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High-energy effective action from scattering of QCD shock waves

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At high energies, the relevant degrees of freedom are Wilson lines—infinite gauge links ordered along straight lines collinear to the velocities of colliding particles. The effective action for these Wilson lines is determined by the scattering of QCD shock waves. I develop the symmetric expansion of the effective action in powers of strength of one of the shock waves and calculate the leading term of the series. The corresponding first-order effective action, symmetric with respect to projectile and target, includes both up and down fan diagrams and pomeron loops.

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I. INTRODUCTION

It is widely believed that the relevant degrees of freedom for the description of high-energy scattering in QCD are Wilson lines—infinite straightline gauge factors. An argument in favor of this goes as follows. As a result of a high-energy collision, we have a shower of produced particles in the whole range of rapidity between the target and the spectator. Let us demonstrate that the interaction of gluons with a different rapidity is described in terms of Wilson lines. Consider the fast particle interacting with some slow gluons. This particle moves along its classical trajectory—a straightline collinear to the velocity, and the only effect of the slow gluons is the phase factor $P\exp\{ig\int dx_\mu A^\mu\}$ ordered along the straightline classical path (here A_μ describes the slow gluons). This picture is reciprocal—in the rest frame of fast particles the fast and slow gluons trade places: former slow gluons move very fast so their propagator reduces to a Wilson-line made from the (former) fast gluons. We see that the particles with different rapidities perceive each other as Wilson lines and therefore these lines must be the relevant degrees of freedom for high-energy scattering. The goal of this approach is to rewrite the original functional integral over gluons (and quarks) as a $2+1$ theory with the effective action written in terms of the dynamical Wilson lines.

For a given interval of rapidity, the effective action is an amplitude of scattering of two QCD shock waves, see Fig. 1. Indeed, let us integrate over the gluons in this interval of rapidity $\eta_1 > \eta > \eta_2$ leaving the gluons with $\eta > \eta_1$ (the "right movers") and with $\eta < \eta_2$ (the "left movers") intact (to be integrated over later). Because of the Lorentz contraction, the left-moving and the right-moving gluons shrink to the two gluon "pancakes" or shock waves. The result of the integration over the rapidities $\eta_1 > \eta > \eta_2$ is the effective action which depends on the Wilson lines made from the left and right movers.

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Due the parton saturation at high energies [2–4], the characteristic scale of the transverse momenta in hadron-hadron collisions is $Q_s \sim e^{c\eta}$ [5–8] and therefore the collision of QCD shock waves can be treated using semi-classical methods [9]. Within the semiclassical approach, the problem of scatering of two shock waves can be reduced to the solution of classical YM equations with sources being the shock waves [10] (see also [11]). At present, these equations have not been solved. There are two approaches discussed in current literature: numerical simulations [12] and expansion in the strength of one of the shock waves. The collision of a weak and a strong shock waves corresponds to the deep inelastic scattering from a nucleus (and scattering of two strong shock waves describes a nucleus-nucleus collision). The first term of the expansion of the strength of one of the waves was calculated in a number of papers [13–15]. Recently, the classical field was calculated up to the second order in a weak source [16]. I will use some formulas of Ref. [16], although the main result for the effective action will be derived independently. The obtained effective action coincides with the expression obtained in Ref. [17] (see also [18,19]) from very different approach—evolution of the Color Glass Condensate in the Hamiltonian picture. The advantage of the derivation is that our method, symmetric with respect to projectile and target, has a "built-in" projectile-target duality (which is a highly nontrivial property of the light-cone Hamiltonian in the framework of the Hamiltonian

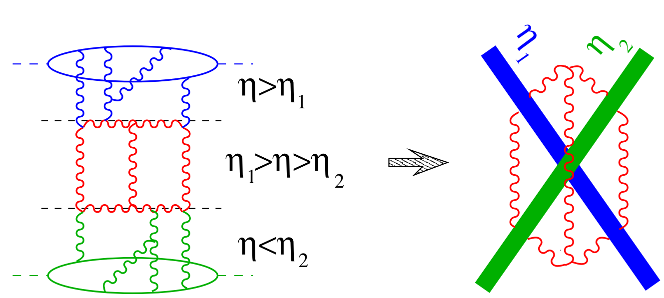

FIG. 1 (color online). High-energy effective action as an amplitude of the collision of two shock waves.

approach [17–21]). In terms of Feynman diagrams the effective action includes both "up" and "down" fan diagrams and therefore it describes pomeron loops which became a topic of intensive discussion in the current literature [17–23].

The paper is organized as follows. Sec. II is devoted to the rapidity factorization which is the starting point of the approach. In Sec. III I define the high-energy effective action as a scattering amplitude of QCD shock waves and develop the expansion in commutators of Wilson lines. In Sec. IV I find the effective action for a given (infinitesimal) range of rapidity in the leading order in this expansion. The corresponding functional integral over the dynamical Wilson-line variables is constructed in Sec. V. The explicit form of the first-order classical fields created by the collision of two shock waves is presented in the Appendix.

II. RAPIDITY FACTORIZATION

The main technical tool of the approach to the high-energy scattering is the rapidity factorization developed in [11,24]. Consider a functional integral for the typical scattering amplitude

$$\int DAJ(p_A)J(p_B)J(-p'_A)J(-p'_B)e^{iS(A)}, \tag{1}$$

where the currents $J(p_A)$ and $J(p_B)$ describe the two colliding particles (say, photons).

Throughout the paper, we use Sudakov variables

$$k = \alpha p_1 + \beta p_2 + k_\perp, \tag{2}$$

and the notations

$$\begin{aligned} x_\bullet &= p_1^\mu x_\mu = \sqrt{\frac{s}{2}}x^-, \qquad x^- = \frac{1}{\sqrt{2}}(x^0 - x^3),\\ x_* &= p_2^\mu x_\mu = \sqrt{\frac{s}{2}}x^+, \qquad x^+ = \frac{1}{\sqrt{2}}(x^0 + x^3). \end{aligned} \tag{3}$$

Here p_1 and p_2 are the lightlike vectors close to p_A and p_B: $p_A = p_1 + (p_A^2/s)p_2$, $p_B = p_2 + (p_B^2/s)p_1$.

Let us take some "rapidity divide" η_1 such that $\eta_A > \eta_1 > \eta_B$ and integrate first over the gluons with the rapidity $\eta > \eta_1$, see Fig. 2. From the viewpont of such particles, the fields with $\eta < \eta_1$ shrink to a shock wave so the result of the integration is presented by Feynman diagrams in the background. With the leading logarithmic approximation (LLA) accuracy, in the Feynman integrals over the gluons with $\eta > \eta_1$ one can set $\eta_1 \to -\infty$ (replace the "rapidity divide" vector $e_1 = p_1 + e^{-\eta_1}p_2$ by the lightlike vector p_2) so the shock wave is infinitely thin and lightlike. In the covariant gauge, this shock wave has the only nonvanishing component $A_\bullet$ which is concentrated near $x_* = 0$. In order to write down factorization we need to rewrite the shock wave in the temporal gauge $A_0 = 0$. In such gauge the most general form of a background is (see Fig. 3)

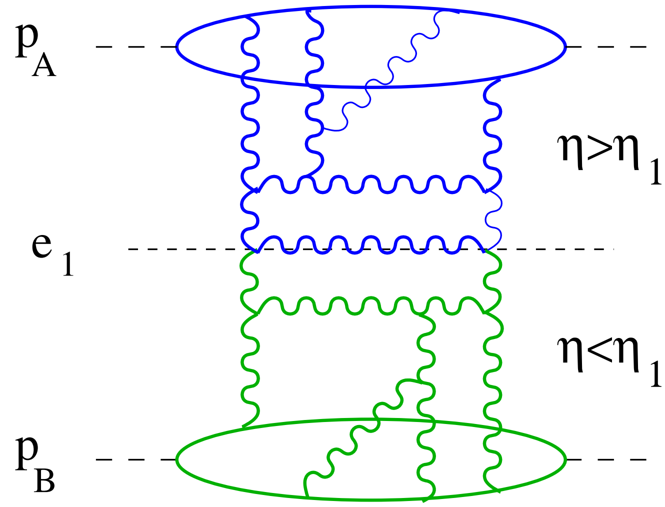

FIG. 2 (color online). Rapidity factorization.

$$A^i = \mathcal{U}_1^i\theta(x_*) + \mathcal{U}_2^i\theta(-x_*), \qquad A_\bullet = A_* = 0, \tag{4}$$

where

$$\mathcal{U}_1^i = U_1^\dagger \frac{i}{g}\partial_i U_1, \qquad \mathcal{U}_2^i = U_2^\dagger \frac{i}{g}\partial_i U_2, \tag{5}$$

are the pure gauge fields (filling the half-spaces $x_* > 0$ and $x_* < 0$). There is a redundant gauge symmetry

$$U_1(x_\perp) \to U_1(x_\perp)\Omega(x_\perp), \qquad U_2(x_\perp) \to U_2(x_\perp)\Omega(x_\perp), \tag{6}$$

related to the fact that gauge-invariant objects like the color dipole

$$\begin{aligned} &\mathrm{Tr}\{[\infty p_1, -\infty p_1]_x[x_\perp - \infty p_1, y_\perp - \infty p_1][-\infty p_1, \infty p_1]_y\\ &\quad\times [y_\perp + \infty p_2, x_\perp + \infty p_2]\} \simeq \mathrm{Tr}\{U_{1x}U_{2x}^\dagger U_{2y}U_{1y}^\dagger\} \end{aligned} \tag{7}$$

depend only on the product $U_{1z}U_{2z}^\dagger$. In papers [1,11] this symmetry was used to gauge away U_2 and simplify the shock wave to $A_i = \mathcal{U}_i\theta(x_*)$ while in Ref. [16] the opposite case $U_1 = 0$ ($A_i = \mathcal{U}_i\theta(-x_*)$) was considered. In the present paper we keep this gauge freedom—as we shall see below it simplifies the effective action for the Wilson-line integral.

The generating functional for the Green functions in the Eq. (4) has the form (cf. [1])

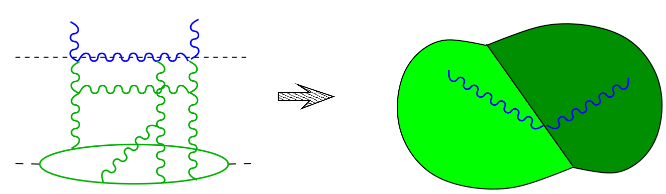

FIG. 3 (color online). Shock wave in the temporal gauge.

$$\int DAJ(p_A)J(-p'_A)e^{iS(A)+i\int d^2z_\perp(0,F_{*i},0)^a_z(\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z}, \quad (8)$$

where ($F_{ei}\equiv e^\mu F_{\mu i}$ etc.)

$$\begin{aligned}(0,F_{ei},0)_z &\equiv \int_{-\infty}^{\infty}du[0,ue]_zF_{ei}(ue+z_\perp)[ue,0]_z\\ &=[0,\infty e]_z\Big(i\frac{\partial}{\partial z^i}+gA_i(\infty e+z_\perp)\Big)[\infty e,0]_z\\ &\quad-[0,-\infty e]_z\Big(i\frac{\partial}{\partial z^i}+gA_i(-\infty e+z_\perp)\Big)\\ &\quad\times[-\infty e,0]_z, \end{aligned} \quad (9)$$

and $(0,F_{\mu i},0)^a\equiv 2\,\mathrm{tr}\,t^a(0,F_{\mu i},0)$. (Throughout the paper, the sum over the Latin indices i,j... runs over the two transverse components while the sum over Greek indices runs over the four components as usual).

It is easy to see that the functional integral (8) generates Green functions in the Eq. (4) background. Indeed, let us choose the gauge $A_*=0$ for simplicity. In this gauge, $(0,F_{*i},0)^a=A_i(\infty p_2+z_\perp)-A_i(-\infty p_2+z_\perp)$ so the functional integral (8) takes the form

$$\int DAJ(p_A)J(-p'_A)\times e^{iS(A)+i\int d^2z_\perp(A_i(\infty p_2+z_\perp)-A_i(-\infty p_2+z_\perp))^a(\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z}. \quad (10)$$

Let us now shift the fields $A_i\to A_i+\bar{A}_i$ and where $\bar{A}^i=\mathcal{U}_1^i\theta(x_*)+\mathcal{U}_2^i\theta(-x_*)$. The only nonzero components of the classical field strength in our case are $\bar{F}_{\bullet i}=(\mathcal{U}_{1i}-\mathcal{U}_{2i})\delta(\frac{2}{s}x_*)$ so we get

$$\begin{aligned}S(A+\bar{A})&=\frac{2}{s}\int d^4zD^i\bar{F}_{i\bullet}A^*\\ &\quad-\frac{2}{s}\int d^2z_\perp dz_*A^i\bar{F}_{\bullet i}\big|^{x_\bullet=\infty}_{x_\bullet=-\infty}\\ &\quad+\frac{1}{2}A^\mu(\bar{D}^2g_{\mu\nu}-2i\bar{F}_{\mu\nu})A^\nu+\ldots. \end{aligned} \quad (11)$$

In the $A_*=0$ gauge the first term in the right-hand side of Eq. (11) vanishes while the second term cancels with the corresponding contribution $\sim-(A_i(\infty p_2+z_\perp)-A_i(-\infty p_2+z_\perp))^a\mathcal{U}^{ai}$ coming from the source in Eq. (8). We obtain

$$\int DAJ(p_A)J(-p'_A)e^{iS(A)+i\int d^2z_\perp(A_i(\infty p_2+z_\perp)-A_i(-\infty p_2+z_\perp))^a(\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z}=\int DAJ(p_A)J(-p'_A)e^{i/2\int d^2zA^\mu(\bar{D}^2g_{\mu\nu}-2i\bar{F}_{\mu\nu})A^\nu}, \quad (12)$$

which gives the Green functions in the Eq. (4) background.

