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Magnetic structure and radiative captures of few-nucleon systems: status and prospects

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Abstract. We review the main ingredients for an *ab-initio* study of few-nucleon reactions of astrophysical interest within the chiral effective field theory approach, with particular attention to radiative captures relevant for Big Bang Nucleosynthesis and stellar evolution. We conclude with an outlook for ongoing and future work.

1. Introduction

There are several astrophysical environments in which nuclear reactions take place and play a significant role. In particular, we consider the nuclear reactions which took place a few minutes after the Big Bang, in the so-called Big Bang Nucleosynthesis (BBN), and those responsible for stellar evolution. Most of these reactions are radiative or weak captures among charged particles, and take place at energies well below the Coulomb barrier. Therefore, their experimental determination is rather difficult, and theory becomes essential for guiding extrapolation of data taken at higher energies, or, in some cases for which experiments are impossible, to provide a reliable estimate. The astrophysics community is eager of more and more accurate predictions in order to test fundamental models. In this regard, a reliable theoretical prediction is required to be model-independent, and possibly combined with a robust determination of the theoretical uncertainty. *Ab-initio* calculations within the chiral effective field theory (χEFT) framework are the answer to these requests. We review the *ab-initio* methods and χEFT in the next two sections. We conclude in Sec. 4 with an outlook of the ongoing work and near and far future developments.

2. *Ab-initio* methods

In an *ab-initio* method, a nucleus is seen as a system of A nucleons interacting with each other and with external electro-weak probes. Therefore, a realistic description of nuclear interactions and electro-weak currents is required. But even if these two ingredients are given, in order to

reach the goal of having reliable predictions, it is necessary to have an accurate method, that is able to solve the A-body quantum mechanical problem, not only for bound but also for scattering states, in an energy region where the Coulomb interaction is crucial. Not many methods are at hand, and only a few of them are available for small values of A, typically $A \leq 4$. One of them is the so-called Hyperspherical Harmonics (HH) method, which we briefly review here. For a detailed discussion of the method we refer to Refs. [1, 2]. In the HH method, the wave function for a bound A-nucleus is written as

$$\Psi_A = \sum_\mu c_\mu \phi_\mu \ , \tag{1}$$

where ϕ_μ is a set of basis functions, which include all the spin-isospin structure of the system and can be expressed in terms of the HH functions [1], while c_μ are unknown coefficients determined applying the Rayleigh-Ritz variational principle. This reduces the problem to solving an eigenvalue-eigenvector problem with standard numerical techniques, and the binding energy and coefficients c_μ can be determined with great accuracy.

Within the HH method, also the scattering problem, below the clusters breakup threshold, can be solved, by writing the scattering wave function Ψ^{LSJJ_z}, having incoming relative orbital angular momentum L and spin S, coupled to total angular momentum JJ_z, as

$$\Psi^{LSJJ_z} = \Psi_C^{LSJJ_z} + \Psi_A^{LSJJ_z} \ . \tag{2}$$

Here $\Psi_C^{LSJJ_z}$ describes the system in the region where the particles are close to each other and the nuclear interaction is strong. Therefore, it vanishes for large inter-cluster distances, and can be expanded as in Eq. (1). The function $\Psi_A^{LSJJ_z}$ describes the relative motion of the two clusters in the asymptotic region, where, for charged particles, only the long-range Coulomb interaction is present. For example, for $A = 3$, i.e. $p + d$, the function $\Psi_A^{LSJJ_z}$ can be written as

$$\Psi_A^{LSJJ_z} = \sum_{L'S'} [\delta_{LL'}\delta_{SS'}\Omega^-_{L'S'JJ_z} - {}^J\mathcal{S}_{LS,L'S'}\Omega^+_{L'S'JJ_z}] \ , \tag{3}$$

where

$$\Omega^\pm_{LSJJ_z} = \sum_{k=1,3} [Y_L(\hat{\boldsymbol{y}}_k) \otimes [\phi_d(ij) \otimes \chi_{s_k}]_S]_{JJ_z} \Big(\frac{\tilde{G}_L(\eta, qy_k)}{qy_k} \pm \mathrm{i} \frac{F_L(\eta, qy_k)}{qy_k} \Big) \ . \tag{4}$$

