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HIGH-ENERGY EFFECTIVE ACTION FROM SCATTERING OF QCD SHOCK WAVES

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At high energies, the relevant degrees of freedom are Wilson lines - infinite gauge links ordered along straight lines collinear to the velocities of colliding particles. The effective action for these Wilson lines is determined by the scattering of QCD shock waves. I develop the symmetric expansion of the effective action in powers of strength of one of the shock waves and calculate the leading term of the series. The corresponding first-order effective action, symmetric with respect to projectile and target, includes both up and down fan diagrams and pomeron loops.

Keywords: Effective action, small-x evolution, Wilson lines.

1. Introduction

It is widely believed that the relevant degrees of freedom for the description of high-energy scattering in QCD are Wilson lines - infinite straight-line gauge factors (for a review see Ref. 1). The particles with different rapidities perceive each other as Wilson lines so these lines may serve as the relevant degrees of freedom for high-energy scattering. The goal of this approach is to rewrite the original functional integral over gluons (and quarks) as a 2+1 theory with the effective action written in terms of the dynamical Wilson lines. For a given interval of rapidity, the effective action is an amplitude of scattering of two QCD shock waves, see Fig. 1. Indeed, let us integrate over the gluons in this interval of rapidity $\eta_1 > \eta > \eta_2$ leaving the gluons with $\eta > \eta_1$ (the "right-movers') and with $\eta < \eta_2$ (the "left-movers') intact (to be integrated over later). Due to the Lorentz contraction, the left-moving and the right-moving gluons shrink to the two gluon "pancakes" or shock waves. The result of the integration over the rapidities $\eta_1 > \eta > \eta_2$ is the effective action which depends on the Wilson lines made from the left-and right-movers.

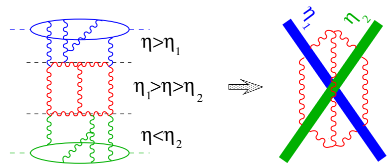

Fig. 1. High-energy effective action as an amplitude of the collision of two shock waves.

2. Rapidity factorization

The main technical tool of the shock-wave approach to the high-energy scattering is the rapidity factorization developed in Ref. 2. Consider a functional integral for the typical scattering amplitude

$$\int DAJ(p_A)J(p_B)J(-p'_A)J(-p'_B)\ e^{iS(A)} \tag{1}$$

where the currents $J(p_A)$ and $J(p_B)$ describe the two colliding particles (say, photons). We use Sudakov variables $k = \alpha p_1 + \beta p_2 + k_\perp$ and the notations $x_\bullet = p_1^\mu x_\mu = \sqrt{\frac{s}{2}}x^-$, $x_* = p_2^\mu x_\mu = \sqrt{\frac{s}{2}}x^+$ Here p_1 and p_2 are the light-like vectors close to p_A and p_B: $p_A = p_1 + \frac{p_A^2}{s}p_2$, $p_B = p_2 + \frac{p_B^2}{s}p_1$.

Let us take some "rapidity divide" η_1 such that $\eta_A > \eta_1 > \eta_B$ and integrate first over the gluons with the rapidity $\eta > \eta_1$, see Fig. 2a. From

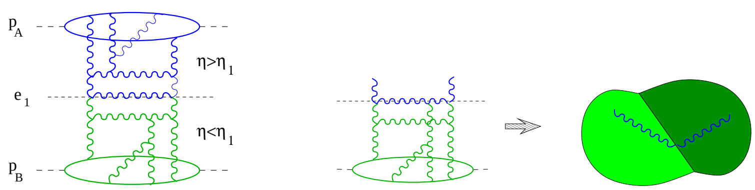

Fig. 2. Rapidity factorization (a) and shock wave in the temporal gauge (b).

the viewpont of such particles, the fields with $\eta < \eta_1$ shrink to a shock wave so the result of the integration is presented by Feynman diagrams in the shock-wave background. In the covariant gauge, this shock wave has the only non-vanishing component $A_\bullet$ which is concentrated near $x_* = 0$. In order to write down factorization we need to rewrite the shock wave in the temporal gauge $A_0 = 0$. In such gauge the most general form of a shock-wave background has the form (see Fig. 2b)

$$A^i = \mathcal{U}_1^i\theta(x_*) + \mathcal{U}_2^i\theta(-x_*), \quad A_\bullet = A_* = 0 \tag{2}$$

where $\mathcal{U}_1^i = U_1^\dagger \frac{i}{g}\partial_i U_1$ and $\mathcal{U}_2^i = U_2^\dagger \frac{i}{g}\partial_i U_2$ are the pure gauge fields (filling the half-spaces $x_* > 0$ and $x_* < 0$). There is a redundant gauge symmetry $U_1(x_\perp) \to U_1(x_\perp)\Omega(x_\perp)$, $U_2(x_\perp) \to U_2(x_\perp)\Omega(x_\perp)$ related to the fact that gauge invariant objects like the color dipole depend only on the product $U_{1z}U_{2z}^\dagger$.