To complete the factorization formula one needs to integrate over the remaining B fields with rapidities $\eta<\eta_1$:

$$\int DAJ(p_A)J(-p'_A)e^{-iS(\mathcal{A})}e^{iS(A)}J(p_B)J(-p'_B)=\int DAJ(p_A)J(-p'_A)\times\int DBJ(p_B)J(-p'_B)e^{iS(A)+iS(B)+i\int d^2z_\perp(0,F_{e_1i},0)^a_z(0,G_{e_1i},0)^a_z}, \quad (13)$$

where the Wilson-line operators $(0,F_{e_1i},0)^a_z$ and $(0,G_{e_1i},0)^a_z$ are the operators (9) made from A and B fields, respectively. (At $(0,G_{*i},0)=\mathcal{U}_{1i}-\mathcal{U}_{2i}$ we recover the generating functional (8)).

Since the Wilson lines in Eq. (13) are lightlike the integration over rapidity in the corresponding Feynman diagrams extends all the way to ∞ and therefore we need to impose the conditions $\eta>\eta_1$ for A fields and $\eta_1>\eta$ for B fields "by hand". As discussed in [1,11,24,25], the better way to cut off tlongitudinal integrations is to change the slope of both Wilson lines to the "rapidity divide" vector $e_{\eta_1}=p_1+e^{-\eta_1}p_2$.:

$$\int DAJ(p_A)J(-p'_A)e^{-iS(\mathcal{A})}e^{iS(A)}J(p_B)J(-p'_B)=\int DAJ(p_A)J(-p'_A)\times\int DBJ(p_B)J(-p'_B)e^{iS(A)+iS(B)+i\int d^2z_\perp(0,F_{e_1i},0)^a_z(0,G_{e_1i},0)^a_z}. \quad (14)$$

Indeed, from the viewpoint of A fields the slope e_1 can be replaced by p_2 with the LLA accuracy. Conversely, from the viewpoint of the B fields the slope e_1 can be replaced by p_1 and we get back the Eq. (13). The advantage of the representation (14) is the "built-in" cutoff in rapidity for the integration over A and B fields.

III. SCATTERING OF OCD SHOCK WAVES

A. Effective action as a scattering amplitude

In this section we define the scattering of the shock waves using the rapidity factorization developed above. Applying the factorization formula (14) 2 times, one gets (see Fig. 4):

$$\int DAJ(p_A)J(p_B)J(-p'_A)J(-p'_B)e^{iS(A)} = \int DAJ(p_A)J(-p'_A)e^{iS(A)}\int DBJ(p_B)J(-p'_B)e^{iS(B)} \times \int DC\exp[iS(C) + i\int d^2z_\perp\{[0, A_{e_1i}, 0]^a_z[0, C_{e_1i}, 0]^a_z + (0, C_{e_2i}, 0)^a_z(0, B_{e_2i}, 0)^a_z\}] \quad (15)$$

where the slope is $e_1 = p_1 + e^{-\eta_1}p_2$ for the $[\ldots]$ Wilson lines and $e_2 = p_1 + e^{-\eta_2}p_2$ for the $(\ldots)$ ones.

The functional integral over the central range of rapidity $\eta_1 > \eta > \eta_2$ is determined by the integral over C field with the sources

$$\begin{aligned}(0, A_{e_1i}, 0)_z &= [0, \infty e_1]_z(i\partial_i + gA_i(\infty e_1 + z_\perp))[\infty e_1, 0]_z - [0, -\infty e_1]_z(i\partial_i + gA_i(-\infty e_1 + z_\perp))[-\infty e_1, 0]_z\\ (0, B_{e_2i}, 0)_z &= [0, \infty e_2]_z(i\partial_i + gB_i(\infty e_2 + z_\perp))[\infty e_2, 0]_z - [0, -\infty e_2]_z(i\partial_i + gB_i(-\infty e_2 + z_\perp))[-\infty e_2, 0]_z,\end{aligned} \quad (16)$$

made from "external" A and B fields. Since $A_i(\pm\infty)$ is a pure gauge these sources can be represented as a difference of a pure-gauge fields $(0, A_{e_1i}, 0)_z = (\mathcal{V}^{ai}_1 - \mathcal{V}^{ai}_2)_z$ and $(0, B_{e_2i}, 0)_z = (\mathcal{U}^{ai}_1 - \mathcal{U}^{ai}_2)_z$ where

$$V_{1,2}(z_\perp) = [0, \pm\infty e_1]_z[\pm\infty e_1 + z_\perp, \pm\infty e_1 + \infty e_\perp] \qquad U_{1,2}(z_\perp) = [0, \pm\infty e_2]_z[\pm\infty e_2 + z_\perp, \pm\infty e_2 + \infty e_\perp]. \quad (17)$$

Since there is no field strength $F_{\mu\nu}$ at infinite time the direction of $e_\perp$ does not matter.

The result of the integration over the C field is an effective action for the $\eta_1 > \eta > \eta_2$ interval of rapidity

$$e^{iS_{\rm eff}(V_1,V_2,U_1,U_2;\eta_1-\eta_2)} = \int DC\exp[iS(C) + i\int d^2z_\perp\{(\mathcal{V}^{ai}_1 - \mathcal{V}^{ai}_2)_z[0, C_{e_1i}, 0]^a_z + (\mathcal{U}^{ai}_1 - \mathcal{U}^{ai}_2)_z(0, C_{e_2i}, 0)^a_z\}]. \quad (18)$$

The amplitude (1) is then the integral over A and B fields with this effective action:

$$\int DAJ(p_A)J(p_B)J(-p'_A)J(-p'_B)e^{iS(A)} = \int DAJ(p_A)J(-p'_A)e^{iS(A)}\int DBJ(p_B)J(-p'_B)e^{iS(B)} \times \exp\{iS_{\rm eff}([0, A_{e_1i}, 0]^a_z, [0, B_{e_2i}, 0]^a_z, \eta_1 - \eta_2)\}. \quad (19)$$

Note that the effective action $iS_{\rm eff}(V_1, V_2, U_1, U_2; \eta_1 - \eta_2)$ defined by Eq. (18) is invariant under the redundant gauge transformations (6)

$$U_{1(2)}(x_\perp) \to U_{1(2)}(x_\perp)\Omega(x_\perp), \qquad V_{1(2)}(x_\perp) \to V_{1(2)}(x_\perp)\Omega'(x_\perp), \quad (20)$$

since this transformation can be absorbed by a gauge rotation of the C fields

$$C_\mu \to \Omega^\dagger\Big(x_\perp, \ln\frac{x_*}{x_\bullet}\Big)C_\mu\Omega\Big(x_\perp, \ln\frac{x_*}{x_\bullet}\Big) + \frac{i}{g}\Omega^\dagger\Big(x_\perp, \ln\frac{x_*}{x_\bullet}\Big)\partial_\mu\Omega\Big(x_\perp, \ln\frac{x_*}{x_\bullet}\Big), \quad (21)$$

where $\Omega(x_\perp, \ln\frac{x_*}{x_\bullet})$ is an arbitrary SU_3 matrix satisfying the conditions $\Omega^\dagger(x_\perp, \eta_1) = \Omega(x_\perp)$ and $\Omega^\dagger(x_\perp, \eta_2) = \Omega'(x_\perp)$.

With a power accuracy $O(m^2/s)$, we can replace e_1 by p_1 and e_2 by p_2:

$$e^{iS_{\rm eff}(V_1,V_2,U_1,U_2;\eta_1-\eta_2)} = \int DC\exp\{iS(C) + i\int d^2z_\perp[(\mathcal{V}^{ai}_1 - \mathcal{V}^{ai}_2)_z[0, C_{\bullet i}, 0]^a_z + (\mathcal{U}^{ai}_1 - \mathcal{U}^{ai}_2)_z(0, C_{*i}, 0)^a_z]\}. \quad (22)$$

One can interpret Eq. (22) as an effective action for scattering of two QCD shock waves defined by the sources (17):

$$\begin{aligned}A^i_{(1)} &= \mathcal{U}^i_1\theta(x_*) + \mathcal{U}^i_2\theta(-x_*), \qquad A_\bullet = A_* = 0\\ A^i_{(2)} &= \mathcal{V}^i_1\theta(x_\bullet) + \mathcal{V}^i_2\theta(-x_\bullet), \qquad A_\bullet = A_* = 0.\end{aligned} \quad (23)$$

The saddle point of the functional integral (22) is determined by the classical equations

$$\frac{\delta}{\delta C^a_\mu}\Big\{S(C) + i\int d^2z_\perp[(\mathcal{V}^{ai}_1 - \mathcal{V}^{ai}_2)_z[0, C_{\bullet i}, 0]^a_z + (\mathcal{U}^{ai}_1 - \mathcal{U}^{ai}_2)_z(0, C_{*i}, 0)^a_z]\Big\} = 0. \quad (24)$$

The solution of this equation determines the classical field $\bar{A}$ created by the collision of two shock waves (23) and the effective action is the expression (22) evaluated at this

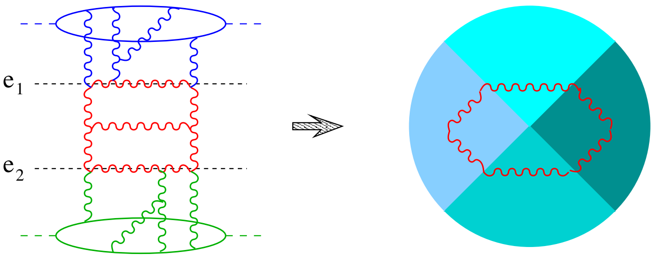

FIG. 4 (color online). Effective action as a scattering of two shock waves.

classical field. Unfortunately, at present it is not known how to solve the Eq. (24) (for the numerical approach see [12]). In the next section we will develop a "perturbation theory" in powers of the parameter $[U,V]\sim g^2[\mathcal{U}_i,\mathcal{V}_j]$. Note that the conventional perturbation theory corresponds to the case when $\mathcal{U}_i,\mathcal{V}_i\sim 1$ while the semiclassical QCD is relevant when the fields are large ($\mathcal{U}_i$ and/or $\mathcal{V}_i\sim 1/g$).

It is worth noting that for the scattering of two heavy nuclei with atomic numbers A and B this parameter is $[U,V]\sim g^4A^{1/6}B^{1/6}$.

B. Expansion in commutators of Wilson lines

The effective action is defined by the functional integral (22) (hereafter we switch back to the usual notation A_μ for the integration variable and $F_{\mu\nu}$ for the field strength)

$$e^{iS_{\rm eff}(V_1,V_2,U_1,U_2;\eta_1-\eta_2)} = \int DA\exp\Big(iS(A) + i\int d^2z_\perp\{(\mathcal{V}_1^{ai}-\mathcal{V}_2^{ai})_z[0,F_{\bullet i},0]_z^a + (\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z(0,F_{*i},0)_z^a\}\Big). \qquad (25)$$

Taken separately, the sources $\sim\mathcal{U}_i$ create a shock wave $\mathcal{U}_{1i}\theta(x_*)+\mathcal{U}_{2i}\theta(-x_*)$ and those $\sim\mathcal{V}_i$ create $\mathcal{V}_{1i}\theta(x_\bullet)+\mathcal{V}_{2i}\theta(-x_\bullet)$ In QED, the two sources $\mathcal{U}_i$ and $\mathcal{V}_i$ do not interact (in the leading order in α) so the sum of the two shock waves

$$\bar A_i^{(0)} = \mathcal{U}_{1i}\theta(x_*)+\mathcal{U}_{2i}\theta(-x_*)+\mathcal{V}_{1i}\theta(x_\bullet) + \mathcal{V}_{2i}\theta(-x_\bullet), \qquad (26)$$
$$\bar A_\bullet^{(0)} = \bar A_*^{(0)} = 0$$

is a classical solution to the set of Eqs. (24). In QCD, the interaction between these two sources is described by the commutator $g[\mathcal{U}_i,\mathcal{V}_k]$ (the coupling constant g corresponds to the three-gluon vertex). The straightforward approach is to take the trial configuration in the form of a sum of the two shock waves and expand the "deviation" of the full QCD solution from the QED-type ansatz (26) in powers of commutators $[U,V]$. This is done rigorously in [16] and the relevant formulas are presented in the Appendix. Here we will use a slightly different zero-order approximation (cf. [1]) which leads to same results in a more streamlined way at a price of some uncertainties (like $\theta(0)$) which, however, do not contribute to the effective action in the leading order.

