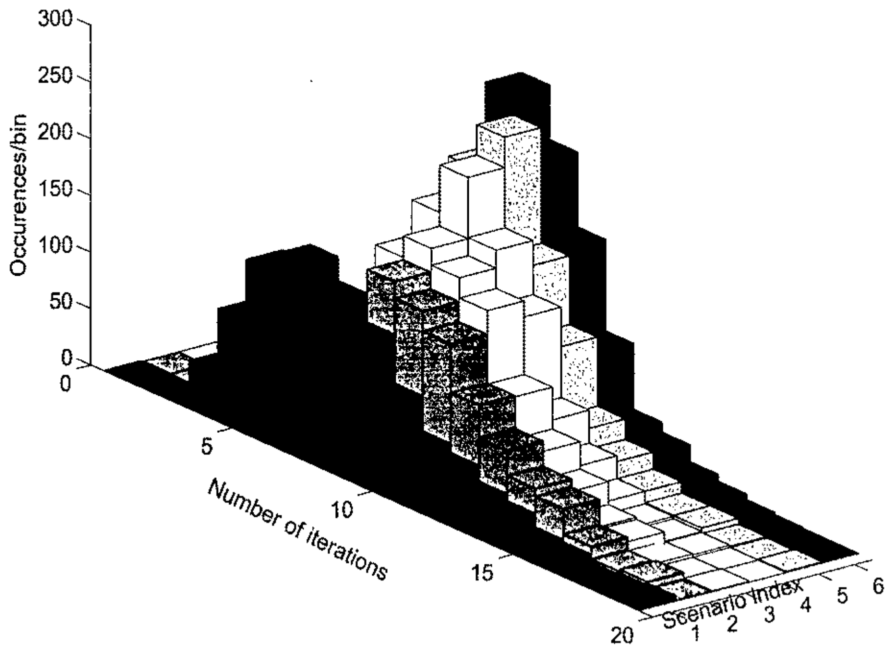

FIG. 5: Histogram Plot for the Eigen Algorithm for the one user added to the system with $\frac{L}{N} = 1.8$.

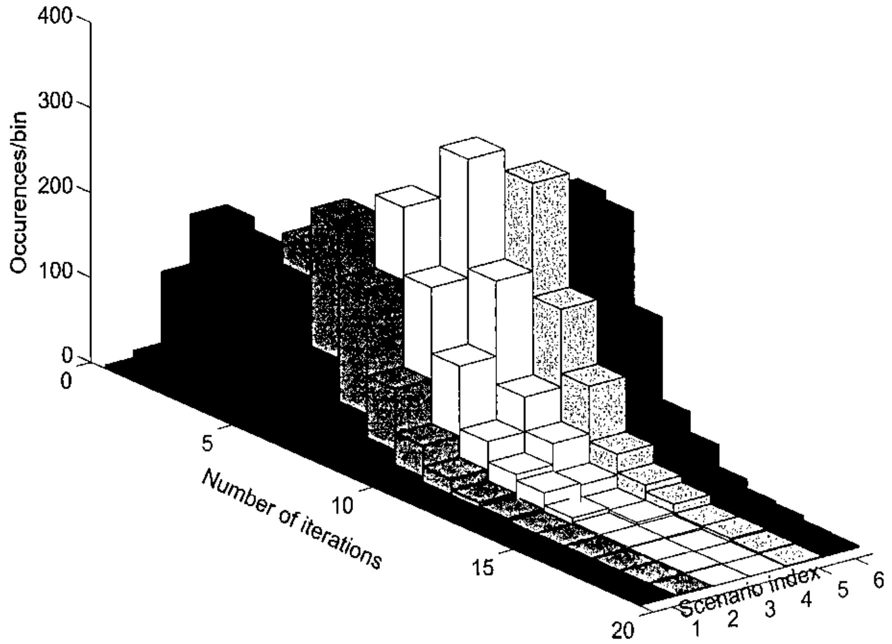

FIG. 6: Histogram Plot for the Eigen Algorithm for the one user dropped from the system with $\frac{L}{N} = 1.8$.

$\frac{L}{N} = 1.2, 1.8$ and 2.4 respectively.

With respect to the data obtained from $1,000$ trials and the corresponding average ensemble iteration value, we observe the three different plots for the MMSE algorithm which is shown in Figures 7, 8 and 9. Figure 7 shows the convergence speed for random initialization, Figure 8 shows the convergence speed when one user was added to the system and Figure 9 represents the scenario when one user was dropped from the system for the MMSE algorithm. Figures 10, 11 and 12 show the histogram plots for the MMSE algorithm respectively for random initialization, one user added to the system and one user dropped from the system.

The codeword and power for previous users were taken from a fixed point, and the codeword and power for newly added users are taken randomly for a new system configuration.

TABLE II: Convergence Speed of the MMSE Algorithm

Average Iterations						
	N=5	N=10	N=15	N=20	N=25	N=30
$\frac{L}{N} = 1.2$	14 10 11	15 11 12	14 13 11	14 14 10	13 15 9	13 16 9
$\frac{L}{N} = 1.8$	9 7 7	7 8 6	7 9 5	7 9 4	6 9 4	6 9 4
$\frac{L}{N} = 2.4$	7 7 6	6 7 5	6 8 4	6 8 4	6 8 4	6 8 4

