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# Solutions for Fermi Questions, January 2022: Question 1: Snow Volume; Question 2: Longbow Arrow Velocity

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# Fermi Questions

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## Solutions for Fermi Questions, January 2022

### Question 1: Snow volume

How much snow falls on the U.S. each winter?

***Answer:*** To answer this we need to estimate the amount of snowfall and the area receiving snow. We might remember that the U.S. is about 5000 km wide (east-west). If we don't remember that, we could calculate it from the time zones (3 time zones is 3/24 = 1/8 of the Earth's circumference) or from a NY to LA plane ride (6 hours at 500 mph). The north-south distance is about half the east-west distance, giving a total area

$$A = (5\times10^3 \text{ km})(2\times10^3 \text{ km}) = 10^7 \text{ km}^2 = 10^{13}\text{m}^2.$$

The amount of the U.S. that receives significant snowfall (more than 1 ft or 0.3 m) is more than 10% and less than 100% of the total, giving an estimate of 30%.

The average amount of snow is more than 0.3 m and less than 10 m, giving an estimate of 2 m. Reality is closer to 1 m, indicating that I spent far too much time in Boston.

This gives a total snow volume of

$$\begin{aligned} V &= fAt \\ &= (0.3)(10^{13}\text{m}^2)(2 \text{ m}) \\ &= 6\times10^{13}\text{m}^3 \\ &= 6\times10^3 \text{ km}^3. \end{aligned}$$

That is a LOT of snow.



### Question 2: Longbow arrow velocity

Arrows from English longbows could penetrate plate armor. What is the velocity of an arrow shot from an English longbow?

***Answer:*** To answer this, we need to estimate the potential energy stored in the longbow and the mass of the arrow. To estimate the potential energy, we need to estimate the maximum force needed to pull the bow and the distance pulled.

A longbow arrow was about 1 yard (1 m) long with a diameter more than 2 and less than 20 mm, so we will take the geometric mean and estimate 6 mm (1/4 in). This gives a volume

$$\begin{aligned} V &= \pi r^2 l \\ &= 3(0.3 \text{ cm})^2(10^2 \text{ cm}) \\ &= 30 \text{ cm}^2. \end{aligned}$$

At a density of about 1 (g/cm$^3$, lb/pint, ton/m$^3$), this gives a mass of 30 g.

A longbow behaves as a giant spring, with $F = -kx$ and stored energy $E = \frac{1}{2}kx^2 = \frac{1}{2}xF_{\text{max}}$.

The maximum pull of a longbow is more than 10 lbs (much too wimpy) and less than $10^3$ lbs (humanly impossible), so we will estimate $10^2$ lbs, or $F = 5\times10^2$N. The distance pulled will be a little less than the arrow length, or about 1m. This gives a stored energy of

$$\begin{aligned} E &= \frac{1}{2}xF_{\text{max}} \\ &= \frac{1}{2}(1 \text{ m})(5\times10^2 \text{ N}) \\ &= 3\times10^2 \text{ J}. \end{aligned}$$

Thus the velocity of the arrow is

$$\begin{aligned} v &= \sqrt{2E/m} \\ &= \sqrt{\frac{2(3\times10^2 \text{ J})}{3\times10^{-2} \text{ kg}}} \\ &= 10^2 \text{ m/s} \end{aligned}$$

or about 200 mph.

By comparison, an AR-15 fires a bullet of mass about 3 g (10 times less than an arrow) with an initial speed of $10^3$ m/s (10 times greater), with about 1500 J of kinetic energy.