Solutions for Fermi Questions, January 2022: Question 1: Snow Volume; Question 2: Longbow Arrow Velocity

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Fermi Questions

Question 1: Falling on a BMW

**Question:** A New Jersey man fell out of a ninth story window, landed on a BMW, and survived. How much force was exerted on him by the car?

**Answer:** To estimate this, we need to estimate the height of the fall and the distance over which the car crumpled. A typical building has about 10 ft (3 m) between floors, so the man fell about 30 m. The roof of the car crumpled about 1 m (more than 10 cm and less than 10 m). The work done by gravity while falling is $mgh$ and the work done by the car to stop him is $W = Fd$, giving

$$F = \frac{mgh}{d} = \frac{mg}{g} \frac{h}{d} = 30mg,$$

so that the force was 30 times his weight. If he has a mass of 80 kg (i.e., a weight of about 180 lb), this means that the force on his body was about $F = 30(80 \text{ kg})(10/\text{kg}) = 24 \text{ kN},$

or about 2.4 tons.

That force was enough to smash the roof of the car and seriously injure the man. However, if he had not hit the roof of the car, then his stopping distance would have been far less, and the resulting forces far greater and far more lethal.


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Question 2: Recycling contact lenses

**Question:** My eye doctor now has a special bin to recycle contact lenses. Is this worth it? How many lenses would need to be recycled to account for the dedicated bin?

**Answer:** To estimate this, we need to estimate the number of recycled contact lenses, the weight of each contact lens, and the weight of the recycling bin. We’ll start with the plastic bin, which is the size of a small trash can. It weighs more than an ounce and less than 10 pounds, so we will estimate one pound (or 0.5 kg). (Alternatively, we could estimate that it masses more than 100 g and less than 10 kg, and get an estimate of 1 kg.)

A daily-use contact lens is thicker than one sheet of paper, but thinner than 10, giving an estimate of three sheets. Since one ream of paper (500 sheets) is about 5-cm thick, one sheet of paper is $10^{-2}$ cm, and one contact lens is $3\times10^{-2}$ cm. The area of a contact lens is about $1 \text{ cm}^2$, giving a volume of:

$$V_{cl} = (3\times10^{-2} \text{ cm})(1 \text{ cm}^2) = 3\times10^{-2} \text{ cm}^3$$

and a mass of 0.03 g (or $3\times10^{-5}$kg). This means that we would need to recycle

$$N = \frac{m_{bin}}{m_{cl}} = \frac{500 \text{ g}}{5\times10^{-3} \text{ g}} = 2\times10^4$$

contact lenses, just so that the mass of recycled material equals the mass of the recycling bin.

However, the contact lenses will also need to be collected, transported, cleaned, and processed, before being turned into some other (far less valuable) form of plastic. Thus, far more than $10^4$ contact lenses will need to be recycled to “pay” for the recycling bin.

Now let’s estimate the value of one contact lens. The value of that recycled plastic will be between one and 10 times its energy content, so we will estimate three times. The energy content of plastic is about equal to the energy content of gasoline. At current prices, gasoline is $3$ per gallon, or about $1$ per liter (or per kg). This means that the value of one contact lens is

$$C_{cl} = (3)(1 \$/\text{kg})(3\times10^{-5} \text{ kg})$$

or about one one-hundredth of a penny.

While I appreciate the efforts of my eye doctor to “go green,” this seems like a total waste of time.

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