Here we have indicated with $\boldsymbol{y}_k$ the distance between the proton (particle k) and the deuteron (particles ij), q is the magnitude of the relative momentum, χ_{s_k} is the spin function of particle k, and ϕ_d is the deuteron wave function. Furthermore, $\tilde{G}_L$ is defined as

$$\tilde{G}_L(\eta, qy) = G_L(\eta, qy) \times [1 - \mathrm{e}^{-\beta y}]^{2L+1} \ , \tag{5}$$

G_L, and F_L in Eq. (4), being the irregular and regular Coulomb functions, with $\eta = 2\mu e^2/q$ (μ is the $p+d$ reduced mass). The parameter β is determined by requiring $\tilde{G}_L \to G_L$ for large values of y, and the function $\tilde{G}_L$ to be regular at small values of y, and goes to G_L for large y. With the definition of Eq. (4), the functions $\Omega^+_{LSJJ_z}$ ($\Omega^-_{LSJJ_z}$) describe the asymptotic outgoing (ingoing) $p + d$ relative motion. The parameters ${}^J\mathcal{S}_{LS,L'S'}$ in Eq. (3) are the S-matrix elements which determine phase-shifts and, for coupled channels, mixing angles at a given energy $E_{c.m.} = q^2/2\mu$.

The unknown quantities in the expansion of Eq. (2) are the coefficients c_μ present in $\Psi_C^{LSJJ_z}$ and the parameters ${}^J\mathcal{S}_{LS,L'S'}$ present in $\Psi_A^{LSJJ_z}$. They are determined applying the Kohn variational principle, which states that the functional

$$\Big[{}^J\mathcal{S}_{LS,L'S'} \Big] = {}^J\mathcal{S}_{LS,L'S'} - \langle \Psi^{L'S'JJ_z} | H - E | \Psi^{LSJJ_z} \rangle \tag{6}$$

must be stationary. To be noticed that $E \equiv E_{c.m.} - B_d$ is the energy of the system, B_d being the deuteron binding energy. By using Eq. (6), a linear set of equations is obtained for ${}^J\mathcal{S}_{LS,L'S'}$ and c_μ, and solved with standard numerical techniques.

As an application of the method, we present in Fig. 1 the latest study [3] of the p+$d \rightarrow {}^3$He+γ astrophysical S-factor, performed using a phenomenological two- and three-nucleon interaction model (namely the Argonne v_{18} [4] two-nucleon and Urbana IX [5] three-nucleon interaction), and a model for the nuclear current derived in Ref. [6], where the main two-nucleon current operators are constructed so as to satisfy the current conservation relation with the given Hamiltonian. The energy range of interest is the one relevant for BBN. For more details see Ref. [3]. From inspection of the figure we can conclude that both calculations of Ref. [6] and [3] are systematically higher than the available experimental data. However, the two calculations, although performed with the same models for interactions and currents, differ by about 8-10 %. In Ref. [3] this difference was traced back to the different degree of accuracy of the $p + d$ scattering wave functions, those of Ref. [3] being checked by calculating the average value of the Hamiltonian in a box (with radius $\simeq$ 40 fm), which has to be equal to the energy E. Such a test was not performed in the previous study of Ref. [6]. From this we can conclude that the degree of accuracy in the solution of the three-body problem, especially for scattering states, is crucial in these studies. Furthermore, it is not possible to quantify in a robust way the uncertainty of the theoretical predictions. This is clearly unsatisfactory. In order to be able to assess the theoretical uncertainty of a given calculation, assuming that the method used to solve the quantum mechanical problem is accurate, as the one presented here, we can work within the χEFT approach, which will be described in the next Section.

