1996

Q^2 Evolution of Chiral-Odd Twist-3 Distributions h_L(�, Q^2) and e(�, Q^2) in Large- N_C QCD

I.I. Balitsky
ibalitsk@odu.edu

V. M. Braun

K. Koike

K. Tanaka

Original Publication Citation
Balitsky, I.I., Baun, V.M., Koike, Y., & Tanaka, K. (1996) Q^2 evolution of chiral-odd twist-3 distributions h_L(�, Q^2) and e(�, Q^2) in Large- N_C QCD. *Physical Review Letters, 77*(15), 3078-3081

Q^2 Evolution of Chiral-Odd Twist-3 Distributions $h_L(x, Q^2)$ and $e(x, Q^2)$ in Large-N_c QCD

I. I. Balitsky,[1] V. M. Braun,[2] Y. Koike,[3] and K. Tanaka[4]
[1]Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
[2]NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
[3]Department of Physics, Niigata University, Niigata, 950-21 Japan
[4]Department of Physics, Juntendo University, Inba-gun, Chiba 270-16, Japan
(Received 29 May 1996)

We prove that the twist-3 chiral-odd parton distributions obey simple Gribov-Lipatov-Altarelli-Parisi evolution equations in the limit $N_c \to \infty$ and give analytic results for the corresponding anomalous dimensions. To this end we introduce an evolution equation for the corresponding three-particle twist-3 parton correlation functions and find an exact analytic solution. For large values of n (operator dimension) we are further able to collect all corrections subleading in N_c, so our final results are valid to $O((1/N_c^2)\ln(n)/n)$ accuracy. [S0031-9007(96)01302-6]

PACS numbers: 11.15.Pg, 11.10.Hi, 12.38.Bx, 13.85.Qk

The increasing precision of experimental data from LEP, HERA, and the Tevatron requires understanding of higher twist corrections induced by correlations of partons in the colliding (produced) hadrons. The twist-3 parton distributions play a distinguished role in spin physics, where they can be measured as leading effects responsible for certain asymmetries. In particular, the E143 Collaboration has reported the measurement of the twist-3 contribution of the polarized structure function $g_2(x, Q^2)$ [1], thus providing us with the first experimental test of quark-gluon correlations in the nucleon. The chiral-odd parton distribution $h_L(x, Q^2)$ is expected to be measurable in the Drell-Yan production [2,3].

The Q^2 evolution of twist-3 distributions is usually believed to be quite sophisticated due to mixing with quark-antiquark-gluon operators, the number of which increases with spin (moment of the structure function). For the case of the chiral-even twist-3 flavor-nonsinglet distribution $g_2(x, Q^2)$ it has been observed by Ali-Braun-Hiller (ABH) [4] that the operators involving gluons effectively decouple from the evolution equation in the large N_c limit, that is, neglecting $O(1/N_c^2)$ corrections [5]. Consequently, $g_2(x, Q^2)$ obeys a simple Gribov-Lipatov-Altarelli-Parisi (GLAP) evolution equation and the corresponding anomalous dimension is known in analytic form [4]. The statement holds true with full account for effects subleading in N_c but for large moments n. Thus the claimed accuracy is, in fact, $O((1/N_c^2)\ln(n)/n)$. The ABH evolution equation gives a guide to the expected small x and large x behavior which is important for experimental extrapolations, and is used [6] to rescale the model predictions to high values of Q^2 of the actual experiments.

Physics of this decoupling is so far not understood, and it is probably not related to usual simplifications of the $N_c \to \infty$ limit. The observed phenomenon appears, however, to be quite general. In this Letter we demonstrate that the same pattern is obeyed by chiral-odd parton distributions $h_L(x, Q^2)$ and $e(x, Q^2)$ [3] as well, albeit with different anomalous dimensions. For all practical purposes this solves the problem of the Q^2 evolution of twist-3 nonsinglet parton distributions, since the corrections $1/N_c^2$ are small.