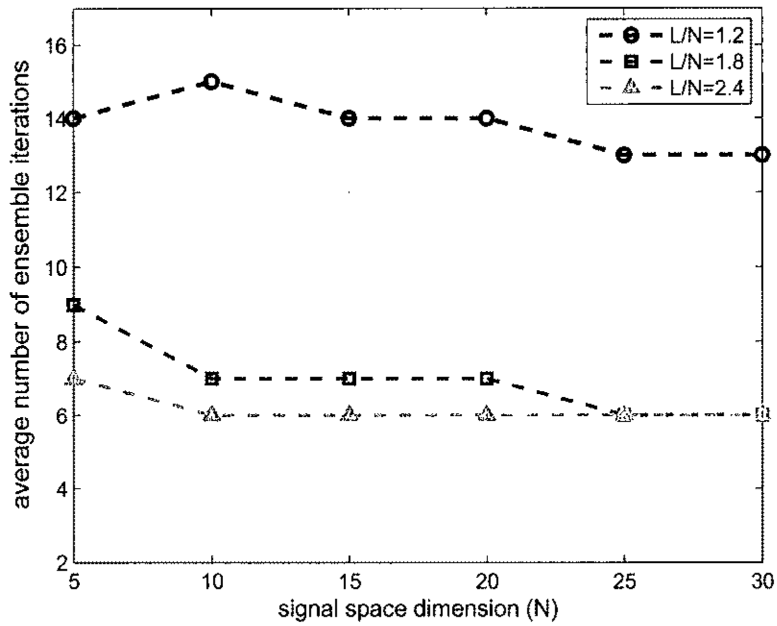

FIG. 7: Convergence speed for the MMSE Algorithm for the random initialization scenario.

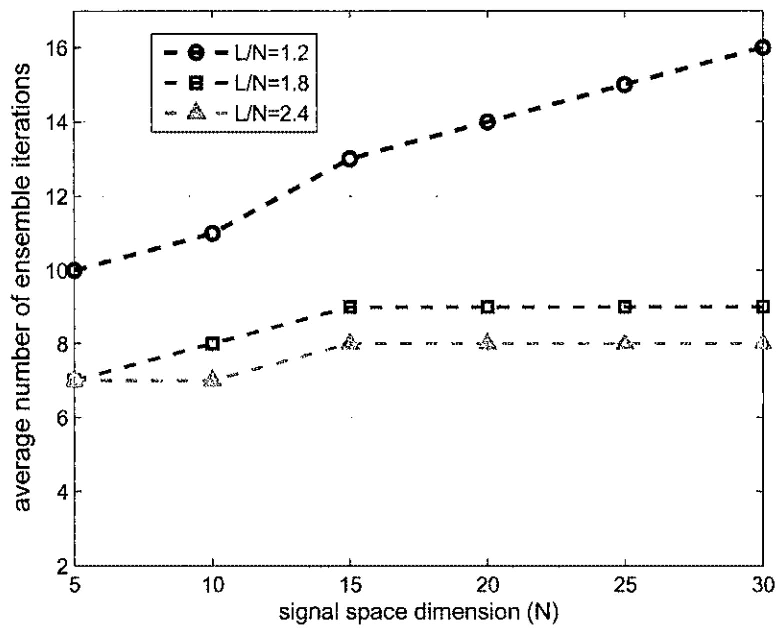

FIG. 8: Convergence speed for the MMSE Algorithm for the one user added to the system scenario.

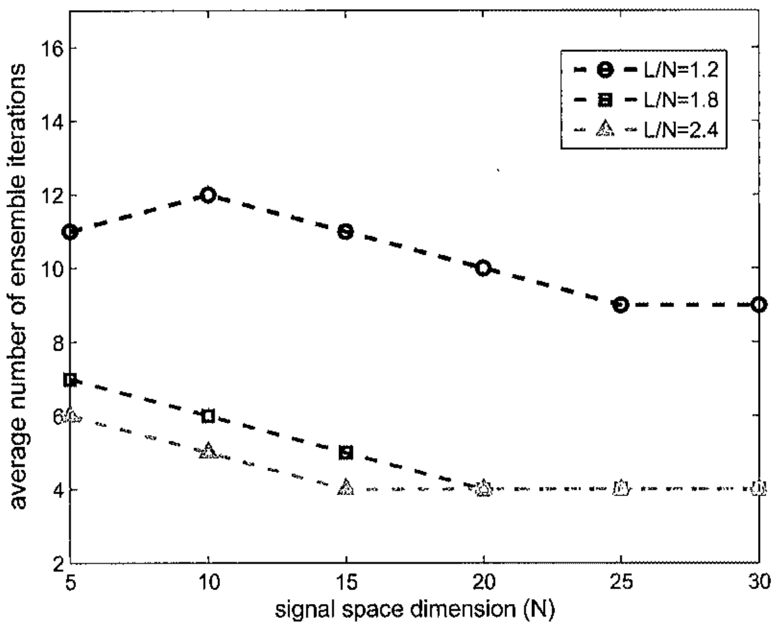

FIG. 9: Convergence speed for the MMSE Algorithm for the one user dropped from the system scenario.