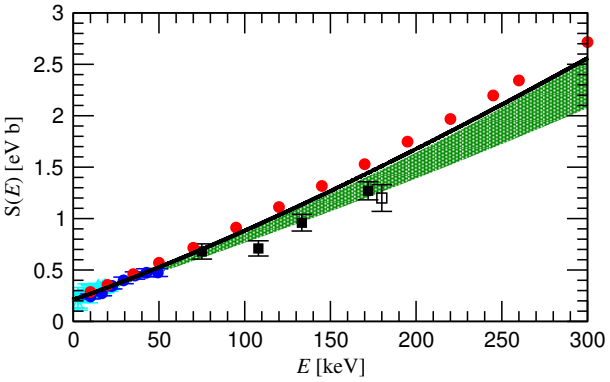

Figure 1. The astrophysical S-factor of the $p + d \rightarrow {}^3\mathrm{He} + \gamma$ radiative capture as function of the center of mass energy. The calculation of Ref. [3] is represented as red dots, the green band corresponds to the polinomial fit of Ref. [7], the black line reports the study of Ref. [6], while the remaining symbols are the experimental data of Refs. [8, 9, 10, 11].

3. The χEFT approach

In this contribution, we provide a brief review of the χEFT approach, emphasizing those features of the formalism which make it suitable to study nuclear reactions of astrophysical interest.

In essence, the χEFT framework can be seen as a formulation of Quantum Chromodynamics (QCD) in terms of effective degrees of freedom suitable for low-energy nuclear physics: pions and nucleons. The symmetries of QCD, in particular its (spontaneously broken) chiral symmetry, severely restrict the form of the interactions of these particles among themselves and with external electroweak fields. As a consequence, it is possible to expand the Lagrangian describing these interactions in powers of Q/Λ_χ, Q being the pion momentum and Λ_χ being the chiral-symmetry-breaking scale (here $\Lambda_\chi \sim 1$ GeV). Then, classes of Lagrangians emerge, each of the order $(Q/\Lambda_\chi)^n$ and each involving a certain number of unknown coefficients, the so-called low-energy constants (LECs), which arise when the high-energy degrees of freedom are integrated out. These LECs are in practice constrained by fits to experimental data. The potentials and currents derived within this framework have power-law behavior for large momenta, and

are regularized by introducing a momentum-cutoff function, with cutoff Λ. The introduction of such a cutoff typically makes the potentials strongly non local (see Ref. [12] and references therein). This does not in principle represent a problem for the HH approach (see Refs. [13, 14]), but it poses challenges for other methods, such as those based on Monte Carlo techniques. A few more observations are in order: (i) in χEFT, two- and three-nucleon interactions arise naturally and the available models have reached a high degree of accuracy in describing few-nucleon systems. (ii) In χEFT it is expected that increasing the chiral order n, reduces the dependence on the cutoff Λ. Therefore, a rough estimate of the theoretical uncertainty is provided by the Λ-dependence of the results. A further estimate of the theoretical uncertainty is the order-by-order convergence of the results. The Λ-dependence and the order-by-order convergence give significant information on the theoretical uncertainty. More sophisticated techniques are also available, as those described for instance in Ref. [15]. (iii) The framework of χEFT is suitable to derive consistently interactions and currents. We will discuss the electromagnetic current operator in χEFT below.

In order to mitigate the non-locality problem alluded to above, we have recently developed a class of χEFT potential models which are minimally non-local, and therefore can be dealt with in coordinate space. We will call these models Norfolk potentials (NVs) and we will briefly describe them in the following. For more details see Refs. [16, 17, 18] and Ref. [19].