Following [3,7] we define the parton distributions in question as nucleon matrix elements of nonlocal light-cone operators [8]

$$\langle PS|\overline{\psi}(z)\psi(-z)|PS\rangle = 2M\int_{-1}^{1} dx\, e^{2i(P\cdot z)x} e(x,\mu^2) \quad (1)$$

and

$$\langle PS|\overline{\psi}(z)\sigma_{\mu\nu}z_\nu i\gamma_5\psi(-z)|PS\rangle = \frac{2}{M} S_{\perp\mu}(P\cdot z)\int_{-1}^{1} dx\, e^{2i(P\cdot z)x} h_1(x,\mu^2) - 2Mz_\mu \frac{(S\cdot z)}{(P\cdot z)} \int_{-1}^{1} dx\, e^{2i(P\cdot z)x} h_L(x,\mu^2). \quad (2)$$

Here z is a lightlike vector $z^2 = 0$, P and S are the nucleon momentum and spin vectors ($P^2 = M^2$, $S^2 = -M^2$, $P\cdot S = 0$), respectively, and $S_{\perp\mu} = S_\mu - P_\mu(S\cdot z)/(P\cdot z) + M^2 z_\mu(S\cdot z)/(P\cdot z)^2$. A thorough discussion of the parton interpretation and of physical relevance of these distributions can be found in Ref. [3].

As it is well known, equations of motion allow us to express the twist-3 quark-antiquark distributions in terms of quark-gluon correlations. This is visualized most explicitly in the form of the operator identities [see Eqs. (27) and (28) in [9]]

$$\overline{\psi}(z)\psi(-z) = \overline{\psi}(0)\psi(0) + \int_0^1 du \int_{-u}^{u} dt\, S(u,t,-u), \quad (3)$$

$$\overline{\psi}(z)\sigma_{\mu\nu}z_\nu i\gamma_5\psi(-z) = [\overline{\psi}(z)\sigma_{\mu\nu}z_\nu i\gamma_5\psi(-z)]_{\text{twist 2}} + iz_\mu\int_0^1 u\,du\int_{-u}^{u} t\,dt\,\tilde{S}(u,t,-u)\,, \tag{4}$$

where we have introduced shorthand notations for the nonlocal operators

$$\begin{aligned} S(u,t,v;\mu^2) &= \overline{\psi}(uz)\sigma_{\mu\xi}gG_{\nu\xi}(tz)z_\mu z_\nu\psi(vz)\,,\\ \tilde{S}(u,t,v;\mu^2) &= \overline{\psi}(uz)i\sigma_{\mu\xi}\gamma_5 gG_{\nu\xi}(tz)z_\mu z_\nu\psi(vz)\,. \end{aligned} \tag{5}$$

The relations in (3) and (4) are exact up to twist-4 corrections and neglecting operators containing total derivatives [9] which are irrelevant for our present purposes.

Note contribution of the local scalar operator in Eq. (3). It gives rise to the nucleon σ term and does not have a partonic interpretation. The remaining contributions give rise to three-particle parton distributions describing interference between scattering from a coherent quark-gluon pair and from a single quark. In particular, one can define

$$\langle PS|S(u,t,-u)|PS\rangle = -8M(p\cdot z)^2\int d\alpha \times \int d\beta\, e^{i(p\cdot z)[\alpha(u-t)+\beta(u+t)]} \times D_g(\alpha,\beta)\,, \tag{6}$$

and a similar quantity $\tilde{D}_g(\alpha,\beta)$ for $\tilde{S}$. The support properties of $D_g(\alpha,\beta), \tilde{D}_g(\alpha,\beta)$ and their parton interpretation are discussed in Ref. [10]. The variables α, β, and $\alpha-\beta$ have physical meaning of the momentum fractions carried by the antiquark, quark, and gluon partons, respectively, and $D_g, \tilde{D}_g$ vanish unless $|\alpha|<1$, $|\beta|<1$, and $|\alpha-\beta|<1$. Combining (3), (4), and (6) one can express the twist-3 structure functions $h_L(x)$ and $e(x)$ in terms of a certain integral of these quark-antiquark-gluon distributions [11]. Note that only such integral (over the gluon momentum) is potentially measurable in inclusive reactions like deep inelastic scattering or Drell-Yan processes.