Let us consider the behavior of the solution of the YM equations at, say, $x_0\to\infty$, x_3 fixed (in the forward quadrant of the space). Since there is no field strength at $t\to\infty$, the field must be a pure gauge. As demonstrated in Ref. [16], this pure-gauge field has the form of a sum of the shock waves plus a correction proportional to their commutator. Technically, for a pair of pure-gauge fields $\mathcal{U}_i(x_\perp)$ and $\mathcal{V}_i(x_\perp)$ we define $\mathcal{W}_i(x_\perp)=\mathcal{U}_i(x_\perp)+\mathcal{V}_i(x_\perp)+gE_i(x_\perp;U,V)$ as a pure-gauge field satisfying the equation $(i\partial_i+g[\mathcal{U}_i+\mathcal{V}_i,)E^i=0$. In the first order in $[U,V]$ this field has the form

$$E_i^a(U,V) = -\Big(x_\perp\Big|U\frac{p^k}{p_\perp^2}U^\dagger + V\frac{p^k}{p_\perp^2}V^\dagger - \frac{p^k}{p_\perp^2}\Big|^{ab} \times[\mathcal{U}_i,\mathcal{V}_k]^b - i\leftrightarrow k) + O([U,V]^2)\Big) \qquad (27)$$

where $[\mathcal{U}_i,\mathcal{V}_k]^a\equiv 2\,{\rm Tr}\,t^a[\mathcal{U}_i,\mathcal{V}_k]$. The second, $[U,V]^2$, term of the expansion (27) can be found in [16] but we do not need it with our accuracy.

Throughout the paper, we use Schwinger notations for the propagator in the external field $(x|1/P^2|y)$. For the bare propagator it reduces to $(x|1/p^2|y)$ and for the two-dimensional propagator in the transverse space we use the notation $(x_\perp|1/p_\perp^2|y_\perp)$ where $p_\perp^2=-p_ip^i$. Also, $|f)$ denotes $\int d^2z_\perp f(z_\perp)|z_\perp)$ and later we will use the notation $|0,f)\equiv\int d^2z_\perp f(z_\perp)|0,z_\perp)$.

The zero-order approximation for the solution of the classical equations for the functional integral (25) can be taken as a superposition of pure-gauge fields in the forward, backward, left, and right quadrants of the space (see Fig. 5):

$$\bar A_\bullet^{(0)} = \bar A_*^{(0)} = 0$$
$$\bar A^{(0)i} = \mathcal{W}_F^i(x_\perp)\theta(x_*)\theta(x_\bullet)+\mathcal{W}_L^i(x_\perp)\theta(-x_*)\theta(x_\bullet) + \mathcal{W}_R^i(x_\perp)\theta(x_*)\theta(-x_\bullet)+\mathcal{W}_B^i(x_\perp)\theta(-x_*)\theta(-x_\bullet), \qquad (28)$$

where

$$\mathcal{W}_F^i = \mathcal{U}_1^i+\mathcal{V}_1^i+E_F^i, \qquad \mathcal{W}_L^i = \mathcal{U}_2^i+\mathcal{V}_1^i+E_L^i,$$
$$\mathcal{W}_R^i = \mathcal{U}_1^i+\mathcal{V}_2^i+E_R^i, \qquad \mathcal{W}_B^i = \mathcal{U}_2^i+\mathcal{V}_2^i+E_B^i, \qquad (29)$$

and $E_F^i(U_1,V_1)$, $E_L^i(U_2,V_1)$, $E_R^i(U_1,V_2)$, and $E_B^i(U_2,V_2)$ are given by Eq. (27). For the trial configuration (28)

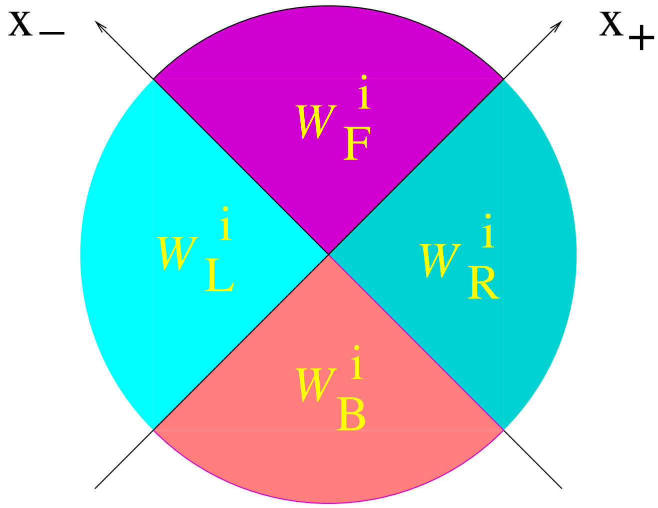

FIG. 5 (color online). Pure-gauge ansatz as a zero-order approximation to the classical field created by the collision of shock waves.

$$\bar{F}^i_{\bullet} = \delta\Big(\frac{2x_*}{s}\Big)\{\theta(x_\bullet)(\mathcal{W}^i_F - \mathcal{W}^i_L) + \theta(-x_\bullet)\times(\mathcal{W}^i_R - \mathcal{W}^i_B)\},$$
$$\bar{F}^i_{*} = \delta\Big(\frac{2x_\bullet}{s}\Big)\{\theta(x_*)(\mathcal{W}^i_F - \mathcal{W}^i_R) + \theta(-x_*)\times(\mathcal{W}^i_L - \mathcal{W}^i_B)\}, \quad (30)$$

so

$$D^i\bar{F}_{\bullet i} = \delta\Big(\frac{2}{s}x_*\Big)([\theta(x_\bullet)(\partial^i - i[\bar{A}^i,)(\mathcal{W}_{Fi} - \mathcal{W}_{Li}) + \theta(-x_\bullet)(\partial^i - i[\bar{A}^i,)(\mathcal{W}_{Ri} - \mathcal{W}_{Bi}),$$
$$D^i\bar{F}_{*i} = \delta\Big(\frac{2}{s}x_\bullet\Big)(\theta(x_*)(\partial^i - i[\bar{A}^i,)(\mathcal{W}_{Fi} - \mathcal{W}_{Ri}) + \theta(-x_*)(\partial^i - i[\bar{A}^i,)(\mathcal{W}_{Li} - \mathcal{W}_{Bi}), \quad (31)$$

and

$$D_*\bar{F}_{\bullet i} = D_*\bar{F}_{\bullet i} = \delta\Big(\frac{2}{s}x_*\Big)\delta\Big(\frac{2}{s}x_\bullet\Big)(\mathcal{W}_{Fi} - \mathcal{W}_{Ri} - \mathcal{W}_{Li} + \mathcal{W}_{Bi}) = \delta\Big(\frac{2}{s}x_*\Big)\delta\Big(\frac{2}{s}x_\bullet\Big)(E_{Fi} - E_{Ri} - E_{Li} + E_{Bi}). \quad (32)$$

Next, one shifts $A \to A + \bar{A}^{(0)}_i$ in the functional integral (25) and obtains

$$e^{iS_{\text{eff}}(V_1,V_2,U_1,U_2;\eta_1-\eta_2)} = \int DA \exp\Big\{iS(\bar{A}) + i\int d^4z\Big(\frac{1}{2}A^\mu\bar{D}_{\mu\nu}A^\nu + T^\mu A_\mu\Big)\Big\}. \quad (33)$$

Here

$$\bar{S} = \frac{1}{2}\int d^2z_\perp\Big\{(\mathcal{V}_1 - \mathcal{V}_2)^a_i(\mathcal{W}^i_F - \mathcal{W}^i_L + \mathcal{W}^i_R - \mathcal{W}^i_B)^{ia} + (\mathcal{U}_1 - \mathcal{U}_2)^a_i(\mathcal{W}_F + \mathcal{W}_L - \mathcal{W}_R - \mathcal{W}_B)^{ia} - \frac{1}{2}(\mathcal{W}_F - \mathcal{W}_L + \mathcal{W}_R - \mathcal{W}_B)^{ia}(\mathcal{W}_F + \mathcal{W}_L - \mathcal{W}_R - \mathcal{W}_B)^a_i\Big\}, \quad (34)$$

is a sum of the action and source contributions due to the trial configuration (28), $D_{\mu\nu} = D^2(\bar{A})g_{\mu\nu} - 2i\bar{F}_{\mu\nu}$ is the inverse propagator in the background-Feynman gauge[1] and T_μ is the linear term for our trial configuration:

$$T_i = 2\delta\Big(\frac{2}{s}x_*\Big)\delta(x_\bullet)(\mathcal{W}^i_F - \mathcal{W}^i_L - \mathcal{W}^i_R + \mathcal{W}^i_B) = 2\delta\Big(\frac{2}{s}x_*\Big)\delta(x_\bullet)(E^i_F - E^i_L - E^i_R + E^i_B)$$
$$T_* = -\frac{i}{2}\delta\Big(\frac{2}{s}x_\bullet\Big)(\theta(x_*)[\mathcal{V}_{1i} - \mathcal{V}_{2i}, E^i_F + E^i_R] + \theta(-x_*)[\mathcal{V}_{1i} - \mathcal{V}_{2i}, E^i_L + E^i_B]) \quad (35)$$
$$T_\bullet = -\frac{i}{2}\delta\Big(\frac{2}{s}x_*\Big)(\theta(x_\bullet)[\mathcal{U}_{1i} - \mathcal{U}_{2i}, E^i_F + E^i_L] + \theta(-x_*)[\mathcal{U}_{1i} - \mathcal{U}_{2i}, E^i_R + E^i_B]).$$

The first line in this equation follows directly from Eq. (32) while the two last lines are obtained by adding Eqs. (31) and the corresponding first derivatives of the sources (24)

[1]Strictly speaking, the inverse propagator is the sum of $D_{\mu\nu}$ and the second variational derivative of the source (8), see Ref. [26].

$$\frac{\delta}{\delta A_*}\int d^2z_\perp(\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z(0,F_{*i},0)_z^a\Big|_{A_*=0}=\delta(x_*)\{[\theta(x_\bullet)(\partial^i-i[\bar{A}^i(\infty p_2+x_\perp),)(\mathcal{U}_{1i}-\mathcal{U}_{2i})+\theta(-x_\bullet)(\partial^i-i[\bar{A}^i(-\infty p_2+x_\perp),)(\mathcal{U}_{1i}-\mathcal{U}_{2i})\}$$
$$\frac{\delta}{\delta A_\bullet}\int d^2z_\perp[(\mathcal{V}_1^{ai}-\mathcal{V}_2^{ai})_z[0,F_{\bullet i},0]_z^a\Big|_{A_\bullet=0}=\delta(x_\bullet)\{\theta(x_*)(\partial^i-i[\bar{A}^i(\infty p_1+x_\perp),)(\mathcal{V}_{1i}-\mathcal{V}_{2i})+\theta(-x_*)(\partial^i-i[\bar{A}^i(-\infty p_1+x_\perp),)(\mathcal{V}_{1i}-\mathcal{V}_{2i})\}. \tag{36}$$

We get

$$\frac{\delta}{\delta A_*}\Big\{S_{\rm QCD}+\int d^2z_\perp(\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z(0,F_{*i},0)_z^a\Big\}\Big|_{A_*=0}=-\delta(x_*)\{\theta(x_\bullet)(\partial^i-i[\bar{A}^i(\infty p_2+x_\perp),)(E_{Fi}-E_{Li})+\theta(-x_\bullet)(\partial^i-i[\bar{A}^i(-\infty p_2+x_\perp),)(E_{Ri}-E_{Bi})\Big\},$$
$$\frac{\delta}{\delta A_\bullet}\Big\{S_{\rm QCD}+\int d^2z_\perp[(\mathcal{V}_1^{ai}-\mathcal{V}_2^{ai})_z[0,F_{\bullet i},0]_z^a\Big\}\Big|_{A_\bullet=0}=-\delta(x_\bullet)\{\theta(x_*)(\partial^i-i[\bar{A}^i(\infty p_1+x_\perp),)(E_{Fi}-E_{Ri})+\theta(-x_*)(\partial^i-i[\bar{A}^i(-\infty p_1+x_\perp),)(E_{Li}-E_{Bi})\}. \tag{37}$$

Using $\theta(0)=1/2$ so that $\bar{A}^i(\infty p_2+x_\perp)=1/2(\mathcal{W}_{Ri}+\mathcal{W}_{Bi})$, $\bar{A}^i(-\infty p_2+x_\perp)=1/2(\mathcal{W}_{Li}+\mathcal{W}_{Bi})$, $\bar{A}^i(\infty p_1+x_\perp)=1/2(\mathcal{W}_{Fi}+\mathcal{W}_{Ri})$, $\bar{A}^i(-\infty p_1+x_\perp)=1/2(\mathcal{W}_{Li}+\mathcal{W}_{Bi})$, and the condition $(i\partial_i+[\mathcal{W}_i,)E^i=0$ one easily obtains Eq. (35).[2]

Expansion in powers of T in the functional integral (33) yields the set of Feynman diagrams in the external fields (28) with the sources (35). The parameter of the expansion is $g^2[\mathcal{U}_i,\mathcal{V}_j]$ ($\sim[U,V]$, see Eq. (5)).