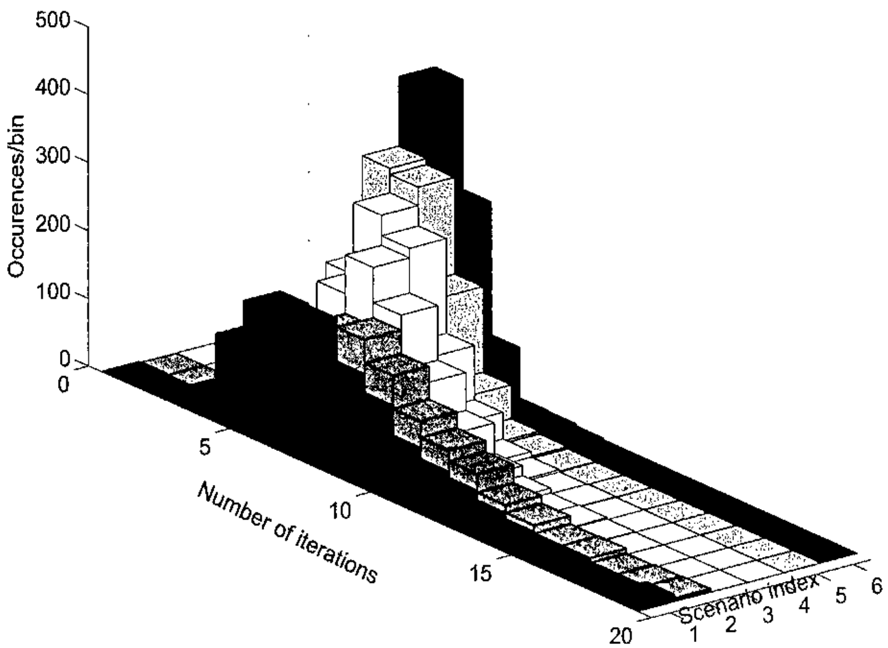

FIG. 10: Histogram Plot for the MMSE Algorithm for random initialization with $\frac{L}{N} = 1.8$.

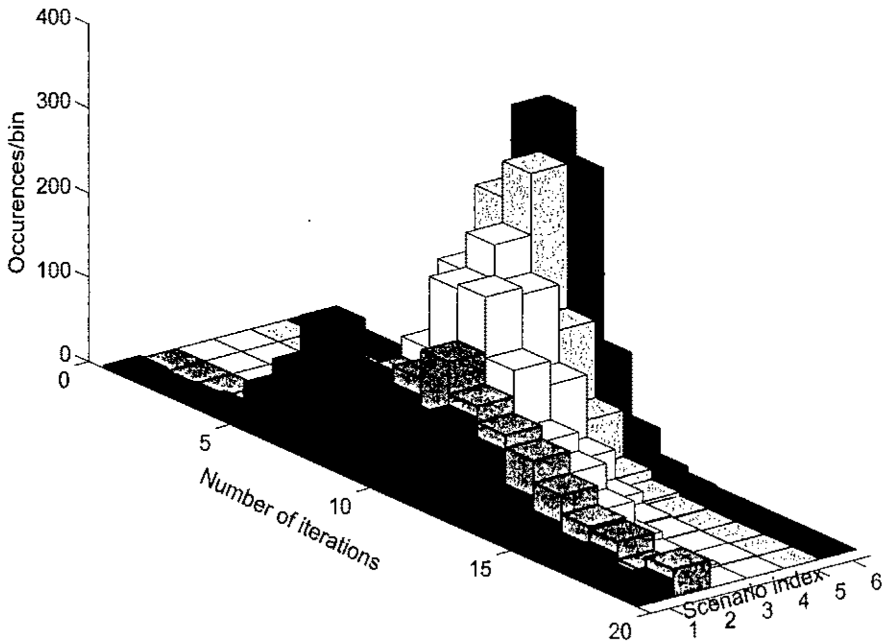

FIG. 11: Histogram Plot for the MMSE Algorithm for the one user added to the system with $\frac{L}{N} = 1.8$.

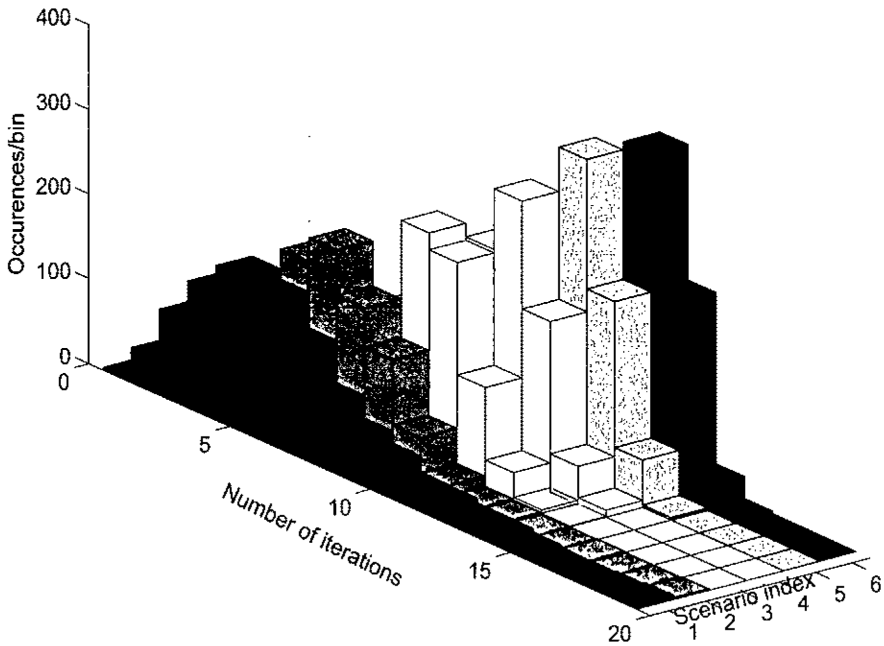

FIG. 12: Histogram Plot for the MMSE Algorithm for the one user dropped from the system with $\frac{L}{N} = 1.8$.

We observed the convergence speed of the algorithm given in Table II.