The NV potential includes among the degrees of freedom, together with nucleons and pions, also Δ-isobar as an effective degree of freedom, since the nucleon-Δ mass difference is only twice the pion mass. By doing so, several contributions, which in the Δ-less theory are at a given chiral order, are promoted at lower order. Then, the NV two-nucleon potentials are written as

$$V_{NN} = v^{EM} + v^{LR} + v^{SR} \,, \tag{7}$$

where v^{EM} is the electro-magnetic component, which includes corrections up to α^2 (α is the fine-structure constant), v^{LR} is the long-range part of the interaction, derived up to the order $n = 3$ [or $(Q/\Lambda_\chi)^3$] including chiral one- and two-pion exchange diagrams with Δ's, and v^{SR} is the short-range interaction which retains contact terms up to $n = 4$. The potential then contains 26 LECs to be fitted to the experimental data. Before doing this, it is necessary to regularize the potential, which is done using a Gaussian cutoff both in the long- and short-range part, i.e.

$$C_{R_X}(r) \propto \mathrm{e}^{-(r/R_X)^2} \tag{8}$$

with $X = L, S$ for long- and short-range respectively, and $(R_S, R_L) = (0.8, 1.2)$ fm or $(R_S, R_L) = (0.7, 1.0)$ fm for the so-called model NVa and NVb respectively. The two sets of values allow to study the cutoff dependence. The 26 LECs are obtained fitting the 2013 Granada database [20]: when such fit is performed up to lab-energy $E_{lab} = 125$ MeV, a χ^2/datum ≤ 1.1 is obtained (model I), otherwise if $E_{lab} = 200$ MeV, χ^2/datum ≤ 1.4 (model II).

The three-nucleon interaction to be used in conjunction with the two-nucleon NV model is derived up to $n = 3$, and includes the diagrams of Fig. 2, i.e. a long-range contribution mediated by two-pion exchange, with and without Δ intermediate states, and a short-range contribution parametrized in terms of two contact interactions. LECs multiply the two-pion exchange Δ-less diagram and the short range contributions. However, the former are already present in the two-nucleon Lagrangian, and therefore constrained consistently with the two-nucleon interaction. The LECs multipling the two short-range contact interactions are denoted c_D and c_E. A first determination of c_D and c_E was performed in Ref. [18], where the triton binding energy and the nd doublet scattering length were used in conjunction with each two-nucleon NV models. We will refer to these models as NVIa, NVIb, NVIIa, and NVIIb. In a second determination [21], the observables chosen for the fit were the trinucleons binding energy and the Gamow-Teller matrix element of tritium β-decay. This has required the development of a nuclear axial current

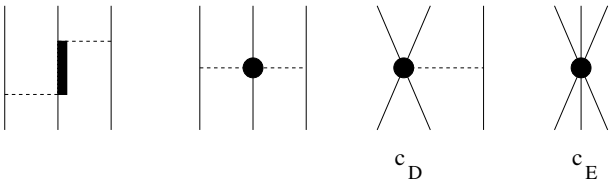

Figure 2. Schematic representation of the contributions to the chiral three-nucleon interaction. Nucleons, Δ's and pions are denoted as solid, thick and dashed lines, respectively. The circles represent vertex involving LECs. See text for more details.

consistent with the NV interaction model. We will not discuss here the construction of the nuclear axial current, as we are mainly focusing on radiative captures. The interested reader can find all the details in Ref. [21]. The two different procedures of fixing c_D and c_E produce rather different values for these LECs, as discussed in Ref. [21]. However, we should notice that the second procedure has two advantages: (i) the well known strong correlation between the trinucleons binding energy and the nd doublet scattering length makes the determination of c_D and c_E of Ref. [18] somewhat problematic; (ii) the value of c_E found in Ref. [21] with the second procedure is smaller than the one obtained with the first procedure of Ref. [18], and furthermore in both cases c_E results negative. A large and negative c_E value leads to a repulsion in light nuclei, but an attraction in neutron matter. As a consequence, neutron matter at even relatively low densities would collapse, and neutron stars of twice solar masses would not exist, in contrast with observational evidences. The potential models obtained with the fitting procedure of Ref. [21] will be labelled as NVIa*, NVIb*, NVIIa*, and NVIIb*.