The Q^2 dependence of the twist-3 distributions is governed by the renormalization group (RG) equation for the corresponding nonlocal operators $S, \tilde{S}$. To leading logarithmic accuracy the evolution of S and $\tilde{S}$ is the same; hence we drop the "tilde" in what follows.

We find it convenient to use a general approach of [12] to write RG equations directly for the nonlocal operators. To this end we introduce the Mellin transformed operators [4,12]

$$S(u,t,v) = \frac{1}{2\pi i}\int_{1/2-i\infty}^{1/2+i\infty} dj(u-v)^{j-2}S(j,\xi)\,, \qquad \xi = \frac{u+v-2t}{u-v}\,. \tag{7}$$

Here j is the complex angular momentum; operators with different j do not mix with each other. The Mellin transformed operators satisfy the RG equation [12]

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)S(j,\xi;\mu) = -\frac{\alpha_s}{2\pi}\int_{-1}^{1}d\eta \times K_j(\xi,\eta)S(j,\eta;\mu)\,, \tag{8}$$

where the kernel $K_j(\xi,\eta)$ is subject to an explicit calculation. Neglecting all contributions which are down by $1/N_c^2$ we get

$$\begin{aligned} \frac{1}{N_c}K_j(\xi,\eta) = {}& \frac{5}{2}\delta(\xi-\eta) - \frac{\theta(\xi-\eta)}{\xi-\eta}\,\frac{1+\eta}{1+\xi}\left[\frac{1+\eta}{1+\xi} + \left(\frac{1-\xi}{1-\eta}\right)^j\right]\\ &+ \delta(\xi-\eta)\int_{-1}^{\xi}\frac{d\eta'}{\xi-\eta'}\,\frac{1+\eta'}{1+\xi}\left[1+\left(\frac{1-\xi}{1-\eta'}\right)^2\right] - \frac{\theta(\eta-\xi)}{\eta-\xi}\,\frac{1-\eta}{1-\xi}\left[\frac{1-\eta}{1-\xi}+\left(\frac{1+\xi}{1+\eta}\right)^j\right]\\ &+ \delta(\eta-\xi)\int_{\xi}^{1}\frac{d\eta'}{\eta'-\xi}\,\frac{1-\eta'}{1-\xi}\left[1+\left(\frac{1+\xi}{1+\eta'}\right)^2\right]\\ &- 4\frac{\theta(\xi-\eta)}{(1+\xi)^3}\left\{\frac{2}{j}\left[1-\left(\frac{1-\xi}{1-\eta}\right)^j\right] - \frac{1-\eta}{j+1}\left[1-\left(\frac{1-\xi}{1-\eta}\right)^{j+1}\right]\right\}\\ &- 4\frac{\theta(\eta-\xi)}{(1-\xi)^3}\left\{\frac{2}{j}\left[1-\left(\frac{1+\xi}{1+\eta}\right)^j\right] - \frac{1+\eta}{j+1}\left[1-\left(\frac{1+\xi}{1+\eta}\right)^{j+1}\right]\right\}. \end{aligned} \tag{9}$$

The kernel turns out to be very similar to the corresponding kernel for the evolution of chiral-even twist-3 operators in Ref. [4]. Derivation of (9) will be given elsewhere [11].

To solve the RG equation in (8) we consider the *conjugate* homogeneous equation

$$\int_{-1}^{1}d\eta\,K_j(\eta,\xi)\phi_j(\eta) = \gamma_j\phi_j(\xi)\,. \tag{10}$$

Each eigenfunction of (8) corresponds to a multiplicatively renormalizable *nonlocal* operator with the anomalous dimension γ_j,

$$\int_{-1}^{1} d\xi\, \phi_j(\xi) S(j,\xi;Q) = [\alpha_s(Q)/\alpha_s(\mu)]^{\gamma_j/b} \times \int_{-1}^{1} d\xi\, \phi_j(\xi) S(j,\xi;\mu)\,, \tag{11}$$

where $b = (11N_c - 2N_f)/3$. To prove this, multiply (8) by $\phi_j(\xi)$ and integrate over ξ. Using (10) one obtains the conventional diagonalized RG equation with the solution in (11).