IV.3 SIMULATION SETUP AND RESULTS FOR ADAPTIVE ALGORITHM

We also vary the number of users L and signal space dimensions N such that the load factor ($\frac{L}{N}$) becomes 1.2, 1.8 and 2.4. For the adaptive algorithm, the admissible target SINRs for all users were chosen to be $\gamma^* = 0.7$. The algorithm constants were chosen to be $\beta = 0.2$, $\mu = 0.01$, the tolerance $\epsilon = 0.001$. From Table III we noted that the average ensemble iterations for random initialization when signal dimension $N = 5$ and active users $L = 6$ is 43. Table III can be referred to the rest of the data with different values of L and N for the load factor $\frac{L}{N} = 1.2, 1.8$ and 2.4.

Similar to previous cases, the first row in Table III gives the average ensemble iterations when user codeword and power were initialized randomly; the second row gives the average ensemble iterations when one user is added without increasing N; the third row gives the average ensemble iterations when one user is dropped from the system when the algorithm reaches the fixed point. All the values of the average ensemble iterations correspond to each load factor. The codeword and power for previous users as in the previous two algorithms defined in Sections $IV.1$ and $IV.2$ were taken from a fixed point, and the codeword and power for a newly added user are initialized randomly and appended to corresponding matrices. We observe the convergence speed of the algorithms given in Table III.

With the help of these data for $1,000$ trials and the average ensemble iteration, we observe the three different plots for the adaptive algorithm case as shown in Figures 13, 14 and 15. Figure 13 shows the convergence speed for random initialization, while Figure 14 shows the convergence speed when one user is added to the system, and Figure 15

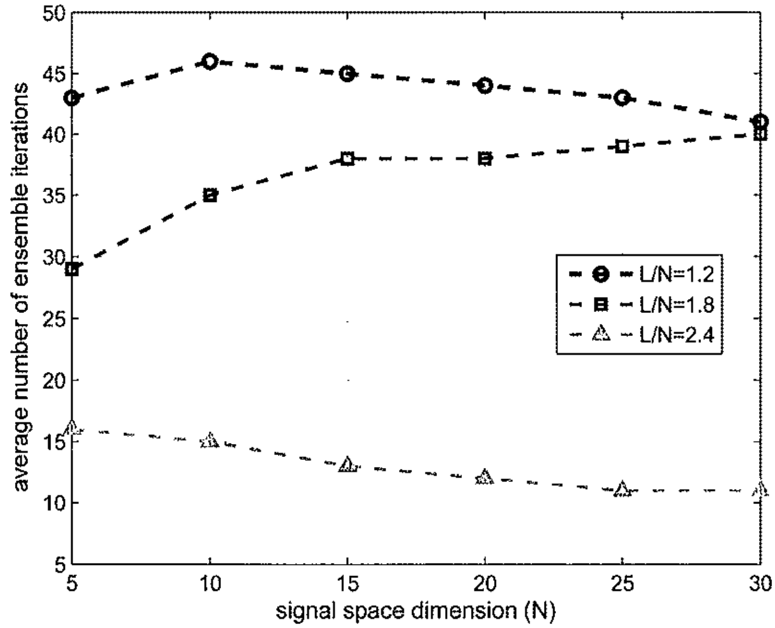

FIG. 13: Convergence speed for the Adaptive Algorithm for the random initialization scenario.

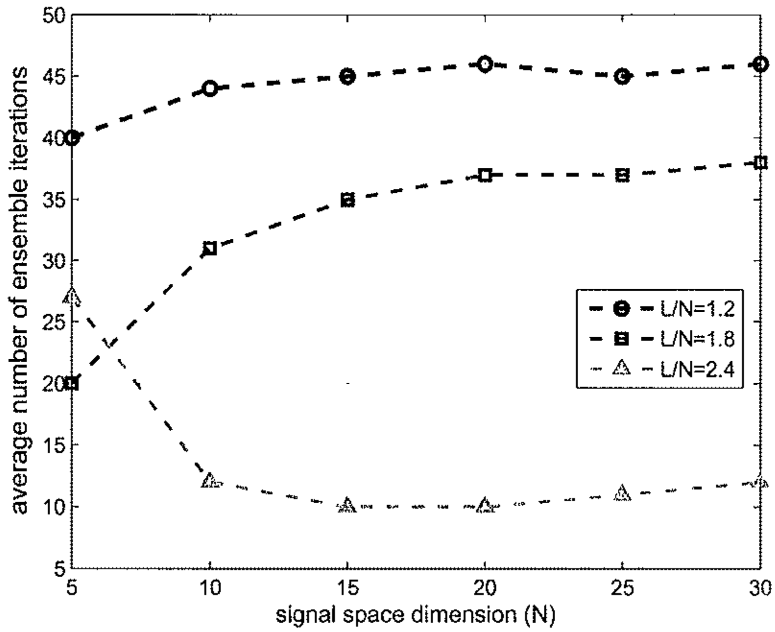

FIG. 14: Convergence speed for the Adaptive Algorithm for the one user added to the system scenario.

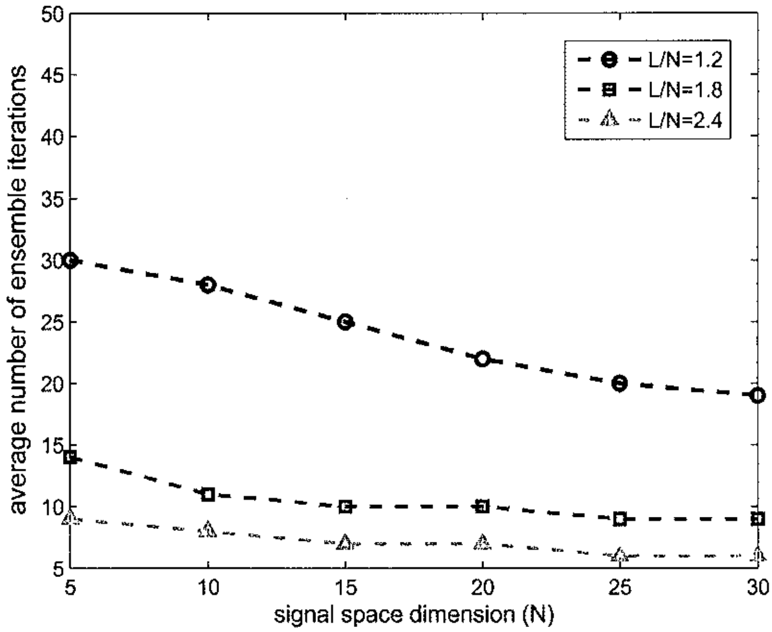

FIG. 15: Convergence speed for the Adaptive Algorithm for the one user dropped from the system scenario.