IV. THE EFFECTIVE ACTION

A. The effective action in the lowest order

The effective action (25) in the first nontrivial order in $[U,V]$ is given by the integration of linear terms (35) with the Green functions in the external field (28)

$$iS_{\rm eff}(U,V)=-\frac{1}{2}\int d^4zd^4z'T_\mu^a(z)\langle A^{\mu a}(z)A^{\nu b}(z')\rangle T_\nu^b(z'). \tag{38}$$

It is easy to see that the term $\sim T_*T_\bullet$ is $\sim[U,V]^3$ so the leading contribution $\sim[U,V]^2$ comes from the product of two T_i's which has the form

$$\frac{i}{2}\int d^2z_\perp d^2z'_\perp L_i^a(z_\perp)\Big(0,z_\perp\Big|\frac{1}{P^2+i\epsilon}\Big|0,z'_\perp\Big)^{ab}L^{bi}(z'_\perp), \tag{39}$$

where

$$L_i\equiv 2(E_F^i-E_L^i-E_R^i+E_B^i)=2(\mathcal{W}_F^i-\mathcal{W}_L^i-\mathcal{W}_R^i+\mathcal{W}_B^i) \tag{40}$$

is actually the transverse part of the Lipatov vertex of the gluon emission by the scattering of two shock waves in the first order in $[U,V]$ (see Appendix). As we shall see below, the main logarithmic contribution to the integral (39) comes from the region $z_\perp\to z'_\perp$ where one can replace the propagator in the background field by the bare propagator. One obtains

$$ig^2\frac{s}{2}\int\frac{d\alpha d\beta}{8\pi^2}\Big(0,L_i^a\Big|\frac{1}{\alpha\beta s-p_\perp^2+i\epsilon}\Big|0,L^{ia}\Big). \tag{41}$$

The integral (41) is formally divergent. Within the LLA accuracy, one can cut the integration off at the width of the shock waves $\lambda\sim\sqrt{(s/m^2)}e^{-\eta_1/2}$, $\rho\sim\sqrt{(s/m^2)}e^{\eta_2/2}$ and obtain:

$$ig^2\frac{s}{2}\int\frac{d\alpha d\beta}{8\pi^2}e^{-i(\alpha\lambda+\beta\rho)}\Big(0,L_i^a\Big|\frac{1}{\alpha\beta s-p_\perp^2+i\epsilon}\Big|0,L^{ia}\Big)=\frac{\alpha_s\Delta\eta}{4}\int d^2z_\perp L_i^a(z_\perp)L^{ia}(z_\perp), \tag{42}$$

where $\Delta\eta=\eta_1-\eta_2$ is our rapidity interval.

In addition, within the LLA approximation the zero-order term (34) can be simplified to

$$\bar{S}=\int d^2z_\perp(\mathcal{V}_1-\mathcal{V}_2)^{ia}(\mathcal{U}_1-\mathcal{U}_2)_i^a. \tag{43}$$

Indeed, it is easy to see that in the right-hand side of Eq. (34) the terms $\sim[U,V]$ cancel while the the terms $\sim[U,V]^2$ are not multiplied by $\Delta\eta$ so they can be omitted in the LLA (the Eq. (42) is $\sim[U,V]^2\Delta\eta$). It is worth noting that Eq. (43) is the usual light-cone lattice action [27] in the

[2] A careful analysis shows that the "formula" $\theta(0)=1/2$ is not valid here. It can be demonstrated that instead of $1/2(E_F+E_L)\int_0^\infty dz_*A_\bullet(z_*)$ one should use $E_F\int_0^\infty dz_*A_\bullet^{(+)}(z_*)+E_L\int_0^\infty dz_*A_\bullet^{(-)}(z_*)$ where $A^{(+)}$ and $A^{(-)}$ are the positive and negative frequency parts of the field A. (With such T one reproduces the correct set of fields A_* and $A_\bullet$ given by Eq. (A8) from the Appendix). Fortunately, the corresponding contribution to the effective action is $\sim T_*T\bullet\sim[U,V]^3$ which exceeds our accuracy.

limit when transverse size of the plaquette vanishes and the longitudinal increases to infinity. Thus, the effective action in the first order can be represented as

$$S_{\rm eff}(U,V)=\int d^2z_\perp\Big\{(\mathcal{V}_1-\mathcal{V}_2)^a_i(\mathcal{U}_1-\mathcal{U}_2)^a_i - i\frac{\alpha_s\Delta\eta}{4}\int d^2z_\perp L^a_i(z_\perp)L^{ia}(z_\perp)\Big\}. \quad (44)$$

We shall see below that L_i is the Lipatov vertex of the gluon emission by the scattering of two shock waves in the first order in $[U,V]$. Note that $S_{\rm eff}$ given by Eq. (44) is invariant with respect to rotation of the sources

$$U_j\to U_j\Omega,\qquad V_j\to V_j\Omega \quad (45)$$

For the first term in the right-hand side of Eq. (44) it is trivial while for the second it follows from the gauge-invariant form discussed in the next section, see Eq. (59).

For future applications we will rewrite the effective action (44) as a Gaussian integration over the auxiliary field λ coupled to Lipatov vertex (A9):

$$e^{iS_{\rm eff}(U,V)}=e^{i\int d^2z_\perp(\mathcal{V}_1-\mathcal{V}_2)^{ia}(\mathcal{U}_1-\mathcal{U}_2)^a_i}\times\int D\lambda\exp\Big\{-\alpha_s\Delta\eta\int d^2z_\perp(\lambda^a_i\lambda^{ai}-L^a_i\lambda^{ai})\Big\}. \quad (46)$$

B. Nonlinear evolution equation from the effective action

Let us prove now that the effective action (46) agrees with the nonlinear evolution equation. To find the evolution of the dipole $U_xU^\dagger_y$, we need to consider the effective action for the weak source V. From Eq. (27) one sees that at small $g\mathcal{V}_i\sim\partial_iV$

$$L^a_i(x_\perp)=-2\Big(x\,\Big|\,U^\dagger_1\frac{p_ip^k}{p^2_\perp}U_1-U^\dagger_2\frac{p_ip^k}{p^2_\perp}U_2\,\Big|^{ab}\times(\mathcal{V}_1-\mathcal{V}_2)^b_k\Big), \quad (47)$$

and Eq. (25) can be rewritten as

$$\int DA\exp\Big\{iS(A)+i\int d^2z_\perp[(\mathcal{V}^{ai}_1-\mathcal{V}^{ai}_2)_z[0,F_{\bullet i},0]^a_z+(\mathcal{U}^{ai}_1-\mathcal{U}^{ai}_2)_z(0,F_{*i},0)^a_z]\Big\} = \int D\lambda e^{\int d^2z_\perp\{-\alpha_s\Delta\eta\lambda^a_i\lambda^{ai}+i(\mathcal{V}_1-\mathcal{V}_2)^a_i(\tilde{\mathcal{U}}^i_1-\tilde{\mathcal{U}}^i_2)^a\}}. \quad (48)$$

Here

$$\tilde U_1=e^{2\alpha_s\Delta\eta(\partial_i/\partial^2_\perp)(U_1\lambda^iU^\dagger_1)}U_1, \quad (49)$$

$$\tilde U_2=e^{2\alpha_s\Delta\eta(\partial_i/\partial^2_\perp)(U_2\lambda^iU^\dagger_2)}U_2, \quad (50)$$

so that

$$\tilde{\mathcal{U}}^a_{1i}=\mathcal{U}^a_{1i}+2\alpha_s\Delta\eta\Big(U^\dagger_1\frac{\partial_i\partial^k}{\partial^2_\perp}U_1\Big)^{ab}\lambda^b_k, \quad (51)$$

$$\tilde{\mathcal{U}}^a_{2i}=\mathcal{U}^a_{2i}+2\alpha_s\Delta\eta\Big(U^\dagger_2\frac{\partial_i\partial^k}{\partial^2_\perp}U_2\Big)^{ab}\lambda^b_k, \quad (52)$$

where we need only the first term in expansion in λ_i in λ.[3]

To find the evolution of the color dipole (7) we should expand Eq. (48) in powers of $\mathcal{V}_{1i}-\mathcal{V}_{2i}$ and use the formula

$$[0,\infty p_1]_x[x_\perp+\infty p_1,y_\perp+\infty p_1][\infty p_1,0]^{\eta_2}_y = Pe^{ig\int_x^y dz_i[0,F_{\bullet i},0]_z}, \quad (53)$$

which results in

$$U^\dagger_{1y}U_{1x}U^\dagger_{2x}U_{2y}=\int D\lambda e^{-\alpha_s\Delta\eta\int d^2z_\perp\lambda^a_i\lambda^{ai}}\tilde U^\dagger_{1y}\tilde U_{1x}\tilde U^\dagger_{2x}\tilde U_{2y}. \quad (54)$$

Performing the Gaussian integration over λ one obtains after some algebra

$$\mathrm{tr}\{U_{1x}U^\dagger_{2x}U_{2y}U^\dagger_{1y}\}=\mathrm{tr}\{U_{1x}U^\dagger_{2x}U_{2y}U^\dagger_{1y}\}+\frac{\alpha_s\Delta\eta}{4\pi^2}\int d^2z_\perp\frac{(x-y)^2_\perp}{(x-z)^2_\perp(z-y)^2_\perp}\times(\mathrm{tr}\{U_{1x}U^\dagger_{2x}U_{2z}U^\dagger_{1z}\}\mathrm{tr}\{U_{1z}U^\dagger_{2z}U_{2y}U^\dagger_{1y}\} - N_c\mathrm{tr}\{U_{1x}U^\dagger_{2x}U_{2y}U^\dagger_{1y}\}) \quad (55)$$

which is the nonlinear evolution equation [26,28] for the Wilson-line operator $U_x=U_{1x}U^\dagger_{2x}=[\infty e,-\infty e]_x$.

[3]To cancel the UV divergence in the gluon-Reggeization term $\sim 2t^aU_xt^b(x|(p_i/p^2_\perp)U^{ab}(p_i/p^2_\perp)|y)$ we need the second-order contribution $c_F(x|1/p^2_\perp|x)U_x+c_F(x|1/p^2_\perp|x)U_y$. However, since the pure divergency is set to zero in the dimensional regularization, at least within this regularization the first term is sufficient.

C. Gauge-invariant representation of the first-order effective action $L_i L^i$

Our expression for the $\Delta\eta$ term in the effective action, proportional to the square of the Lipatov vertex $(\mathcal{W}^i_F - \mathcal{W}^i_L - \mathcal{W}^i_R + \mathcal{W}^i_B)^a$, was obtained in the axial-type gauges. It can be rewritten it in the gauge-invariant "diamond" form of trace of four Wilson lines at $x_{\bullet,*} = \pm\infty$ (see Fig. 6) as suggested in a recent paper [17]. The diamond Wilson loop is defined as follows

$$\begin{aligned}\diamond(x_\perp) \equiv{}& \mathrm{tr}\{[-\infty p_1, F_{\bullet i}, \infty p_1]_{\infty p_2}[\infty p_2, F_{*i}, -\infty p_2]_{\infty p_1}[\infty p_1, -\infty p_1]_{-\infty p_2}[-\infty p_2, \infty p_2]_{-\infty p_1} + \mathrm{tr}[-\infty p_1, \infty p_1]_{\infty p_2}\\ &\times[\infty p_2, F_{*i}, -\infty p_2]_{\infty p_1}[\infty p_1, F_{\bullet i}, -\infty p_1]_{-\infty p_2}[-\infty p_2, \infty p_2]_{-\infty p_1} + \mathrm{tr}[-\infty p_1, \infty p_1]_{\infty p_2}[\infty p_2, -\infty p_2]_{\infty p_1}\\ &\times[\infty p_1, F_{\bullet i}, \infty p_1]_{-\infty p_2}[-\infty p_2, F_{*i}, \infty p_2]_{-\infty p_1} + \mathrm{tr}[-\infty p_1, F_{\bullet i}, \infty p_1]_{\infty p_2}[\infty p_2, -\infty p_2]_{\infty p_1}[\infty p_1, -\infty p_1]_{-\infty p_2}\\ &\times[-\infty p_2, F_{*i}, \infty p_2]_{-\infty p_1}\},\end{aligned} \tag{56}$$

where the transverse arguments in all Wilson lines are $x_\perp$. Next, define this diamond as a function of the sources

$$\diamond(U_1, U_2, V_1, V_2) \equiv \mathcal{N}^{-1}\int DA\diamond(A)\exp\Big(iS(A) + i\int d^2z_\perp\{(\mathcal{V}^{ai}_1 - \mathcal{V}^{ai}_2)_z[0, F_{e_1 i}, 0]^a_z + (\mathcal{U}^{ai}_1 - \mathcal{U}^{ai}_2)_z(0, F_{e_2 i}, 0)^a_z\}\Big). \tag{57}$$

(In the Appendix we demonstrate that the trace of four Wilson lines

$$\mathrm{tr}\{[-\infty e_1, \infty e_1]_{\infty e_2}[\infty e_2, -\infty e_2]_{\infty e_1}[\infty e_1, -\infty e_1]_{-\infty e_2}[-\infty e_2, \infty e_2]_{-\infty e_1}\} = 1 \tag{58}$$

is trivial in the leading order.)

Note that $\diamond(U, V)$ is invariant with respect to the rotation of all sources by one gauge matrix $\Omega(x_\perp)$

$$\diamond(U_1\Omega, U_2\Omega, V_1\Omega, V_2\Omega) = \diamond(U_1, U_2, V_1, V_2), \tag{59}$$

since it can be absorbed by gauge transformation of the fields $A_\mu \to \Omega^\dagger A_\mu \Omega + \frac{i}{g}\Omega^\dagger\partial_\mu\Omega$ in the functional integral Eq. (57).