We were able to find two solutions for the equation in (10) with eigenfunctions

$$\phi^+(\xi) = 1, \qquad \phi^-(\xi) = \xi \tag{12}$$

(they do not depend on j, so we drop the subscript). The corresponding eigenvalues (anomalous dimensions) equal

$$\gamma_j^+ = 2N_c\left\{\psi(j+1) + \gamma_E - \frac{1}{4} - \frac{1}{2(j+1)}\right\}, \tag{13}$$

$$\gamma_j^- = 2N_c\left\{\psi(j+1) + \gamma_E - \frac{1}{4} + \frac{3}{2(j+1)}\right\}, \tag{14}$$

where $\psi(z) = (d/dz)\ln\Gamma(z)$ and γ_E is the Euler constant. Validity of Eqs. (13) and (14) can be checked by a straightforward calculation. By comparison with results of the numerical evaluation of anomalous dimensions for integer $j = n$ (see below) we conclude that our solutions always correspond to operators with the *lowest* anomalous dimension in the spectrum.

The superscript $\pm$ corresponds to the "parity" under $\xi \to -\xi$: due to the symmetry of the kernel one can look for separate solutions in the space of functions which are even (odd) with respect to the substitution $\xi \to -\xi$. From Eqs. (3) and (4), one sees that the relevant quantities for $e(x)$ and $h_L(x)$ are even and odd "ξ-parity" pieces of the nonlocal operators.

Substituting the definition (7) into (3) and (4), and changing the integration variable t to $\xi = -t/u$, we observe that the nonlocal operators in (11) with the particular choices of the weight function (12) are precisely those which give rise to twist-3 quark-antiquark operators at the tree level. Taking the nucleon matrix elements, we get for the moments

$$\mathcal{M}_n[e](Q) = L^{\gamma_n^+/b}\mathcal{M}_n[e](\mu)\,, \tag{15}$$

$$\mathcal{M}_n\left[\tilde{h}_L\right](Q) = L^{\gamma_n^-/b}\mathcal{M}_n\left[\tilde{h}_L\right](\mu)\,, \tag{16}$$

where $\mathcal{M}_n[\tilde{h}_L] \equiv \int_{-1}^{1} dx\, x^n \tilde{h}_L(x)$, $\mathcal{M}_n[e] \equiv \int_{-1}^{1} dx \times x^n e(x)$, and $L \equiv \alpha_s(Q)/\alpha_s(\mu)$. $\tilde{h}_L$ is the genuine twist-3 contribution to h_L, after subtracting out the twist-2 piece [3]. The solutions for the evolution in (15) and (16) present the principal result of our paper.

Expansion of nonlocal operators at small quark-antiquark separations generates the series of local operators of increasing dimension. In particular, expansion of $S(j,\xi)$ for positive integers $j = n \geq 2$ generates the local operators $\theta_{nk} = \overline{\psi}(\overleftarrow{D}\cdot z)^{k-2}\sigma G(z\cdot\overrightarrow{D})^{n-k}\psi$ $(k = 2,3,\ldots)$. The substitution $\xi \to -\xi$ for the nonlocal operators corresponds to $k \to n-k+2$ for the local operators. From the kernel (9) one can calculate the mixing matrix for the local operators even as well as odd under $k \to n-k+2$. One can check that the result for the odd case coincides with Eqs. (3.14)–(3.16) in Ref. [13] to the stated $1/N_c^2$ accuracy. In particular, neglecting the $1/N_c^2$ terms in the mixing matrix of [13] we obtain the evolution equation (16) with

$$\mathcal{M}_n\left[\tilde{h}_L\right](\mu) = \sum_{k=2}^{[n+1/2]}\left(1 - \frac{2k}{n+2}\right)b_{n,k}(\mu)\,, \tag{17}$$

