2021

Solutions to Fermi Questions, Feb. 2021

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Original Publication Citation
Weinstein, L. (2021). Solutions to Fermi Questions, Feb. 2021. *The Physics Teacher, 59*(2), 26-27, Article A141. https://doi.org/10.1119/10.0003484

Fermi Questions

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Light bulb photons

Question: How many visible photons per second does a light bulb emit? (*Thanks to Arav Singh, a REYES student from Arlington, Virginia for suggesting the question.*)

Answer: To estimate this we can either estimate the light output of a bulb (typically about 10^3 lumens for a 75 or 100 W equivalent bulb) and the energy content of a photon and then figure out (or look up) the conversion factors, or we can estimate the energy input, the bulb efficiency, and the photon energy content. In either case, we need the photon energy.

We can estimate the typical visible photon energy in one of two ways. If we remember the wavelength of light (λ = 500 nm), Planck's constant ($h = 6\times10^{-34}$ Js), and the photon energy ($E = h\nu = hc/\lambda$), then it is straightforward to calculate:

$$\begin{aligned} E &= hc/\lambda \\ &= \frac{(6\times10^{-34}\,\text{Js})(3\times10^{8}\,\text{m/s})}{5\times10^{-7}\,\text{m}} \\ &= 4\times10^{-19}\,\text{J}. \end{aligned}$$

However, normal people (and even "normal" physicists) do not generally remember all that.

Alternatively, while ultraviolet photons can break molecular bonds (and give us a sunburn), visible photons cannot. This means that visible photons have slightly less energy than the typical molecular binding energy. We can estimate this energy in one of two ways. A typical 1.5-V battery (AAA, AA, C or D) converts chemical energy to electrical energy. Since it gives each electron 1.5 electron-Volts (eV), the chemical bond energy must also be about 1.5 eV. Alternatively, the molecular binding energy will be more than 1% and less than 100% of the Hydrogen atomic binding energy, so we will estimate that it is 10% of 13.7 eV or also about 1.5 eV. Thus:

$$\begin{aligned} E &= (1.5e\text{V})\times(1.6\times10^{-19}\,\text{C}/e), \\ &= 2\times10^{-19}\,\text{J} \end{aligned}$$

which is reasonably close to our more precise calculation.

Let's start with the light bulb power and efficiency. A 100-W incandescent light bulb has a very poor efficiency. It emits more than 1% and less than 10% of its energy in the form of visible light, so we will estimate 3%. This means that it emits 3 W = 3 J/s of visible photons or about

$$\begin{aligned} N &= \frac{3\ \text{W}}{4\times10^{-19}\,\text{J/photon}} \\ &\approx 10^{19}\,\text{photons/s}. \end{aligned}$$

That is a lot of photons. But then each photon is rather small.

And no. I do not remember the definition of a lumen, except that it is closely related to the candela, which is the light output (per steradian) of a "standard" wax candle.

Light bulbs

Question: How much does the U.S. spend on residential light bulbs (both the bulbs and the electricity) every year? (*Thank to Hollis Williams of the University of Warwick, UK for suggesting the question.*)

Answer: To estimate this we need to estimate the number of residences, the number of light bulbs per residence, the electrical energy consumed per year per light bulb, and the replacement cost of the light bulbs.

Before we do that, let's try a simpler approach. The number of people per household is more than 1 and fewer than 10, so we will estimate three. This is convenient, since it gives 10^8 households. The typical electrical bill in the U.S. is more than \$10 and less than \$1000 per month, so we will estimate \$100. The electricity used for lighting is more than 1% and less than 10% of the electrical bill, so we will estimate 3%. This gives an electrical cost of

$$\begin{aligned} C_e &= (10^8)(10^2\,\$/\text{mo})(10\ \text{mo/yr})(0.03) \\ &= 3\times10^9\,\$/\text{yr} \end{aligned}$$

or about \$30 per household. If we double this to include the lightbulb costs, then we get $\$6\times10^9$ per year.

Back to the more complicated approach. There are more than 10 and fewer than 100 lightbulbs per residence, giving an estimate of 30. However, we really need to estimate the number of lit lightbulbs. The fraction of lit lightbulbs will be more than 10% and less than 100%, giving an estimate of 30%, or 10 lit bulbs (not everyone is careful about turning off the light in empty rooms). In normal times, people spend 1/3 of the day at work, 1/3 at home, and 1/3 sleeping, so that the 10 bulbs are lit for 8 hours per day.

We will estimate the costs separately for incandescent bulbs and for LED bulbs.

Incandescent bulbs will use 100 W each, last about 1000 hours, and cost $0.5 to replace. Electricity costs about $0.1 per kilowatt-hour. Thus the total number of lightbulb-hours will be

$$T = (10^8)(10 \text{ bulbs})(8 \text{ hr/day})(400 \text{ day/yr}) \\ = 4\times10^{11} \text{ hr/yr}$$

and the total electrical cost will be

$$C_e = T(0.1 \text{ kW/bulb})(0.1 \text{ \$/kW-hr}) \\ = 4\times10^{9} \text{ \$/yr}$$

which is almost the same as our previous (simpler) estimate. The yearly bulb replacement costs will be

$$C_{\text{bulb}}^{\text{inc}} = \frac{T(0.5 \text{ \$/bulb})}{10^3 \text{ hr/bulb}} = \$2\times10^8,$$

which is far less than the energy cost.

LED bulbs use about five times less electrical power for the same illumination and cost more than $1 and less than $10 per bulb, so we will estimate $3. They also last about 10 times longer than incandescent bulbs. Thus the electrical costs will be five times less

$$C_e^{\text{LED}} = 10^9 \text{ \$/yr}$$

and the replacement costs will be about two times smaller (six times more expensive, but replaced 10 times less often), or about

$$C_{\text{bulb}}^{\text{LED}} = \frac{1}{2} C_{\text{bulb}}^{\text{inc}} = 10^8 \text{ \$/yr}$$

which is still far less than the energy cost.

Thus, the cost of residential lighting was only about $40 per household per year with incandescent bulbs and has decreased to about $10 per household per year with LED bulbs. That's only 1% of our total residential electricity usage. What an incredibly inexpensive luxury!