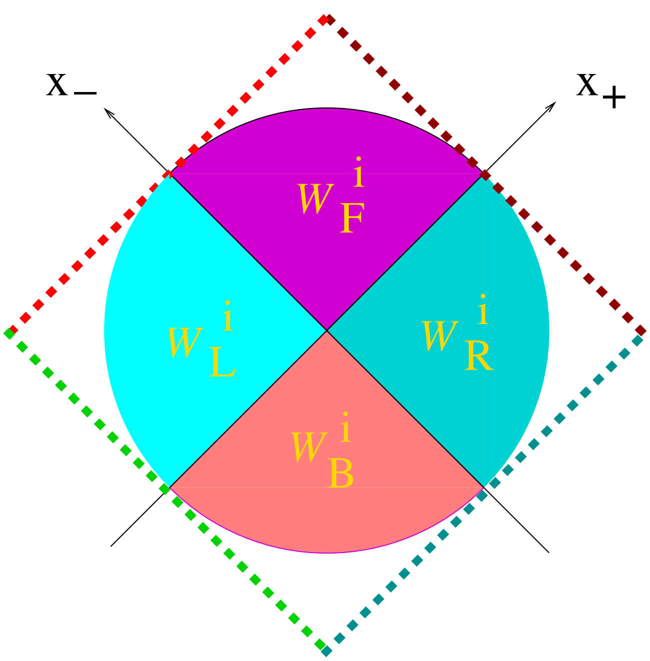

FIG. 6 (color online). Gauge-invariant form of the effective action.

Now we can prove that the square of Lipatov vertex can be expressed as the diamond Wilson loop:

$$\frac{1}{4}L^a_i(U, V)L^{ai}(U, V) = \diamond(U, V). \tag{60}$$

Indeed, it is easy to see that for the trial configuration (28) the Eq. (56) reduces to

$$\begin{aligned}&(\mathcal{W}_{Fi} - \mathcal{W}_{Li})^a(\mathcal{W}^i_F - \mathcal{W}^i_R)^a + (\mathcal{W}_{Ri} - \mathcal{W}_{Fi})^a\\ &\quad\times(\mathcal{W}^i_R - \mathcal{W}^i_B)^a + (\mathcal{W}_{Bi} - \mathcal{W}_{Ri})^a(\mathcal{W}^i_B - \mathcal{W}^i_L)^a\\ &\quad+ (\mathcal{W}_{Li} - \mathcal{W}_{Bi})^a(\mathcal{W}^i_L - \mathcal{W}^i_F)^a\\ &= (\mathcal{W}_{Fi} - \mathcal{W}_{Li} - \mathcal{W}^i_R + \mathcal{W}_{Bi})^a(\mathcal{W}^i_F - \mathcal{W}^i_L - \mathcal{W}^I_R\\ &\quad+ \mathcal{W}^i_B)^a,\end{aligned}$$

which coincide with the left-hand side of the Eq. (60). In the covariant-type gauges where $A_i \to 0$ as $x_\parallel \to \infty$ the right-hand side of the Eq. (56) can be rewritten as

$$\begin{aligned}&\partial_i M_1\partial^i M_2 M^\dagger_3 M^\dagger_4 + M_1\partial_i M_2\partial^i M^\dagger_3 M^\dagger_4\\ &M_1 M_2\partial_i M^\dagger_3\partial_i M^\dagger_4 + \partial_i M_1 M_2 M^\dagger_3\partial_i M^\dagger_4,\end{aligned} \tag{61}$$

where $M_1 = [-\infty p_1, \infty p_1]_{\infty p_2}$, $M_2 = [\infty p_2, -\infty p_2]_{\infty p_1}$, $M_3 = [\infty p_1, -\infty p_1]_{-\infty p_2}$, and $M_4 = [-\infty p_2, \infty p_2]_{-\infty p_1}$. Equation (61) is the expression obtained recently in [17] in the framework of the Hamiltonian approach (see also [18,19]). The corresponding form of our effective action is the following:

$$\frac{1}{4}L_i^a(U,V)L^{ai}(U,V) = \partial_i(W_LW_F^\dagger)\partial^i(W_FW_R^\dagger)W_RW_B^\dagger W_BW_L^\dagger + W_LW_F^\dagger\partial^i(W_FW_R^\dagger)\partial_i(W_RW_B^\dagger)W_BW_L^\dagger + W_LW_F^\dagger W_FW_R^\dagger\partial_i(W_RW_B^\dagger)\partial^i(W_BW_L^\dagger) + \partial_i(W_LW_F^\dagger)W_FW_R^\dagger W_RW_B^\dagger\partial^i(W_BW_L^\dagger). \tag{62}$$

The Eq. (60) links the representation in terms of the effective degrees of freedom (Wilson lines in our case) with the representation in terms of gluons of the underlying Yang-Mills theory via Eq. (57). The remarkable feature of the gauge-invariant form (60) is its universality—if one writes the effective action in terms of some other degrees of freedom (say, Reggeized gluons [29]) one should recover Eq. (60) once these new effective degrees of freedom are expressed in terms of gluons.

V. FUNCTIONAL INTEGRAL OVER THE DYNAMICAL WILSON LINES

A. Effective action as the integral over the Wilson lines

In this section we will rewrite the functional integral for the effective action (25) in terms of Wilson-line variables. To this end, let us use the factorization formula (14) n times as shown in Fig. 7. The effective action factorizes then into a product of n independent functional integrals over the gluon fields labeled by index k:

$$e^{iS_{\text{eff}}(U,V;\eta)} = \int DA^1DA^2\ldots DA^{n+1}\exp i\{(\mathcal{V}_1^i - \mathcal{V}_2^i)(\mathcal{U}_{1i}^{n+1} - \mathcal{U}_{2i}^{n+1}) + S(A_{n+1}) + (\mathcal{V}_1^{n+1,i} - \mathcal{V}_2^{n+1,i})(\mathcal{U}_1^{ni} - \mathcal{U}_2^{ni}) + S(A_n) + \ldots + (\mathcal{V}_1^{2i} - \mathcal{V}_2^{2i})(\mathcal{U}_{1i}^1 - \mathcal{U}_{2i}^1) + S(A_1) + (\mathcal{V}_1^{1i} - \mathcal{V}_2^{1i})(\mathcal{U}_{1i} - \mathcal{U}_{1i})\}, \tag{63}$$

where the integrals over $x_\perp$ and summation over the color indices are implied. As usually, $\mathcal{U}_j^{k,i} = \frac{i}{g}U_j^{\dagger k}\partial^iU_j^k$ and $\mathcal{V}_j^{k,i} = \frac{i}{g}V_j^{\dagger k}\partial^iV_j^k$ where

$$U_{1(2)}^k(x_\perp) = Pe^{ig\int_0^{\pm\infty}dun_k^\mu A_{k,\mu}(un^k + x_\perp)}, \qquad V_{1(2)}^k(x_\perp) = Pe^{ig\int_0^{\pm\infty}dun_{k-1}^\mu A_{k,\mu}(un^{k-1} + x_\perp)}, \tag{64}$$

and the vectors n_k are ordered in rapidity: $\eta_0 > \eta_n > \eta_{n-1}\ldots\eta_2 > \eta_1 > \eta_0'$. To disentangle integrations over different A^k, we rewrite $\exp[i(\mathcal{V}_1^{k+1,i} - \mathcal{V}_2^{k+1,i})(\mathcal{U}_1^{k,i} - \mathcal{U}_2^{k,i})]$ at each "rapidity divide" η_k as an integral over the auxiliary group variables $\hat{V}_{1,2}^{k+1}$ and $\hat{U}_{1,2}^k$ using the formula

$$e^{i\int dx_\perp V_iU^i} = \det(\partial_i - ig\mathcal{V}_i)(\partial^i - ig\mathcal{U}^i)\int D\hat{V}(x_\perp)D\hat{U}(x_\perp)e^{i\int dx_\perp\mathcal{V}_i\hat{\mathcal{U}}^i + i\int dx_\perp\hat{\mathcal{V}}_i\mathcal{U}^i - i\int dx_\perp\hat{\mathcal{V}}_i\hat{\mathcal{U}}^i}. \tag{65}$$

(where $\hat{\mathcal{V}}_i \equiv \hat{V}^\dagger\frac{i}{g}\partial_i\hat{V}$ and $\hat{\mathcal{U}}_i \equiv \hat{U}^\dagger\frac{i}{g}\partial_i\hat{U}$). The determinant gives the perturbative nonlogarithmic corrections of the same order as the corrections to the factorization formula (14). In the LLA they can be ignored, consequently, we obtain

$$\begin{aligned}e^{iS_{\text{eff}}(U_1,U_2,V_1,V_2,\eta_1-\eta_2)} = \int &\Pi_{k=0}^{n+1}DA^k\Pi_{k=0}^nD\hat{U}_1^kD\hat{U}_2^kD\hat{V}_1^kD\hat{V}_2^k\exp i\{(\mathcal{V}_1^i - \mathcal{V}_2^i)(\mathcal{U}_{1i}^{n+1} - \mathcal{U}_{1i}^{n+1}) + S(A^{n+1})\\ &+ (\mathcal{V}_{1i}^{n+1} - \mathcal{V}_{2i}^{n+1})(\hat{\mathcal{U}}_1^{ni} - \hat{\mathcal{U}}_2^{ni}) - (\hat{\mathcal{V}}_{1i}^n - \hat{\mathcal{V}}_{2i}^n)(\hat{\mathcal{U}}_1^{n,i} - \hat{\mathcal{U}}_2^{n,i}) + \ldots\\ &+ (\mathcal{V}_{1i}^3 - \mathcal{V}_{2i}^3)(\hat{\mathcal{U}}_1^{2i} - \hat{\mathcal{U}}_2^{2i}) - (\hat{\mathcal{V}}_1^{2i} - \hat{\mathcal{V}}_2^{2i})(\hat{\mathcal{U}}_{1i}^2 - \hat{\mathcal{U}}_{2i}^2) + (\hat{\mathcal{V}}_{2i}^2 - \hat{\mathcal{V}}_{1i}^2)(\mathcal{U}_1^{2i} - \mathcal{U}_2^{2i})\\ &+ S(A^2) + (\mathcal{V}_{1i}^2 - \mathcal{V}_{2i}^2)(\hat{\mathcal{U}}_1^{1i} - \hat{\mathcal{U}}_2^{1i}) - (\hat{\mathcal{V}}_{1i}^1 - \hat{\mathcal{V}}_{2i}^1)(\hat{\mathcal{U}}_1^{1i} - \hat{\mathcal{U}}_2^{1i}) + (\hat{\mathcal{V}}_{1i}^1 - \hat{\mathcal{V}}_{2i}^1)(\mathcal{U}_1^{1i} - \mathcal{U}_2^{1i})\\ &+ S(A^1) + (\mathcal{V}_1^{1i} - \mathcal{V}_2^{1i})(U_{1i} - U_{2i})\}.\end{aligned} \tag{66}$$

Now we can integrate over the gluon fields A_k. Using the results of the previous Section, we get

$$\int DA_k\exp\{i(\hat{\mathcal{V}}_{1i}^k - \hat{\mathcal{V}}_{2i}^k)(\mathcal{U}_1^{k,i} - \mathcal{U}_2^{k,i}) + iS(A_k) + i(\mathcal{V}_{1i}^k - \mathcal{V}_{2i}^k)(\hat{\mathcal{U}}_1^{k-1,i} - \hat{\mathcal{U}}_2^{k-1,i})\} = e^{iS_{\text{eff}}(\hat{V}_1^k,\hat{V}_2^k,\hat{U}_1^{k-1},\hat{U}_2^{k-1};\Delta\eta)}, \tag{67}$$

where at sufficiently small $\Delta\eta$

$$S_{\text{eff}}(\hat{V}_1^k,\hat{V}_2^k,\hat{U}_1^{k-1},\hat{U}_2^{k-1};\Delta\eta) = (\hat{\mathcal{V}}_{1i}^k - \hat{\mathcal{V}}_{2i}^k)(\hat{\mathcal{U}}_1^{k-1,i} - \hat{\mathcal{U}}_2^{k-1,i}) - i\frac{\alpha_s\Delta\eta}{4}L_i(\hat{\mathcal{V}}^k,\hat{\mathcal{U}}_{k-1})L^i(\hat{\mathcal{V}}^k,\hat{\mathcal{U}}_{k-1}). \tag{68}$$

Performing the integrations over A^k we get

$$
\begin{aligned}
e^{iS_{\rm eff}(U,V,\eta_1-\eta_2)} = \int \Pi_{k=0}^{n} D\hat{U}_1^k D\hat{U}_2^k D\hat{V}_1^k D\hat{V}_2^k \exp i\Big\{ & (\mathcal{V}_1^i - \mathcal{V}_2^i)(\hat{\mathcal{U}}^n_{1i} - \hat{\mathcal{U}}^n_{2i}) - \frac{i\alpha_s}{4} L^2(V, \hat{U}^n)\Delta\eta \\
& - (\hat{\mathcal{V}}^n_{1i} - \hat{\mathcal{V}}^n_{2i})(\hat{\mathcal{U}}_1^{n,i} - \hat{\mathcal{U}}_2^{n,i}) + \ldots - (\hat{\mathcal{V}}_1^{2i} - \hat{\mathcal{V}}_2^{2i})(\hat{\mathcal{U}}^2_{1i} - \hat{\mathcal{U}}^2_{2i}) + (\hat{\mathcal{V}}^2_{1i} - \hat{\mathcal{V}}^2_{2i})(\hat{\mathcal{U}}_1^{1i} - \hat{\mathcal{U}}_2^{1i}) \\
& - \frac{i\alpha_s}{4} L^2(\hat{V}^2, \hat{U}^1)\Delta\eta - (\hat{\mathcal{V}}^1_{1i} - \hat{\mathcal{V}}^1_{2i})(\hat{\mathcal{U}}_1^{1i} - \hat{\mathcal{U}}_2^{1i}) + (\hat{\mathcal{V}}^1_{1i} - \hat{\mathcal{V}}^1_{2i})(U_{1i} - U_{2i}) - \frac{i\alpha_s}{4} L^2(\hat{V}^1, U)\Delta\eta\Big\}.
\end{aligned}
\tag{69}
$$

In the continuum limit $n \to \infty$ we obtain the following functional integral for the effective action

$$
\begin{aligned}
e^{iS_{\rm eff}(U_1(x),U_2(x),V_1(x),V_2(x);\eta_1-\eta_2)} = & \int \Pi_{j=1,2} DV_j(x,\eta) DU_j(x,\eta)|_{U_j(x,\eta_2)=U_j(x)} \exp\Big[\int d^2x\Big(i[\mathcal{V}^a_{1i}(x) - \mathcal{V}^a_{2i}(x)] \\
& \times [\mathcal{U}_1^{ai}(x,\eta) - \mathcal{U}_2^{ai}(x,\eta)] + \int_{\eta_0'}^{\eta_0} d\eta\Big\{-i[\mathcal{V}^a_{1i}(x,\eta) - \mathcal{V}^a_{2i}(x,\eta)]\frac{\partial}{\partial\eta} \\
& \times [\mathcal{U}_1^{ai}(x,\eta) - \mathcal{U}_2^{ai}(x,\eta)] + \frac{\alpha_s}{4} L_i^a(V(x,\eta), U(x,\eta)) L^{ai}(V(x,\eta), U(x,\eta))\Big\}\Big)\Big],
\end{aligned}
\tag{70}
$$

where we displayed the color indices explicitly and removed the hat from the notation of the integration variables. This looks like the functional integral over the canonical coordinates U and canonical momenta V with the (nonlocal) Hamiltonian $L^2(V,U)$. The rapidity η serves as the time variable for this system. The above representation of the effective action as an integral over the dynamical Wilson-line variables is the main result of this paper.