where $b_{n,k}(\mu)$ $(k = 2,\ldots,[(n+1)/2])$ are reduced matrix elements of the independent quark-antiquark-gluon operators in the notation of [3,13]. This is the consequence of the fact that in this limit we have

$$\sum_{k=2}^{[n+1/2]}\left(1 - \frac{2k}{n+2}\right)X_{kl} = -\left(1 - \frac{2l}{n+2}\right)\gamma_n^-\,, \tag{18}$$

where X_{kl} is the mixing matrix in the notation of [13] as

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g)\frac{\partial}{\partial g}\right)b_{n,k}(\mu) = \frac{\alpha_s}{2\pi}\sum_{l=2}^{[n+1/2]} X_{kl}b_{n,l}(\mu)\,. \tag{19}$$

The coefficients $1 - 2k/(n+2)$ correspond to $\phi^-(\xi)$ of (12) in the nonlocal operator language. Equation (17) is precisely the operator giving $\tilde{h}_L$ at the tree level. All operators with higher anomalous dimensions decouple from the Q^2 evolution in the $N_c \to \infty$ limit.

The complete spectrum of anomalous dimensions in the $N_c \to \infty$ limit obtained by the numerical diagonalization of the mixing matrix in [13] is shown in Fig. 1, together with our analytic solution for the lowest eigenvalue.

To illustrate numerical accuracy of the leading-N_c approximation, consider the exact result [13] (including $1/N_c^2$ corrections) for the evolution of the $n = 5$ moment of h_L, which is the lowest moment in which mixing appears,

$$\mathcal{M}_5\left[\tilde{h}_L\right](Q) = [0.416b_{5,2}(\mu) + 0.913b_{5,3}(\mu)]L^{12.91/b} + [0.013b_{5,2}(\mu) - 0.050b_{5,3}(\mu)]L^{18.05/b}. \tag{20}$$

This is reduced in the large N_c limit to

$$\mathcal{M}_5\left[\tilde{h}_L\right](Q) = \left[\tfrac{3}{7}b_{5,2}(\mu) + \tfrac{1}{7}b_{5,3}(\mu)\right]L^{13.7/b}. \tag{21}$$

One observes the following: (i) the lowest anomalous dimension and the coefficients in front of the two terms in this contribution are close to their exact values; (ii) the contribution of the operator with the higher anomalous dimension is small ($\sim 1/N_c^2$). It is the latter observation which is crucial for the phenomenological importance of

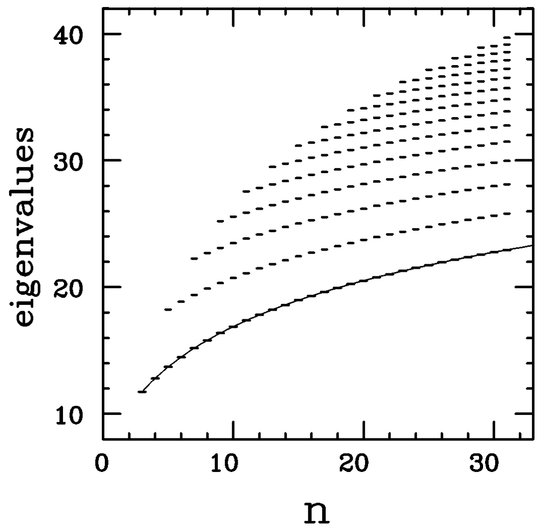

FIG. 1. Spectrum of anomalous dimensions of twist-3 operators for $h_L(x)$ in the limit $N_c \to \infty$. The solid line shows the analytic solution (14).

our results, since it means that the description of each moment of the twist-3 distribution requires one single nonperturbative parameter. A similar check for $e(x)$ [i.e., (18)–(21)] has also been done in [14].