represents the scenario when one user is dropped from the system for the adaptive algorithm as in the previously defined algorithms. We note that the adaptive interference avoidance algorithm is the slowest one in terms of convergence speed. Figures 16, 17 and 18 show the histogram plots for the adaptive algorithm respectively for random initialization, one user added to the system and one user dropped from the system.

Numerically, the fixed point for all three algorithms is reached when the Euclidian distance between a given codeword and its corresponding replacement is within the tolerance ϵ. These algorithms converge to generalized Welch Bound Equality (WBE) sequences. We performed another simulation to find out how the different admissible target SINR affect the convergence speed of the adaptive algorithm. For this example, we have taken $L = 6$ users in signal space of dimension $N = 5$ (i.e. $\frac{L}{N} = 1.2$ initially) and applied the algorithm

TABLE III: Convergence Speed of the Adaptive Algorithm

Average Iterations						
	N=5	N=10	N=15	N=20	N=25	N=30
	43	46	45	44	43	41
$\frac{L}{N} = 1.2$	40	44	45	46	45	46
	30	28	25	22	20	19
	29	35	38	39	39	40
$\frac{L}{N} = 1.8$	20	31	35	37	37	38
	14	11	10	10	9	9
	16	15	13	12	11	11
$\frac{L}{N} = 2.4$	27	12	10	10	11	12
	9	8	7	7	6	6

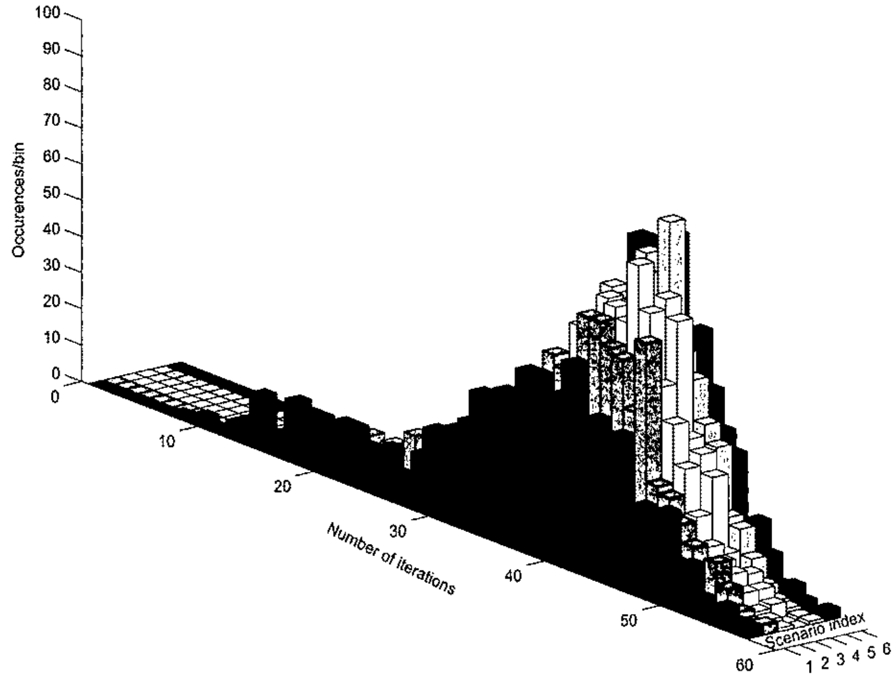

FIG. 16: Histogram Plot for the Adaptive Algorithm for random initialization with $\frac{L}{N}$ = 1.8.

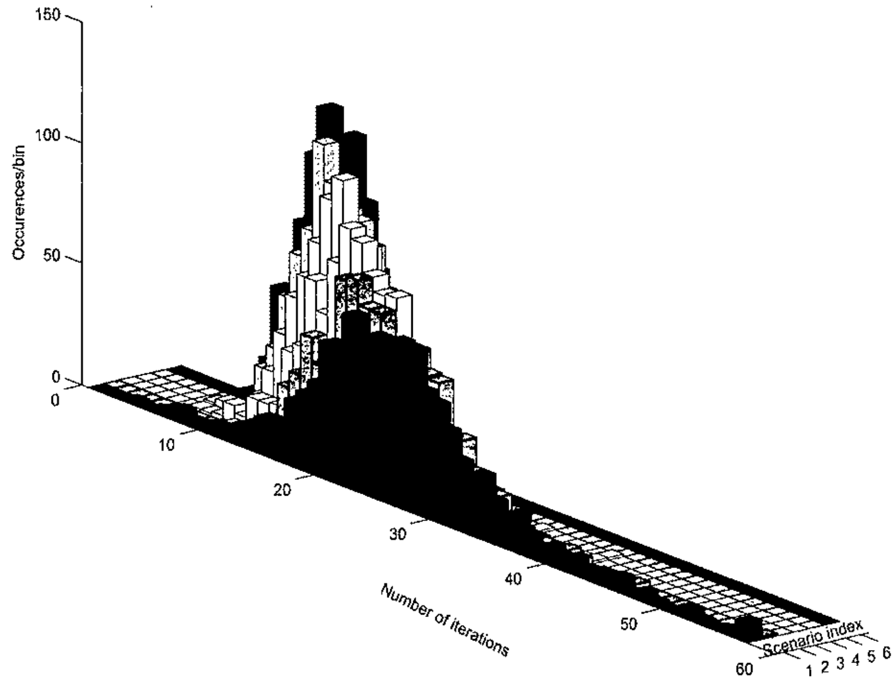

FIG. 17: Histogram Plot for the Adaptive Algorithm for the one user added to the system with $\frac{L}{N}$ = 1.8.