Note that the L_iL^i term in the exponent in (70) is invariant under the rotations (20)

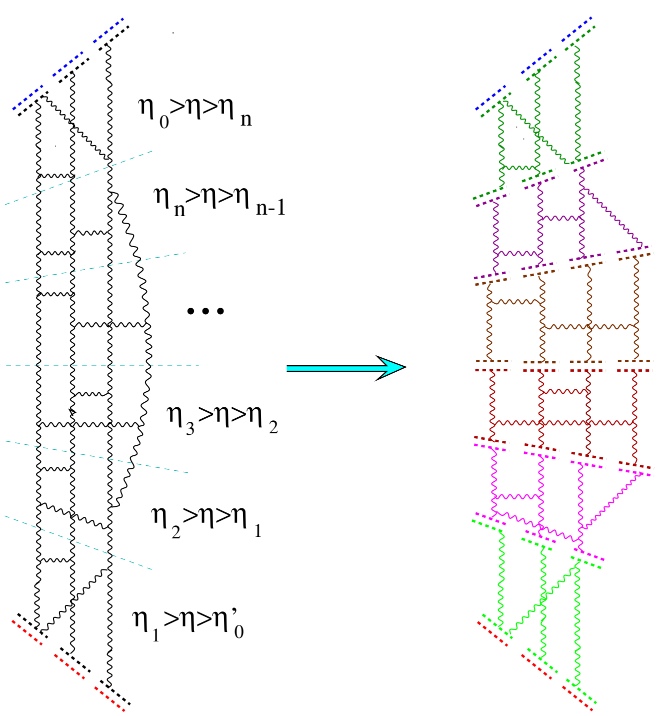

FIG. 7 (color online). Effective action factorized in n functional integrals.

$$
\begin{aligned}
U_j(x,\eta) &\to U_j(x,\eta)\Omega(x,\eta), \\
V_j(x,\eta) &\to V_j(x,\eta)\Omega(x,\eta),
\end{aligned}
\tag{71}
$$

(see. Equation (59)), but the term $\sim \mathcal{U}\frac{\partial}{\partial\eta}\mathcal{V}$ preserves only the η-independent symmetry

$$
U_j(x,\eta) \to U_j(x,\eta)\Omega(x), \qquad V_j(x,\eta) \to V_j(x,\eta)\Omega(x). \tag{72}
$$

This probably means that the term $\sim \mathcal{U}\frac{\partial}{\partial\eta}\mathcal{V}$ should be adjusted by a $\sim[U,V]^2$ correction (not important in the LLA) so that the full symmetry (71) is restored.

The idea how to use the factorization formula to rewrite the functional integral in terms of Wilson lines was formulated in Ref. [1] where the first-order effective action was obtained (the expression in terms of square of Lipatov vertex is given in [16]). However, the additional redundant gauge symmetry (20) was fixed by the requirement that there is no field at $t \to -\infty$ which correspond to the choice $U_2 = 0$ and $V_2 = 0$ for the two colliding shock waves. In this case, one obtains the functional integral in terms of only two variables, U and V, at a price of a more complicated form of the effective action [1].

It should be noted that $L_i^2(U,V)$ is only the first term of the expansion of the true high-energy effective action $K(U,V)$ in powers of $[U,V]$. An example of the next-order, $\sim[U,V]^3$, contribution to $K(V,U)$ which is missing in the effective action (70) is presented in Ref. [1], see Fig. 8.

B. Functional integral for the nonlinear evolution

It is instructive do demonstrate that the functional integral (70) reproduces the nonlinear evolution in the case of one small source. Basically, we recast the arguments of the Sec. IV B in the language of functional integrals.

First, note that at small V the functional integral over $\mathcal{V}$ is Gaussian (see the Eq. (47)). It is convenient to introduce the "Gaussian noise" associated with the Lipatov vertex and rewrite the functional integral (70) as:

$$e^{iS_{\rm eff}(U_1(x),U_2(x),V_1(x),V_2(x);\eta_1-\eta_2)} = \int \Pi_{j=1,2} DV_j(x,\eta)DU_j(x,\eta)|_{U_j(x,\eta_2)=U_j(x)} D\lambda_i^a(x,\eta)\exp\int d^2x\Big(i[\mathcal{V}^a_{1i}(x)-\mathcal{V}^a_{2i}(x)] \times[\mathcal{U}^{ai}_1(x,\eta)-\mathcal{U}^{ai}_2(x,\eta)] + \int_{\eta_0'}^{\eta_0} d\eta\Big\{-\alpha_s\lambda_i^a(x,\eta)\lambda^{ai}(x,\eta) - i[\mathcal{V}^a_{1i}(x,\eta)-\mathcal{V}^a_{2i}(x,\eta)]\frac{\partial}{\partial\eta}[\mathcal{U}^{ai}_1(x,\eta)-\mathcal{U}^{ai}_2(x,\eta)] - 2\alpha_s\lambda^{ai}(x,\eta)\Big(x|U_1^\dagger\frac{p_ip^k}{p_\perp^2}U_1-(1\leftrightarrow 2)|^{ab}(\mathcal{V}_1^\eta-\mathcal{V}_2^\eta)^b_k\Big)\Big\}\Big). \tag{73}$$

(When convenient, we use the notation $(\ldots)^\eta \equiv (\ldots)(\eta)$ for brevity). The integral over $\mathcal{V}$ gives the δ-function of the form

$$\delta\Big(\partial_i\Big(\frac{\partial}{\partial\eta}[\mathcal{U}^{ai}_1(x,\eta)-\mathcal{U}^{ai}_2(x,\eta)]\Big)\Big) - 2i\alpha_s\Big(x|U_1^\dagger\frac{p_ip^k}{p_\perp^2}U_1 - U_2^\dagger\frac{p_ip^k}{p_\perp^2}U_2|^{ab}\lambda_k^{b\eta}\Big),$$

which restricts U in a following way

$$\frac{\partial}{\partial\eta}[\mathcal{U}^{ai}_1(x,\eta)-\mathcal{U}^{ai}_2(x,\eta)] = 2i\alpha_s\Big(x|U_1^\dagger\frac{p_ip^k}{p_\perp^2}U_1 - U_2^\dagger\frac{p_ip^k}{p_\perp^2}U_2|^{ab}\lambda_k^{b\eta}\Big). \tag{74}$$

It is convenient to rewrite Eq. (74) in the integral form (cf. Eq. (50)):

$$U_i(x,\eta) = Te^{2i\alpha_s t^a\int_{\eta_2}^{\eta}d\eta'(x|(p^k/p_\perp^2)U_i(\eta')^{ab}|\lambda_k^b(\eta'))}U_i(x_\perp,\eta_2), \tag{75}$$

where T means ordering in rapidity (= our "time"). The remaining integral over λ is Gaussian with the "propagator"

$$\langle\lambda_i^a(x_\perp,\eta)\lambda_j^b(y_\perp,\eta')\rangle = g_{ij}\delta^{ab}\frac{1}{2\alpha_s}\delta(x_\perp-y_\perp)\delta(\eta-\eta'). \tag{76}$$

The evolution of the dipole can be represented as

$$U^{\dagger\eta}_{1y}U^\eta_{1x}U^{\dagger\eta}_{2x}U^\eta_{2y} = \int D\lambda e^{-\alpha_s\int_{\eta_2}^{\eta_1}d\eta\int d^2z_\perp\lambda_{iz}^{a\eta}\lambda_z^{ai\eta}} U^{\dagger\eta_2}_{1y}\bar{T}e^{-2i\alpha_s t^a\int_{\eta_2}^{\eta}d\eta'(y|(p^k/p_\perp^2)U_1^{\eta'}|^{ab}\lambda_k^{b\eta'})} Te^{2i\alpha_s t^a\int_{\eta_2}^{\eta}d\eta'(x|(p^k/p_\perp^2)U_1^{\eta'}|^{ab}\lambda_k^{b\eta'})} \times U^{\eta_2}_{1x}U^{\dagger\eta_2}_{2x}\bar{T}e^{-2i\alpha_s t^a\int_{\eta_2}^{\eta}d\eta'(x|(p^k/p_\perp^2)U_2^{\eta'}|^{ab}\lambda_k^{b\eta'})} Te^{2i\alpha_s t^a\int_{\eta_2}^{\eta}d\eta'(y|(p^k/p_\perp^2)U^{\eta'}|^{ab}\lambda_k^{b\eta'})}U^{\eta_2}_{2y}, \tag{77}$$

where $\bar{T}$ denotes the inverse rapidity ordering. Taking the derivative with respect to η we get

$$\frac{\partial}{\partial\eta}U^{\dagger\eta}_{1y}U^\eta_{1x}U^{\dagger\eta}_{2x}U^\eta_{2y} = \int D\lambda e^{-\alpha_s\int_{\eta_2}^{\eta_1}d\eta\int d^2z_\perp\lambda_{iz}^{a\eta}\lambda_z^{ai\eta}}2i\alpha_s \times\Big(U^{\dagger\eta}_{1y}\Big[t^a\Big(x\Big|\frac{p^k}{p_\perp^2}U_1^\eta\Big|^{ab}\lambda_k^{b\eta}\Big) - x\leftrightarrow y\Big]U^\eta_{1x}U^{\dagger\eta}_{2x}U^\eta_{2y} - U^{\dagger\eta}_{1y}U^\eta_{1x}U^{\dagger\eta}_{2x} \times\Big[t^a\Big(x\Big|\frac{p^k}{p_\perp^2}U_2^\eta\Big|^{ab}\lambda_k^{b\eta}\Big)-x\leftrightarrow y\Big] \times U^\eta_{2y}\Big).$$

Using the contraction

$$\langle\lambda_k^a(z,\eta)U(x,\eta)\rangle = -\frac{1}{2}i\Big(z\Big|U^\dagger(\eta)\frac{p_k}{p_\perp^2}\Big|x\Big)^{ab}t^bU(x,\eta)$$
$$\langle\lambda_k^a(z,\eta)U^\dagger(x,\eta)\rangle = -\frac{1}{2}i(z|U^\dagger(\eta)\frac{p_k}{p_\perp^2}|x)^{ab}U^\dagger(x,\eta)t^b, \tag{78}$$

one gets the nonlinear evolution Eq. (55) after some algebra.

The factor 1/2 in the Eq. (78) comes from $\theta(0)=1/2$. To avoid this uncertainty, one should first calculate the correlations in λ and then differentiate with respect to rapidity (cf. Ref. [30])

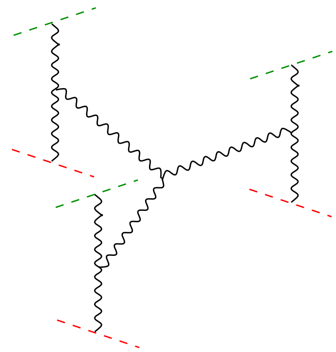

FIG. 8 (color online). Feynman diagram for a typical $[U, V]^3$ term.

$$\frac{\partial}{\partial\eta}\int_{\eta_2}^{\eta} d\eta' d\eta'' \langle\lambda_i^a(x_\perp,\eta')\lambda_j^b(y_\perp,\eta'')\rangle f(\eta')g(\eta'') = -\frac{1}{2\alpha_s}\delta(x-y)_\perp g_{ij} f(\eta)g(\eta)$$

$$\frac{\partial}{\partial\eta}\int_{\eta_2}^{\eta} d\eta' \int_{\eta_2}^{\eta'} d\eta'' \langle\lambda_i^a(x_\perp,\eta')\lambda_j^b(y_\perp,\eta'')\rangle f(\eta')f(\eta'') = -\frac{1}{4\alpha_s}\delta(x-y)_\perp g_{ij} f(\eta)f(\eta).$$

The first line in the above equation should be used to make contractions between different T and $\bar{T}$ in Eq. (77) while the second line takes care of the contractions within same T or $\bar{T}$. It is easy to check that the result is consistent with taking $\theta(0) = 1/2$ in the Eq. (78).