We can make this point even stronger by observing that admixture of operators with higher anomalous dimensions is suppressed at large n for arbitrary values of N_c. The full evolution equation with account of all $1/N_c^2$ terms is complicated and will be given elsewhere [11]. However, it simplifies drastically in the limit $j \to \infty$ and coincides with the large-j evolution equation considered in [4]. We get

$$\gamma_j \phi(\xi) = 4C_f[\ln j + \gamma_E - 3/4]\phi(\xi) + N_c \phi(\xi) + N_c \int_{-1}^{\xi} d\eta \left(\frac{1-\xi}{1+\eta}\right)^2 \frac{\phi(\xi) - \phi(\eta)}{\eta - \xi} + N_c \int_{\xi}^{1} d\eta \left(\frac{1+\xi}{1+\eta}\right)^2 \frac{\phi(\xi) - \phi(\eta)}{\eta - \xi}, \quad (22)$$

with $C_f = (N_c^2 - 1)/2N_c$. Comparing to the large-j limit of the kernel in (9), the only difference is in the replacement $2N_c \to 4C_f$ in the first term. Thus, the functions in (12) still provide the solution, and the anomalous dimensions are shifted by

$$\gamma_j^{\pm} \to \gamma_j^{\pm} + (4C_f - 2N_c)[\ln j + \gamma_E - 3/4]. \quad (23)$$

With this modification of the anomalous dimensions, the results in (16) are valid to the $O((1/N_c^2)\ln(n)/n)$ accuracy.

To summarize, solutions in the present paper provide a powerful framework both in confronting with experimental data and for model building. From a general point of view, they are interesting as providing us with an example of an interacting three-particle system in which one can find an exact energy of the lowest state. For phenomenology, the main lesson is that inclusive measurements of twist-3 distributions are complete (to our accuracy) in the sense that knowledge of the distribution at one value of Q_0^2 is enough to predict its value at arbitrary Q^2, in the spirit of GLAP evolution equation. This allows us to relate results of different experiments to each other, and to compare with model calculations which typically are given at a very low scale.

As shown above, the $1/N_c^2$ corrections are not large for $n = 5$ and further decrease as $\ln(n)/n$ at large n. Thus, the only possibility for sizable $1/N_c^2$ effects might be for small x behavior, in case the location of the singularity in the complex j plane of the exact anomalous dimension happens to be on the right of the singularity of the leading N_c result. A detailed study of the small x behavior goes beyond the tasks of this Letter.

We thank the Institute for Nuclear Theory at the University of Washington for its hospitality and the DOE for partial support during our visit which initiated this study.

[1] K. Abe *et al.*, Phys. Rev. Lett. **76**, 587 (1996).
[2] J.P. Ralston and D.E. Soper, Nucl. Phys. **B152**, 109 (1979); J.L. Cortes, B. Pire, and J.P. Ralston, Z. Phys. C **55**, 409 (1992).
[3] R.L. Jaffe and X. Ji, Nucl. Phys. **B375**, 527 (1992).
[4] A. Ali, V.M. Braun, and G. Hiller, Phys. Lett. B **266**, 117 (1991).
[5] The leading terms in the evolution equation are proportional to N_c while the neglected terms are of order $1/N_c$. Thus the accuracy is $O(1/N_c^2)$.
[6] M. Stratmann, Z. Phys. C **60**, 763 (1993); X. Song, Report No. INPP-UVA-95-04 (hep-ph/9604264).
[7] J.C. Collins and D.E. Soper, Nucl. Phys. **B194**, 445 (1982).
[8] We do not show the gauge factors connecting the quark and the antiquark fields.
[9] V.M. Braun and I.E. Filyanov, Z. Phys. C **48**, 239 (1990).
[10] R.L. Jaffe, Nucl. Phys. **B229**, 205 (1983).
[11] I.I. Balitsky, V.M. Braun, Y. Koike, and K. Tanaka, (to be published).
[12] I.I. Balitsky and V.M. Braun, Nucl. Phys. **B311**, 541 (1988/89).
[13] Y. Koike and K. Tanaka, Phys. Rev. D **51**, 6125 (1995).
[14] Y. Koike and N. Nishiyama, hep-ph/9609207.