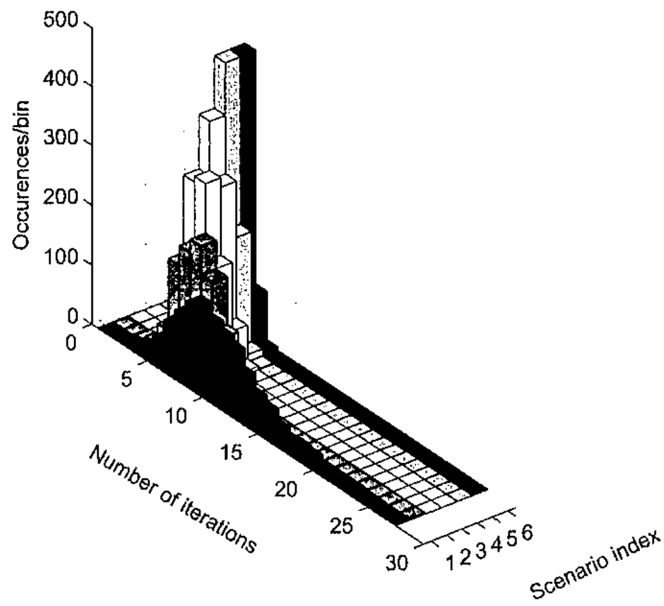

FIG. 18: Histogram Plot for the Adaptive Algorithm for the one user dropped from the system with $\frac{L}{N} = 1.8$.

for $1,000$ trials for different values of γ^* and noted the corresponding average ensemble iterations. For each target SINR, when the algorithm reached a fixed point one user was added to the system so that the total active number of users in the system was $L = 7$ in signal space of dimension $N = 5$.

The adaptive algorithm was applied again for a new configuration with target SINR taken as in the previous case for $1,000$ simulation trials, and we noted the corresponding average ensemble iterations as shown in Table IV. Finally, when the algorithm reached a fixed point, we dropped one user from the system so that there were $L = 6$ users in the signal space of dimensions $N = 5$ by taking the first six columns of **S** and **P** matrices. We have observed that the algorithm changes the system configuration after adding new active users in the system, dropping the last user from the system or changing the target SINRs

TABLE IV: Convergence Speed of the Adaptive Algorithm with Different Target SINR

Average Iterations									
$\gamma^* \rightarrow$	0.5	1	1.5	2.0	2.5	3.0	3.5	4.0	4.5
$L = 6$	42	47	50	52	54	54	55	56	56
$L = 7$	41	41	41	41	41	41	41	41	41
$L = 6$	29	29	31	31	31	31	31	32	32

of the active users. The algorithm adds/drops the user codewords to/from the system when there is a change after updating the codeword and power matrices(S and P) in the system.

We note that for larger target SINR, the algorithm needs a larger more number of average ensemble iterations to converge to the fixed point. As in the previous example, we also note that for the same load factor, 1.2 (i.e. the first row) in Table IV, the average ensemble iterations is higher for the random initialization of codewords and powers than that of the dropped user case (i.e. the third row in Table IV). This is because one user was dropped from the system when the algorithm reached a fixed point (i.e. codewords were placed almost in their suitable positions), and we applied the algorithm for that set up. Therefore, the algorithm adjusts the codewords with fewer ensemble iterations.

We observe that all three algorithms considered need more iterations for convergence in the case of lower values of the system load factor $\frac{L}{N}$ and fewer iterations for higher load factors. This behavior may be explained by the fact that in the case of light system load there are more degrees of freedom available to users and algorithms need more time to

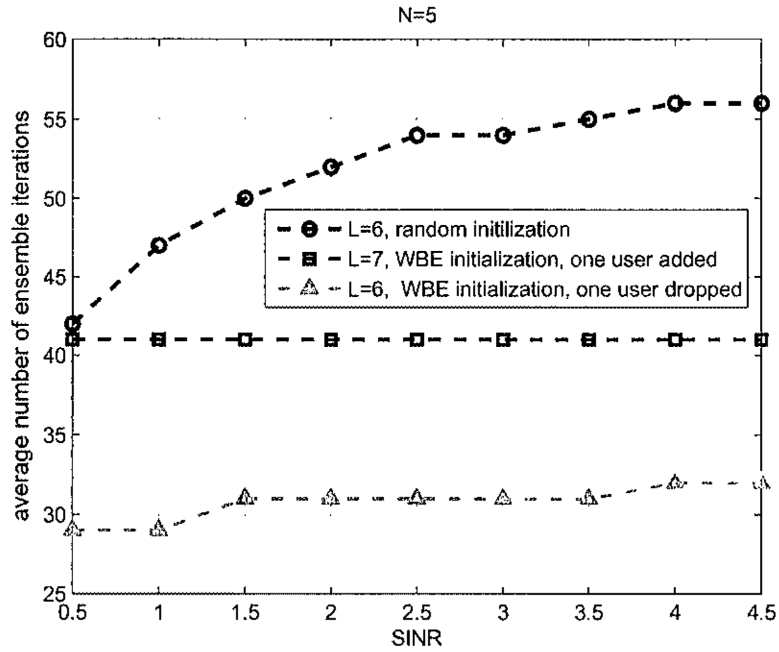

FIG. 19: Convergence of the Adaptive IA Algorithm for fixed signal dimension and increasing target SINRs.