Similarly one can demonstrate that all the hierarchy of the evolution equations for Wilson lines [26,31] ($\equiv$ JIMWLK equation [32]) is reproduced.

C. Classical equations for the Wilson-line functional integral

As we discussed above, the characteristic fields in the functional integral are large but the coupling constant $\alpha_s(Q_s)$ is small due to the saturation. In this case, we can try to calculate the functional integral (70) semiclassically. Using the approximate formula

$$\delta\mathcal{W}_i^a(U,V) \simeq -\left(W^\dagger\frac{p_i p^j}{p_\perp^2}W\right)^{ab}(\delta\mathcal{U}_j^b + \delta\mathcal{V}_j^b), \tag{79}$$

we get the classical equations for the functional integral (70) in the form

$$\begin{aligned}
(\partial^i - ig\mathcal{V}_1^i)^{ab}(\dot{\mathcal{U}}_{1i} - \dot{\mathcal{U}}_{2i})^b &= 2i\alpha_s(\partial^i - ig\mathcal{V}_1^i)^{ab}\left(W_F^\dagger\frac{p^ip^j}{p_\perp^2}W_F - W_L^\dagger\frac{p^ip^j}{p_\perp^2}W_L\right)^{bc}E_j^c,\\
(\partial^i - ig\mathcal{V}_2^i)^{ab}(\dot{\mathcal{U}}_{1i} - \dot{\mathcal{U}}_{2i})^b &= 2i\alpha_s(\partial^i - ig\mathcal{V}_2^i)^{ab}\left(W_R^\dagger\frac{p^ip^j}{p_\perp^2}W_R - W_B^\dagger\frac{p^ip^j}{p_\perp^2}W_B\right)^{bc}E_j^c,\\
(\partial^i - ig\mathcal{U}_1^i)^{ab}(\dot{\mathcal{V}}_{1i} - \dot{\mathcal{V}}_{2i})^b &= -2i\alpha_s(\partial^i - ig\mathcal{U}_1^i)^{ab}\left(W_F^\dagger\frac{p^ip^j}{p_\perp^2}W_F - W_R^\dagger\frac{p^ip^j}{p_\perp^2}W_R\right)^{bc}E_j^c,\\
(\partial^i - ig\mathcal{U}_2^i)^{ab}(\dot{\mathcal{V}}_{1i} - \dot{\mathcal{V}}_{2i})^b &= -2i\alpha_s(\partial^i - ig\mathcal{V}_2^i)^{ab}\left(W_L^\dagger\frac{p^ip^j}{p_\perp^2}W_L - W_B^\dagger\frac{p^ip^j}{p_\perp^2}W_B\right)^{bc}E_j^c,
\end{aligned} \tag{80}$$

with the initial conditions

$$U(\eta) = U \text{ at } \eta = \eta_2, \qquad V(\eta) = V \text{ at } \eta = \eta_1. \tag{81}$$

At small $\mathcal{V}_i$ these equations reduce to (cf. Equation (74))

$$\begin{aligned}
(\dot{\mathcal{U}}_{1i} - \dot{\mathcal{U}}_{2i})^a &= 2i\alpha_s\left(U_1^\dagger\frac{p^ip^j}{p_\perp^2}U_1 - U_2^\dagger\frac{p^ip^j}{p_\perp^2}U_2\right)^{ab}E_j^b\\
\dot{\mathcal{V}}_{1i} - \dot{\mathcal{V}}_{2i} &= O([U,V]^2),
\end{aligned} \tag{82}$$

while in the opposite case of small $\mathcal{U}_i$ they are

$$\begin{aligned}
(\dot{\mathcal{V}}_{1i} - \dot{\mathcal{V}}_{2i})^a &= -2i\alpha_s\left(V_1^\dagger\frac{p^ip^j}{p_\perp^2}V_1 - V_2^\dagger\frac{p^ip^j}{p_\perp^2}V_2\right)^{ab}E_j^b\\
\dot{\mathcal{U}}_{1i} - \dot{\mathcal{U}}_{2i} &= O([U,V]^2).
\end{aligned} \tag{83}$$

It is instructive to rewrite the Eqs. (82) and (83) in terms of $\dot{W}$'s.

$$\begin{aligned}
\dot{\mathcal{W}}_{Fi}^a - \dot{\mathcal{W}}_{Bi}^a &= 2i\alpha_s\left(W_R^\dagger\frac{p^ip^j}{p_\perp^2}W_R - W_L^\dagger\frac{p^ip^j}{p_\perp^2}W_L\right)^{ab}E_j^c\\
\dot{\mathcal{W}}_{Ri}^a - \dot{\mathcal{W}}_{Li}^a &= 2i\alpha_s\left(W_F^\dagger\frac{p^ip^j}{p_\perp^2}W_F - W_B^\dagger\frac{p^ip^j}{p_\perp^2}W_B\right)^{ab}E_j^c.
\end{aligned} \tag{84}$$

From the viewpoint of the functional integral (70) the W's are the (nonlocal) functions of U and V variables given in the first order by Eq. (27). It would be very interesting to rewrite the Eq. (25) of the W variables themselves, that is, to construct the functional integral over the W variables with a saddle-point equations given by the Eq. (84).

VI. CONCLUSION

As mentioned in the Introduction, the popular idea of how to solve QCD at high energies is to reformulate it in

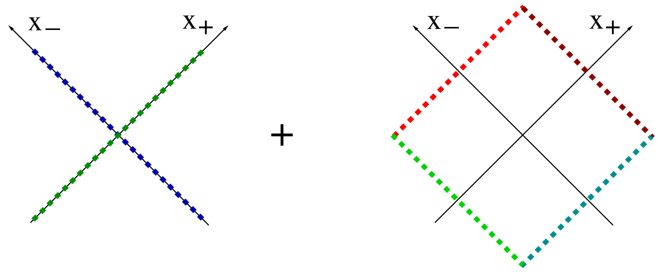

FIG. 9 (color online). Wilson-line structure of the effective action .

terms of the relevant high-energy degrees of freedom—Wilson lines. The functional integral (70) gives an example of such $2+1$ theory where 2 stands for the transverse coordinates and 1 for d the rapidity serving as a time variable. The structure of the effective action is presented in Fig. 9. Note that the two terms in the exponent in the effective action, shown in Fig. 9, are both local in $x_\perp$ but differ with respect to the longitudinal coordinates: the first (kinetic) term is made from the Wilson lines located at $x_+ = 0$ or $x_- = 0$ while the second term is made from the Wilson lines at $x_\pm = \pm\infty$. Unfortunately, the transition between these Wilson lines is nonlocal in $x_\perp$ (see Eq. (27)) and so the resulting effective action is a nonlocal function of the dynamical variables U and V.

In should be emphasized that Eq. (70) is only a model - the genuine effective action for the $2+1$ high-energy theory of Wilson lines must include all the contributions $\sim[U,V]^n$ (as we mentioned above, an example of a $[U,V]^3$ term which is missing in Eq. (25) is presented in Fig. 8). However, this model is correct in the case of weak projectile fields and strong target fields, and vice versa. In terms of Feynman diagrams, the effective action (70) includes both up and down fan ladders and the pomeron loops, see Fig. 10. In the dipole language, it describes both multiplication and recombination of dipoles (see the discussion in [20,21]). In conclusion I would like to emphasize that the effective action (70) summarizes all present knowledge about the high-energy evolution of Wilson lines in a way symmetric with respect to projectile and target and hence it may serve as a starting point for future analysis of high-energy scattering in QCD.

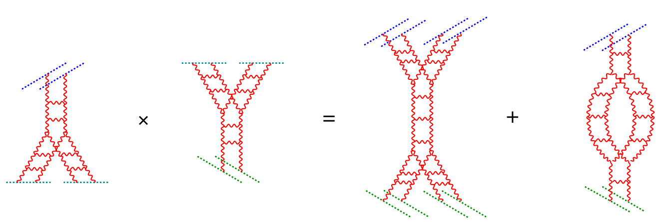

FIG. 10 (color online). Typical Feynman diagrams included in the effective action.

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APPENDIX: CLASSICAL FIELDS IN THE FIRST ORDER IN $[U, V]$

Following Ref. [16], we take the zero-order approximation in the form of the sum of the two shock waves (26)

$$\bar{A}_i^{(0)} = \mathcal{U}_{1i}\theta(x_*) + \mathcal{U}_{2i}\theta(-x_*) + \mathcal{V}_{1i}\theta(x_\bullet) + \mathcal{V}_{2i}\theta(-x_\bullet), \qquad \bar{A}_\bullet^{(0)} = \bar{A}_*^{(0)} = 0. \tag{A1}$$

We will expand the deviation of the full QCD solution from the QED-type ansatz (A1) in powers of commutators $[U,V]$. To carry this out, we shift $A \to A + \bar{A}_i^{(0)}$ in the functional integral (25) and obtain

$$\int DA \exp\left\{i\int d^4z\left(\frac{1}{2}A^\mu \bar{D}_{\mu\nu}A^\nu + gT^\mu A_\mu\right)\right\}. \tag{A2}$$

Here $D_{\mu\nu} = D^2(\bar{A})g_{\mu\nu} - 2i\bar{F}_{\mu\nu}$ is the inverse propagator in the background-Feynman gauge and T_μ is the linear term for the trial configuration (A1). Since the only nonzero component of the field strength for the ansatz (A1) is

$$\bar{F}_{ik}^{(0)} = -i[U_{1i}, V_{1k}]\theta(F) - i[U_{1i}, V_{2k}]\theta(R) - i[U_{2i}, V_{1k}]\theta(L) - i[U_{2i}, V_{2k}]\theta(B) - (i \leftrightarrow k), \tag{A3}$$

the linear term $T_\mu = \bar{D}^\rho \bar{F}_{\rho\mu}^{(0)}$ is

$$\begin{aligned} T_* &= T_\bullet = 0,\\ T^i &= (i\partial_k + g[U_{1k} + V_{1k},)([U_1^i, V_1^k] - i \leftrightarrow k)\theta(F)\\ &\quad + (i\partial_k + g[U_{1k} + V_{2k},)([U_1^i, V_2^k] - i \leftrightarrow k)\theta(R)\\ &\quad + (i\partial_k + g[U_{2k} + V_{2k},)([U_2^i, V_1^k] - i \leftrightarrow k)\theta(L)\\ &\quad + (i\partial_k + g[U_{2k} + V_{2k},)([U_2^i, V_2^k] - i \leftrightarrow k)\theta(B), \end{aligned} \tag{A4}$$

where $\theta(F) \equiv \theta(z_*)\theta(z_\bullet)$, $\theta(R) \equiv \theta(z_*)\theta(-z_\bullet)$, $\theta(L) \equiv \theta(-z_*)\theta(z_\bullet)$, and $\theta(B) \equiv \theta(-z_*)\theta(-z_\bullet)$

Expanding in powers of T in the functional integral (A2) one gets the set of Feynman diagrams in the external fields

(26) with the sources (A4). The parameter of the expansion is $g^2[U_i, V_j]$ ($\sim [U, V]$, see Eq. (5)).

The general formula for the classical solution in the first order in $[U, V]$ has the form

$$\bar{A}_\mu^{(1)a}(x) = ig \int d^4z \langle A^a_\mu(x) A^{b\nu}(z)\rangle_{\bar{A}} T^b_\nu(z). \tag{A5}$$

The Green functions in the background of the Eq. (26) field can be approximated by cluster expansion

$$\langle A_\mu(x)A^\nu(z)\rangle_{\bar{A}} = \langle A_\mu(x)A^\nu(z)\rangle_U + \langle A_\mu(x)A^\nu(z)\rangle_V - \langle A_\mu(x)A^\nu(z)\rangle_0 + O([U, V]), \tag{A6}$$

where $\langle A_\mu(x)A^\nu(z)\rangle_0$ is the perturbative propagator and

$$\langle A^a_\mu(x)A^b_\nu(y)\rangle_U \overset{x_*>0, y_*<0}{=} \int dz \delta\Big(\frac{2}{s}z_*\Big)\Big\{U^\dagger_{1x}\Big(x\,\Big|\frac{1}{p^2+i\epsilon}\Big|\,z\Big) \times \Big(2\alpha g_{\mu\nu}U_1U_2^\dagger + \frac{4i}{s}(p_{2\nu}\partial_\mu(U_1U_2^\dagger)_z + \mu\leftrightarrow\nu) - \frac{4p_{2\mu}p_{2\nu}}{\alpha s^2}\partial_\perp^2(U_1U_2^\dagger)_z\Big) \times \Big(z\,\Big|\frac{1}{p^2+i\epsilon}\Big|\,y\Big)U_{2y}\Big)^{ab} \tag{A7}$$

is the propagator in the background of the shock wave U (the propagator in the V background is obtained by the replacement $U \leftrightarrow V$, $p_2 \leftrightarrow p_1$.