The electromagnetic current operator to be used consistently with the NV potential model has been derived in Ref. [22]. The leading-order (LO) and next-to-leading order (NLO) contributions, which scale respectively as Q^{-2} and Q^{-1} in the power counting, are the "standard" single-nucleon and one-pion-exchange currents, already present in the phenomenological studies of, for instance, Refs. [6, 3]. The N2LO terms arise from relativistic corrections to the LO current, and from contributions involving Δ-isobars in the intermediate states. The N3LO terms are written as sum of an isoscalar one-pion-exchange (OPE), an isovector two-pion-exchange, and both isoscalar and isovector contact contributions. The explicit expressions for all these terms are listed in Ref. [22]. Here we mention that there are 5 LECs, three in the N3LO OPE contribution, and two in the non-minimal N3LO term (see Ref. [22] for more details). At a first attempt, which we will refer to as "Fit 1", by assuming saturation of the OPE contribution with the tree-level Δ current, the LECs reduce to three, and are fitted reproducing the deuteron and the isoscalar and isovector combination μ_S and μ_V of the $A = 3$ magnetic moments. The description of the $A = 2$ and 3 electromagnetic structure is in reasonable agreement with the experimental data, up to values of the momentum transfer Q of $2 - 3$ fm^{-1}. However, some long-standing puzzles remain, as the failure to reproduce the first diffraction region of the $A = 3$ magnetic form factors. These are shown in the first row of Fig. 3, from which we can see also a large model-dependence especially for $Q > 3$ fm^{-1} (see Ref. [22] for more details). In an attempt to solve this problem, we present here a different fitting procedure, which we refer to as "Fit 2". In this case, all the 5 LECs are considered and are obtained by fitting the $A = 2, 3$ magnetic moments and the deuteron electro-disintegration cross section at threshold. This leads clearly to completely different values for 3 of the 5 LECs, i.e. for the isovector ones, since the two LECs in front of the isoscalar operators are still fixed using the deuteron and magnetic moment and μ_S. We report here for completeness the values for the 3 LECs found with the "Fit 2" procedure, in the notation of Ref. [22], using the NVIa* (NVIb*) potentials: $d_1^V = -0.0374(-0.0416)$, $d_2^V = 0.939(0.324)$, and $d_3^V = 0.118(0.0894)$. A far better description of the $A = 3$ magnetic form factors is obtained, as it can be seen in the second row of Fig. 3. Furthermore, the model-dependence is significantly reduced, since the "full" results, obtained including all the contributions up to N3LO, essentially

overlap for all the four considered potentials models up to quite large values of the momentum transfer Q.