settle into an optimal codeword ensemble, while for heavily loaded systems there is not much choice available and an optimal ensemble is easily found. We also note that the average number of ensemble iterations needed for convergence when one user is added to the system is larger than the corresponding number of iterations when one user is removed from the system. This may be explained by noting that when one user is added to the system the total amount of interference in the system is increased making it more difficult for the algorithms to find the new optimal WBE ensemble with minimum mutual interference. In contrast, when one user is removed from the system, the total amount of interference in the system is decreased making it easier for the algorithms to find the new optimal WBE ensemble.

The adaptive IA algorithm allows joint codeword adaptation and power control and enables users to achieve specified target SINR values implied by specific QoS requirements. Motivated by this observation we have also performed a separate experiment in which we looked at variation of the convergence speed of the adaptive IA algorithm in 1,000 trials with fixed dimension of the signal space and increasing value of the target SINRs for the same scenarios: convergence from random codeword ensembles to optimal ensembles as well as convergence from an optimal ensemble with L users to a new optimal ensemble where one user is added to/removed from the system. Results of this experiment for $N = 5$, $L = 6$ and the same SINR values for users ranging from 0.5 to 4.5 are shown in Figure 19.

However, we note that the first two algorithms may lead to abrupt changes of the user codeword and/or power that are not desirable in a real time wireless system since the system may not be able to adapt to the sudden changes. This may lead to increased probability of error at the receiver or loss of communication link. An adaptive interference avoidance algorithm allows the receiver to continue detecting transmitted symbols with high accuracy. As we expected, the algorithms converge faster when one user was dropped since the codewords were at almost optimal positions when the algorithm reached a fixed point.

IV.4 SUMMARY

The simulation setup and numerical results of the distributed codeword algorithms is discussed in detail in this chapter. We note that the number of ensemble iterations needed for convergence from random to optimal ensembles increases as the target SINR values increases, but does not change for convergence from an optimal ensemble to a new optimal ensemble when one user is added to/removed from the system. We note that the eigen algorithm needs the least number of average ensemble iterations and therefore the fastest algorithm among the three, and the adaptive interference avoidance algorithm is the slowest one in terms of convergence speed.

CHAPTER V

CONCLUSION AND FUTURE WORK

In this thesis we have presented a side-by-side comparision of three algorithms for distributed codeword adaptation in uplink of a CDMA system based on IA: the eigen algorithm, the MMSE update, and the adaptive IA algorithms. The performance of CDMA wireless systems is analyzed for random initialization scenario and one user added to the system as well as one user dropped from the system. We have compared the convergence speed for these three scenarios for the three different algorithms mentioned above. We have also included in this thesis a list of simulation histogram plots that are of practical importance and which relate to the implementation of the proposed three IA algorithms that should be further investigated in order to make interference avoidance a useful tool in the design of future wireless communication systems. The comparision is based on the numerical results obtained in extensive simulations which show that, on average, the number of ensemble iterations needed for the convergence is smallest for the eigen algorithm and largest for the adaptive IA algorithm.

The numerical results have also shown that increasing the load factor ($\frac{L}{N}$) and/or dimensionality of the signal space N does not result in significant increase in the average number of ensemble iterations needed for convergence by the three different IA algorithms. The obtained simulation plots of the ensemble iterations of all three algorithms follow the Gaussian distribution with the mean/average value of ensemble iteration.

Future work related to this thesis should consider the addition of the channel in the system to study the convergence speed of the distributed codeword adaptation algorithms in conjunction with realistic channel models for wireless systems. Future work may also include a study of convergence speed for IA algorithms in a downlink of a CDMA system as well as in collaborative multibase systems [13–15].

BIBLIOGRAPHY

[1] P. Anigstein and V. Anantharam, "Ensuring Convergence of the MMSE iteration for Interference Avoidance to the global optimum," *IEEE Transactions on Information Theory*, vol. 49, no. 4, pp. 873–885, April 2003.

[2] R. M. Buehrer, *Code Division Multiple Access (CDMA).* San Rafael, CA: Morgan & Claypool Publishers, 2006.

[3] A. B. Carleial, "Outer Bounds on the Capacity of Interference Channels," *IEEE Transactions on Information Theory*, vol. 29, no. 4, pp. 602–606, July 1983.

[4] T. M. Cover and J. A. Thomas, *Elements of Information Theory.* New York, NY: Wiley-Interscience, 1991.

[5] D. C. Popescu and C. Rose, *Interference Avoidance Methods for Wireless Systems.* NY: Kluwer, 2004.

[6] R. G. Gallager, *Information Theory and Reliable Communication.* New York, NY: Wiley, 1968.

[7] D. J. Love, R. W. Heath, V. K. N. Lau, D. Gesbert, B. D. Rao, and M. Andrews, "An Overview of Limited Feedback in Wireless Communication Systems," *IEEE Journals on Selected Areas in Communications*, vol. 26, no. 8, pp. 1–24, October 2008.