Substituting Eq. (A7) and (A4) into the above equation, one obtains (the details of the calculations can be found in Ref. [16] and here we present only the final set of gauge fields):

$$A^\mu = \theta(F)\Big\{\mathcal{W}^{\mu\perp}_F(x_\perp) - gt^a\Big(W_F^\dagger\frac{1}{\partial^2 - i\epsilon}W_F\Big)^{ab}L^{\mu b}_F(x)\Big\} + (F\leftrightarrow L) + (F\leftrightarrow R) + (F\leftrightarrow B), \tag{A8}$$

where $L^i_F = L^i_L = L^i_R = L^i_B = 2E^i$ and

$$\begin{aligned}
L_{F*} &= 2\Big[V_{1i} - V_{2i}, \frac{1}{\beta+i\epsilon}E^i_R - \frac{1}{\beta-i\epsilon}E^i_B\Big]\\
L_{F\bullet} &= 2\Big[U_{1i} - U_{2i}, \frac{1}{\alpha+i\epsilon}E^i_L - \frac{1}{\alpha-i\epsilon}E^i_B\Big]\\
L_{L*} &= 2\Big[V_{1i} - V_{2i}, \frac{1}{\beta+i\epsilon}E^i_R - \frac{1}{\beta-i\epsilon}E^i_B\Big]\\
L_{L\bullet} &= 2\Big[U_{1i} - U_{2i}, \frac{1}{\alpha+i\epsilon}E^i_F - \frac{1}{\alpha-i\epsilon}E^i_R\Big]\\
L_{R*} &= 2\Big[V_{1i} - V_{2i}, \frac{1}{\beta+i\epsilon}E^i_F - \frac{1}{\beta-i\epsilon}E^i_L\Big]\\
L_{R\bullet} &= 2\Big[U_{1i} - U_{2i}, \frac{1}{\alpha+i\epsilon}E^i_L - \frac{1}{\alpha-i\epsilon}E^i_B\Big]\\
L_{B*} &= 2\Big[V_{1i} - V_{2i}, \frac{1}{\beta+i\epsilon}E^i_F - \frac{1}{\beta-i\epsilon}E^i_L\Big]\\
L_{B\bullet} &= \Big[U_{1i} - U_{2i}, \frac{1}{\alpha+i\epsilon}E^i_F - \frac{1}{\alpha-i\epsilon}E^i_R\Big],
\end{aligned} \tag{A9}$$

where $(2/s)/(\alpha \pm i\epsilon)\mathcal{O}(x) \equiv i\int_0^{\pm\infty} du\mathcal{O}(x + up_2)$ and $\frac{2/s}{\beta\pm i\epsilon}\mathcal{O}(x) \equiv i\int_0^{\pm\infty} du\mathcal{O}(x+up_1)$. It is easy to check the background-Feynman gauge condition $(i\partial_\mu + g[\mathcal{W}^\mu_F,)L_{F\mu} = 0$ (and similarly for three other quadrants of the space).

The transverse part E_i agrees with the results of Sec. III B while the longitudinal part (A9) does not literally agree with (35) (see the footnote after that equation). It should be emphasized that, unlike the calculations with trial configuration (28), the Feynman diagrams in the background of the ansatz (A1) are free from uncertainties like $\theta(0)$.

Let us rederive now the effective action (42) starting from the ansatz (A1) and the fields (A8). Since the only nonzero component of the field strength for the ansatz (A1) is transverse (see Eq. (A3)), we have

$$\begin{aligned}
S_{\rm eff} &= -\frac{1}{4}\int d^4z\bar{F}^{(0)a}_{ik}\bar{F}^{(0)a,ik} + \frac{i}{2}\int d^4z d^4z' T^a_i(z)T^b_j(z')\langle A^{ai}(z)A^{bj}(z')\rangle\\
&= -\frac{1}{4}\int d^4z\Big(\bar{F}^{(0)a}_{ik}\bar{F}^{(0)a,ik} + i\int d^4z'\bar{F}^{(0)a}_{ik}(z)\langle(\bar{D}^iA^{ak}(z) - i\leftrightarrow k)A^{bj}(z')\rangle T^b_j(z')\Big) = -\frac{1}{4}\int d^4z\bar{F}^{(0)a}_{ik}\bar{F}^{(1)a,ik},
\end{aligned} \tag{A10}$$

where $\bar{F}^{(1)a,ik}$ is a field strength in the first order in $[U, V]$. Using the fields (A8) we obtain

$$F^{(1)a}_{ik}(z) = -2g\theta(F)\Big(z\,\Big|W_F^\dagger\frac{\partial_i}{\partial^2 - i\epsilon}W_F\Big|^{ab}0, E^b_k\Big) + (F\leftrightarrow L) + (F\leftrightarrow R) + (F\leftrightarrow B) - (i\leftrightarrow k), \tag{A11}$$

(where $|0, E_i) \equiv \int d^2z'_\perp|0, z'_\perp)E_i(z'_\perp)$) and therefore

$$\begin{aligned}
S_{\rm eff} = -ig^2\int d^4z\Big\{&\theta(F)([U^i_1, V^k_1] - i\leftrightarrow k)^a\Big(W_F^\dagger\frac{\partial_i}{\partial^2}W_F\Big)^{ab}E^b_k + \theta(R)([U^i_1, V^k_2] - i\leftrightarrow k)^a\Big(W_R^\dagger\frac{\partial_i}{\partial^2}W_R\Big)^{ab}E^b_k\\
&+ \theta(L)([U^i_2, V^k_1] - i\leftrightarrow k)^a\Big(W_L^\dagger\frac{\partial_i}{\partial^2}W_L\Big)^{ab}E^b_k + \theta(L)([U^i_2, V^k_2] - i\leftrightarrow k)^a\Big(W_B^\dagger\frac{\partial_i}{\partial^2}W_B\Big)^{ab}E^b_k\Big\}.
\end{aligned} \tag{A12}$$

A typical integral in the above equation has the form

$$\int d^4z d^2z'_\perp \theta(z_*)\theta(z_\bullet)f(z_\perp)\Big(z\Big|\frac{p_i}{p^2+i\epsilon}\Big|0,z'_\perp\Big)g(z'_\perp) = \frac{i}{2\pi}\int_0^\infty \frac{d\alpha}{\alpha}\int d^2z_\perp d^2z'_\perp f(z_\perp)\Big(z_\perp\Big|\frac{p_i}{p_\perp^2}\Big|z'_\perp\Big)g(z'_\perp), \quad \text{(A13)}$$

In the LLA, the integral $\int_0^\infty d\alpha/\alpha$ is replaced by $1/2\Delta\eta$. More accurately, one should remember that the slopes of Wilson lines are $e_1 = p_1 + e^{-\eta_1}p_2$ and $e_2 = p_2 + e^{\eta_2}p_1$ as shown in Eq. (15). In this case, $\theta(z_*)\theta(z_\bullet)$ in the integrand of Eq. (A13) will be replaced by $\theta(z_* + e^{\eta_2}z_\bullet)\theta(z_\bullet + e^{-\eta_1}z_*)$ so one obtains

$$\begin{aligned}\int d^4z d^2z'_\perp \theta(z_*)\theta(z_\bullet)f(z_\perp)\Big(z\Big|\frac{p_i}{p^2+i\epsilon}\Big|0,z'_\perp\Big)g(z'_\perp) &= -\int d^2z_\perp d^2z'_\perp f(z_\perp)g(z'_\perp)\\ &\quad\times\int\frac{d\alpha d\beta d^2p_\perp}{16\pi^4}e^{i(p,z-z')_\perp}\frac{p_i}{\alpha\beta s - p_\perp^2 + i\epsilon}\\ &\quad\times\frac{1}{(\alpha + e^{\eta_2}\beta - i\epsilon)(\beta + e^{-\eta_1}\alpha - i\epsilon)}\\ &= -i\int d^2z_\perp d^2z'_\perp f(z_\perp)g(z'_\perp)\int_0^\infty\frac{d\alpha}{\alpha}\\ &\quad\times\frac{d^2p_\perp}{8\pi^3}e^{i(p,z-z')_\perp}\Big(\frac{p_i}{e^{-\eta_1}\alpha^2 s + p_\perp^2} - \frac{p_i}{e^{-\eta_2}\alpha^2 s + p_\perp^2}\Big)\\ &= -\frac{i}{4\pi}\Delta\eta\int d^2z_\perp d^2z'_\perp f(z_\perp)\Big(z_\perp\Big|\frac{p_i}{p_\perp^2}\Big|z'_\perp\Big)g(z'_\perp), \end{aligned}\quad \text{(A14)}$$

where $\Delta\eta = \eta_1 - \eta_2$. Performing the integrations over $z_*, z_\bullet$ in Eq. (A12) we get

$$\begin{aligned} S_{\text{eff}} = -\alpha_s\Delta\eta\int d^2z_\perp\Big\{&([U_1^i, V_1^k] - i\leftrightarrow k)^a\Big(W_F^\dagger\frac{\partial_i}{\partial_\perp^2}W_F\Big)^{ab} - ([U_1^i, V_2^k] - i\leftrightarrow k)^a\Big(W_R^\dagger\frac{\partial_i}{\partial_\perp^2}W_R\Big)^{ab}\\ &- ([U_2^i, V_1^k] - i\leftrightarrow k)^a\Big(W_L^\dagger\frac{\partial_i}{\partial_\perp^2}W_L\Big)^{ab} + ([U_2^i, V_2^k] - i\leftrightarrow k)^a\Big(W_B^\dagger\frac{\partial_i}{\partial_\perp^2}W_B\Big)^{ab}\Big\}E_k^b = -i\alpha_s\Delta\eta\int d^2z_\perp E_i^a E^{ai}. \end{aligned}\quad \text{(A15)}$$

which coincides with Eq. (42).

Finally, let us demonstrate that the diamond trace of four (nondifferentiated) Wilson lines is trivial (this is related to the fact that the field strength F_{+-} vanishes in the leading order, see Eqs. (A8) and (A9)). To regularize the corresponding expressions, we consider the ''original'' tilted Wilson loop shown in Fig. 11 for the finite size L. We need to prove that

$$\begin{aligned}\lim_{L\to\infty}\text{tr}\{&[-Le_1 + Le_2 + x_\perp, Le_1 + Le_2 + x_\perp][Le_2 + Le_1 + x_\perp, -Le_2 + Le_1 + x_\perp][Le_1 - Le_2 + x_\perp, -Le_1 - Le_2 + x_\perp]\\ &\times[-Le_2 - Le_2 + x_\perp, Le_2 - Le_2 + x_\perp]\} = 1\end{aligned}\quad \text{(A16)}$$

in the leading nontrivial order in $[U, V]$.

Consider the case $\mathcal{U}_i \ll 1$, $\mathcal{V}_i \sim 1$ (the opposite case $\mathcal{V}_i \ll 1$, $\mathcal{U}_i \sim 1$ is similar). It is easy to see from Eq. (A9) that $[Le_2 \pm Le_1 + x_\perp, -Le_2 \pm Le_1 + x_\perp] \sim [U, V]^2$ so we are left with

$$\lim_{L\to\infty}\text{tr}\{[-Le_1 + Le_2 + x_\perp, Le_1 + Le_2 + x_\perp]\times[Le_1 - Le_2 + x_\perp, -Le_1 - Le_2 + x_\perp]\}. \quad \text{(A17)}$$

At this point, we can take the limit $L \to \infty$ in the e_1 direction. We obtain:

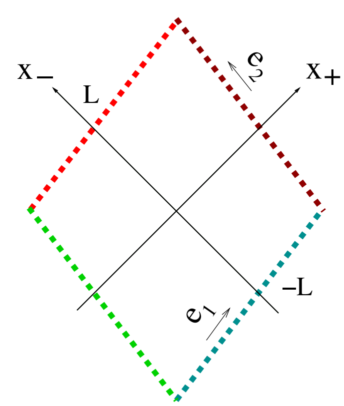

FIG. 11 (color online). Trace of four Wilson lines.

$$[x_\perp + Le_2, x_\perp + Le_2 + \infty e_1][x_\perp - Le_2 + \infty e_1, x_\perp - Le_2] - 1$$
$$= \frac{i}{\pi^2}\int d\alpha d\beta \frac{\sin\alpha L}{(\alpha + e^{\eta_2}\beta - i\epsilon)(\beta + e^{-\eta_1}\alpha - i\epsilon)}\left(x_\perp \left| U_1^\dagger \frac{1}{\alpha\beta s - p_\perp^2 + i\epsilon} U_1 \right|^{ab} \right|[U_{1i} - U_{2i}, E_L^i - E_2^i]^b\Big)$$
$$= -\frac{2}{\pi}\int_0^\infty d\alpha \frac{\sin\alpha L}{\alpha}\left(x_\perp | U_1^\dagger \left(\frac{1}{e^{-\eta_1}\alpha^2 s + p_\perp^2} - \frac{1}{e^{-\eta_2}\alpha^2 s + p_\perp^2}\right) U_1 |^{ab} | [U_{1i} - U_{2i}, E_L^i - E_2^i]^b\right). \tag{A18}$$

We see now that in the limit $L \to \infty$ the right-hand side of Eq. (A19) vanishes so the left-hand side is at best $\sim [U, V]^2$. Similarly,

$$\lim_{L\to\infty}[x_\perp + Le_2, x_\perp + Le_2 - \infty e_1][x_\perp - Le_2 - \infty e_1, x_\perp - Le_2] = 1, \tag{A19}$$

and therefore the trace (A17), which is product of left-hand side of Eq. (A18) and (A19), is equal to 1 in the leading order.

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