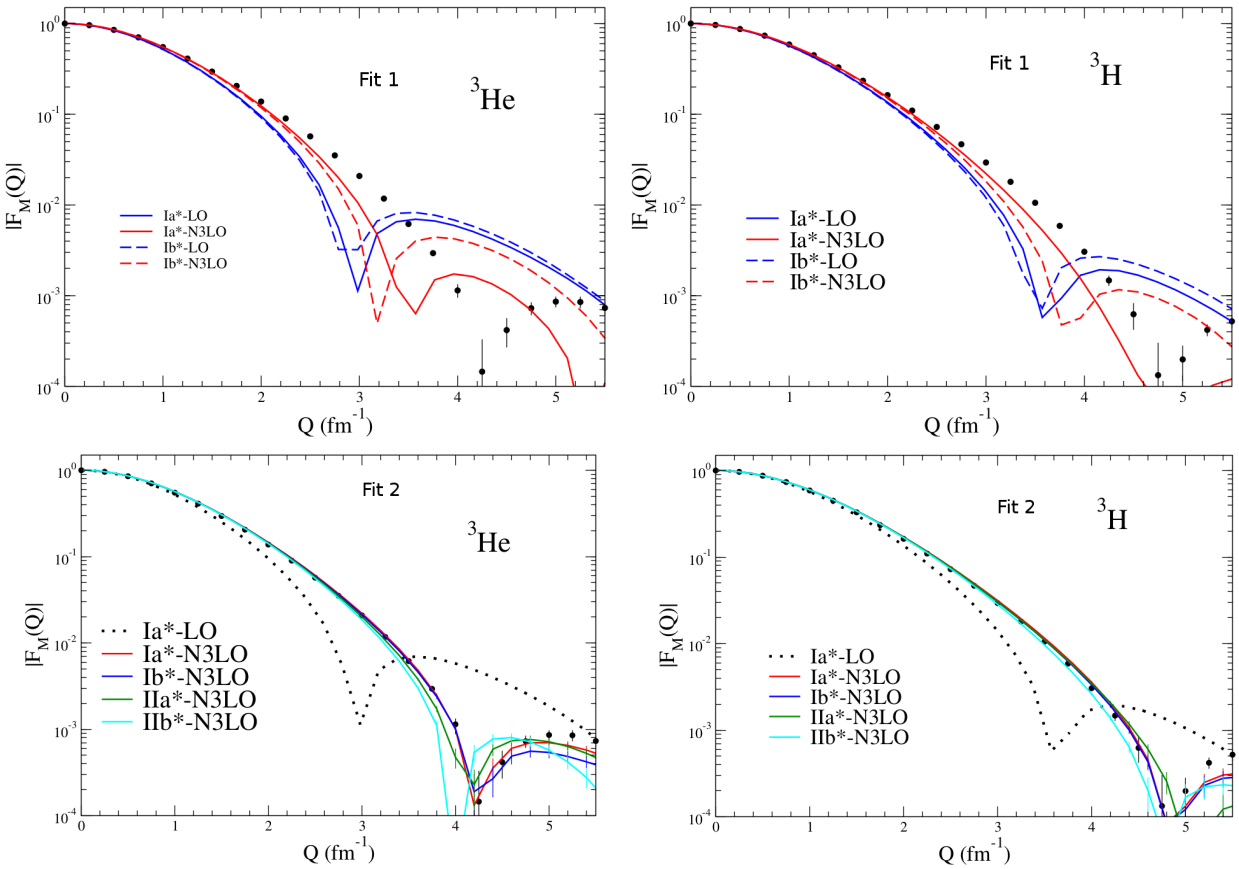

Figure 3. $A = 3$ magnetic form factors as obtained applying the "Fit 1" (first row) and "Fit 2" (second row) fitting procedure for the LECs present in the electromagnetic current operator (see text for more details). In the first row, we show the results obtained with the NVIa* and NVIb* models only, while in the second one all the 4 NV potential models are used. The LO contribution is also shown as indicated in the labels. The "full" contribution is labelled N3LO.

In conclusion, we have now at hand a theoretical framework which allows us to reproduce the electromagnetic structure of light nuclei, and, by studying the model-dependence, gives us a way to estimate the theoretical uncertainty (which seems to be quite small in the case of "Fit 2"). We are ready to address radiative captures of astrophysical interest.

4. Outlook

In the previous sections we have outlined a theoretical framework which allows us to perform *ab-initio* calculations of nuclear reactions in an energy regime of interest for astrophysics, and to assign to the theoretical predictions an uncertainty, whose determination follows well established procedures and can be considered robust. We are now in the process of studying radiative captures of interest for stellar evolution and BBN, like neutron capture on proton and deuteron, and the $p + d$ reaction already mentioned in Sec. 2. But we can approach more reactions: in fact, as already mentioned in Sec. 3, in order to fit the c_D and c_E LECs using the Gamow-Teller matrix element of tritium β-decay, it has been necessary to develop a consistent axial current operator, with a procedure similar to the one outlined here for the electromagnetic one. This

has been done in Ref. [21]. Therefore, also weak capture reactions can be addressed, as the proton weak capture on proton (the *pp* reaction) and ^{3}He (the so-called *hep* reaction).

In the future we also plan to perform a systematic study of radiative and weak captures involving systems with $A > 4$, for esample the $\alpha + d \rightarrow {}^6\mathrm{Li} + \gamma$ radiative capture, which is also of interest for BBN, since it determines the ^{6}Li primordial abundance. In order to do that, the HH method needs to be extended to the $A = 6$ nuclear systems. The first steps to calculate the ground state of ^{6}Li within the HH method have been in fact already performed [23], with very promising results.

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