[8] C. Lăcătuş and D. C. Popescu, "Adaptive Interference Avoidance for Dynamic Wireless Systems: A Game-Theoretic Approach." *IEEE Journal on Selected Topics in Signal Processing*, vol. 1, no. 1, pp. 189–202, June 2007, special Issue on adaptive waveform design for agile sensing and communications.

[9] A. B. MacKenzie and S. B. Wicker, "Game theory in communications: Motivation, explanation, and application to power control," in *In Proceedings 2001 IEEE Global TeleCommunications Conf. -GLOBECOM'01*, vol. 2, San Antonio, TX, pp. 821–826, November 2001.

[10] U. Madhow and M. L. Honig, "MMSE Interference Suppression for Direct-Sequence Spread-Spectrum CDMA," *IEEE Transactions on Communications*, vol. 42, no. 12, pp. 3178–3188, December 1994.

[11] S. Maharjan, D. B. Rawat, and D. C. Popescu, "A Simulation Study of Convergence Speed for Distributed Codeword Adaptation Algorithms in CDMA Wireless Systems," in *Proceedings 2009 IEEE Radio and Wireless Symposium - RWS'09*, San Diego, CA, pp. 425–428, January 2009.

[12] J. L. Massey and T. Mittelholzer, *Welch's Bound and Sequence Sets for Code-Division Multiple Access Systems*. New York, NY: In Sequences II: Methods in Communication, Security and Computer Science, Springer-Verlag, 1991.

[13] D. C. Popescu, "Interference Avoidance for Downlink CDMA," in *In Proceedings* 64th *IEEE Vehicular Technology Conference VTC 2006, Fall, vol 4, Montreal, Canada,*, pp. 1563–1566, September 2006.

[14] D. C. Popescu, O. Popescu, and S. Abraham, "Downlink Transmitter Adaptation Based On Greedy SINR Maximization," in *Proceedings 22nd IEEE Canadian Conference on Electrical and Computer Engineering - CCECE 2009*, St. John's, Newfoundland, Canada, pp. 94–98, May 2009.

[15] O. Popescu and C. Rose, "Greedy SINR Maximization in Collaborative Multi-Base Wireless Systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2004, no. 2, pp. 201–209, December 2004, special Issue on Multiuser MIMO Networks.

[16] Popescu, D. C. and Rose, C., "Interference Avoidance and Multiuser MIMO Systems," *International Journal of Satellite Communications and Networking*, vol. 21, no. 1, pp. 143–161, January 2003.

[17] J. G. Proakis and M. Salehi, *Communication Systems Engineering*. New York, NY: McGraw-Hill Companies, 2000.

[18] C. Rose, "Sum Capacity and Interference Avoidance: Convergence Via Class Warfare," in *In Proceedings 2000 Conference on Information Sciences and Systems - CISC 2000*, Princeton, NJ, pp. WA3–11 – WA3–16, March 2000.

[19] C. Rose, "CDMA Codeword Optimization : Interference Avoidance and Convergence Via Class Warfare," *IEEE Transactions on Information Theory*, vol. 47, no. 6, pp. 2368–2382, September 2001.

[20] C. Rose, S. Ulukus, and R. Yates, "Wireless Systems and Interference Avoidance," *IEEE Transactions on Wireless Communications*, vol. 1, no. 3, pp. 415–428, July 2002.

[21] M. Rupf and J. L. Massey, "Optimum Sequence Multisets for Synchronous Code Division Multiple Access Channels," *IEEE Transactions on Information Theory*, vol. 40, no. 4, pp. 1261–1266, July 1994.

[22] K. C. Sharman, "Adaptive Algorithms for Estimating the Complete Covariance Eigenstructure," in *In Proceedings 1986 International Conference on Acoustics, Speech, and Signal Processing - ICASSP '86*, Tokyo, Japan, pp. 27.15.1 – 27.15.4, 1986.

[23] B. Sklar, *Digital Communications.* New Jersy, NJ: Prentice Hall, 2001.

[24] Y. Sun and M. L. Honig, "Minimum feedback rates for multicarrier transmission with correlated frequency-selective fading," in *In Proc. IEEE Glob. Telecom. Conf.*, vol. 3, pp. 1628-1632, December 2003.

[25] D. Torrieri, *Principles of Spread-Spectrum Communication Systems.* New York, NY: Springer, 2005.

[26] S. Ulukus and R. Yates, "Iterative Construction of Optimum Signature Sequence Sets in Synchronous CDMA Systems," *IEEE Transactions Information Theory*, vol. 47, no. 5, pp. 1989–1998, July 2001.

[27] S. Verdu, "Capacity Region of Gaussian CDMA Channels. The Symbol-Synchronous Case." in *In Proceedings* 24^{th} *Allerton Conference on Communication, Control,and Computing*, pp. 1025–1034, October 1986.

[28] P. Viswanath and V. Anantharam, "Optimal Sequences and Sum Capacity of Synchronous CDMA systems," *IEEE Transactions Information Theory*, vol. 45, no. 6, pp. 1984–1991, September 1999.

[29] P. Viswanath and V. Anantharam, "Optimal Sequences for CDMA Under Colored Noise: A Schur-Saddle Function Property," *IEEE Transactions on Information Theory*, vol. 48, no. 6, pp. 1295–1318, June 2002.

[30] P. Viswanath, V. Anantharam, and D. Tse, "Optimal Sequences, Power Control and User Capacity of Synchronous CDMA Systems with Linear MMSE Multiuser Receivers," *IEEE Transactions on Information Theory*, vol. 45, no. 6, pp. 1968–1983, September 1999.

[31] L. R. Welch, "Lower Bounds on the Maximum Cross Correlation of Signals," *IEEE Transactions on Information Theory*, vol. IT-20, no. 3, pp. 397–399, May 1974.

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