Solutions for Fermi Questions, April 2021

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Original Publication Citation
Weinstein, L. (2021). Solutions for Fermi questions, April 2021. *The Physics Teacher*, 59(4), A299-A300. doi:10.1119/10.0004173

Fermi Questions

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Question 1: Blooming trees

How many trees began to bloom (flower) today in the U.S.? *(Thanks to Radiolab for suggesting the question.)*

Answer: To estimate this, we need to estimate the number of trees in the U.S., the fraction of trees that bloom, and time period during which they can bloom. As usual, we will estimate lower and upper bounds and take the geometric mean for our estimate. We can approximate the U.S. as a rectangle and estimate its area from the east-west and north-south distances. It is three time zones from New York to LA, or about 1/8 of the circumference of the globe. Alternatively, it is a 6-hour plane flight at 500 mph. Either way, we get an East-West distance of 3000 miles or 5000 km. If we use 2000 km for our North-South distance, then the aspect ratio is a reasonable 2.5:1, the arithmetic is trivial, and we get an area of 10^7 km^2 (which is amazingly accurate).

Trees cover more than 10% and less than 100% of the U.S., so we will estimate 30%, giving a treed area of

$$A_{\text{trees}} = 3\times10^6 \text{ km}^2.$$

To get the number of trees, we need to estimate their spacing. Trees are spaced more than 1 m apart and less than 10^2 m (a football field) apart, giving a separation of 10 m. This gives an area per tree $a_{\text{tree}} = 10^2$ m^2 and a total number of trees

$$N_{\text{trees}} = \frac{A_{\text{tree}}}{a_{\text{tree}}} = \frac{3\times10^6 \text{ km}^2}{10^2 \text{ m}^2}\times\frac{10^6 \text{ m}^2}{1 \text{ km}^2} = 3\times10^{10}.$$

That's a big number. Since there are 3×10^8 Americans, that is 100 trees per person. Checking with reality (or as close as we can come on the internet), the U.S. appears to have 10 times as many. This would imply a tree spacing of 3 m (10 ft), rather than 10 m. (I hope that you estimated this better than I did. In retrospect, a 10-m average spacing seems much too large.)

Estimating the fraction of trees that bloom is a bit less well-defined. Do we just consider trees with spectacular flowers or do we consider all trees with flowers? If the latter, then we must include all trees, since trees reproduce by flowering. However, let's just consider the number of trees with pretty flowers that attract humans as well as pollinators. The fraction of trees with pretty blooms must be more than 1% and less than 100%, so we will estimate 10%. This gives us 3×10^9 flowering trees.

Now we need to estimate the time period over which the trees bloom. Spring begins in late February in the southern states but not until August in New England. Maybe I exaggerate. It definitely felt like spring took FOREVER to arrive, but it must have arrived by May. This gives a flowering season of 3 to 4 months or 100 days.

Thus, about 3×10^7 trees began to bloom today. And if we had estimated the total of trees more accurately, then we would estimate that 3×10^8 trees began to bloom today.

Pretty!

Question 2: Human energy

How much energy does a human use in his/her lifetime? (*Thanks to Robert Wiringa of Argonne National Lab for suggesting the question.*)

Answer: To estimate this we need to consider both the internal energy we generate (i.e., food energy) and the external energy we use (for heating and cooling, transportation, writing this column, etc). Let's start with food. We each consume about 2000 to 3000 food calories per day. Since each food calorie is 4000 Joules, that is 10^7 J/day or 4×10^9 J/year. At a lifespan of 80 years, this gives

$$E_{\text{food}} = (4\times10^9 \text{ J/yr})(80 \text{ yr}) = 3\times10^{11} \text{ J}.$$

At an energy density of 4×10^7 J/kg, that is the energy contained in about 10 tons of gasoline (or olive oil). TNT actually has an energy density 10 times less than gasoline (this is not too surprising since TNT is optimized for rapid oxidation and includes all the oxidant it needs). Thus, we use the energy equivalent of 100 tons of TNT during our lifetimes. That much energy released very rapidly would make a rather large BOOM.

However, we use far more energy than we eat. We use energy for a myriad of purposes. Let's estimate our energy use in several different ways. The easiest is to consider the problem as a whole and just estimate that our external energy use is more than 10 and less than 10^4 times our food use, giving an estimate of 300 times more (i.e., the equivalent of having 300 human servants to cater to our every whim).

Now let's break it down and see what we get. We typically drive 10^4 miles per year at an efficiency of 20 miles per gallon, giving a gasoline consumption of 500 gallons

$= 2\times10^3$ L which has a mass of about 2×10^3 kg. Over our lifetime, that gives a total gasoline consumption of 2×10^5 kg = 200 tons, or about 20 times more than our food consumption.

If our electrical bill is about $100 per month (more than $10 and less than $1000), and the rate is about $0.1 per kilowatt-hour, then we consume 10^4 kW-hr per year of electricity. At 4×10^6 J/kW-hr, that is 4×10^{10} J/yr or a lifetime usage of

$$E_{elec} = 3\times10^{12} \text{ J}.$$

Thus, we use 20 times more energy from gasoline than from food and we use 10 times more energy from electricity. That is a total of only 30 times more energy. If we also include commercial energy usage, then we can double that to a factor of 60. This is lower than our crude estimate of 300 times, but not terribly so. That gives a total lifetime energy consumption, including everything, of about

$$\begin{aligned} E_{total} &= 60\times E_{food} \\ &= 60\times(3\times10^{11} \text{ J}) \\ &= 2\times10^{13} \text{ J}. \end{aligned}$$

If our lifetime food consumption is the equivalent of 0.1 kilotons of TNT, then our total lifetime energy consumption is the equivalent of 5 kilotons. That would definitely make a very very large BOOM.