The generating functional for the Green functions in the Eq. (2) background is given by[2]

$$\int DAJ(p_A)J(-p'_A)\ e^{iS(A)+i\int d^2z_\perp (0,F_{*i},0)^a_z(\mathcal{U}_1^{ai}-\mathcal{U}_2^{ai})_z}, \tag{3}$$

$$(0,F_{ei},0)_z \equiv \int_{-\infty}^{\infty} du\ [0,ue]_z F_{ei}(ue+z_\perp)[ue,0]_z\ =\ [0,\infty e]_z(i\frac{\partial}{\partial z^i}$$
$$+\ gA_i(\infty e+z_\perp))[\infty e,0]_z -\ [0,-\infty e]_z(i\frac{\partial}{\partial z^i}+gA_i(-\infty e+z_\perp))[-\infty e,0]_z$$

where $F_{ei} \equiv e^\mu F_{\mu i}$ and $(0,F_{\mu i},0)^a \equiv 2\mathrm{tr}\ t^a(0,F_{\mu i},0)$. [a]

To complete the factorization formula one needs to integrate over the remaining B fields with rapidities $\eta < \eta_1$:

$$\int DAJ(p_A)J(-p'_A)e^{-iS(\mathcal{A})}\ e^{iS(A)}J(p_B)J(-p'_B)\ =\ \int DAJ(p_A)J(-p'_A)$$
$$\times \int DBJ(p_B)J(-p'_B)e^{iS(A)+iS(B)+i\int d^2z_\perp (0,F_{e_1 i},0)^a_z(0,G_{e_1 i},0)^a_z} \tag{4}$$

where the Wilson-line operators $(0,F_{e_1 i},0)^a_z$ and $(0,G_{e_1 i},0)^a_z$ are the operators (3) made from A and B fields, respectively.

3. The effective action

3.1. *Scattering of QCD shock waves*

Applying the factorization formula (4) two times, one gets (see Fig. 3):

$$\int DA\ J(p_A)J(p_B)J(-p'_A)J(-p'_B)\ e^{iS(A)}\ = \tag{5}$$

$$\int DAJ(p_A)J(-p'_A)e^{iS(A)}\int DB\ J(p_B)J(-p'_B)\ e^{iS(B)}$$
$$\times \int DC\ e^{iS(C)+i\int d^2z_\perp\{[0,A_{e_1 i},0]^a_z[0,C_{e_1 i},0]^a_z+(0,C_{e_2 i},0)^a_z(0,B_{e_2 i},0)^a_z]\}}$$

where the slope is $e_1 = p_1 + e^{-\eta_1}p_2$ for the [...] Wilson lines and $e_2 = p_1 + e^{-\eta_2}p_2$ for the (...) ones.

[a]The sum over the Latin indices $i, j...$ runs over the two transverse components while the sum over Greek indices runs over the four components as usual

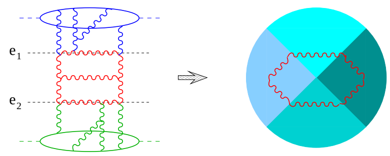

Fig. 3. Effective action as a scattering of two shock waves.

The functional integral over the central range of rapidity $\eta_1 > \eta > \eta_2$ is determined by the integral over C field with the sources made from "external" A and B fields: $(0, A_{e_1 i}, 0)_z = (\mathcal{V}_1^{ai} - \mathcal{V}_2^{ai})_z$ and $(0, B_{e_2 i}, 0)_z = (\mathcal{U}_1^{ai} - \mathcal{U}_2^{ai})_z$ where

$$\begin{aligned} V_{1,2}(z_\perp) &= [0, \pm\infty e_1]_z[\pm\infty e_1 + z_\perp, \pm\infty e_1 + \infty e_\perp] \\ U_{1,2}(z_\perp) &= [0, \pm\infty e_2]_z[\pm\infty e_2 + z_\perp, \pm\infty e_2 + \infty e_\perp] \end{aligned} \tag{6}$$

Since there is no field strength $F_{\mu\nu}$ at infinite time the direction of $e_\perp$ does not matter.

The result of the integration over the C field (the last line in the Eq. (5) is an effective action for the $\eta_1 > \eta > \eta_2$ interval of rapidity. The amplitude (1) is then the integral over A and B fields with this effective action.

3.2. *Expansion in commutators*

The effective action is defined by the last line in the functional integral (5) (hereafter we switch back to the usual notation A_μ for the integration variable and $F_{\mu\nu}$ for the field strength)

$$\begin{aligned} e^{iS_{\rm eff}(V_1,V_2,U_1,U_2;\eta_1-\eta_2)} &= \int DA \exp\Big(iS(A) \\ &+ i\int d^2z_\perp\Big\{(\mathcal{V}_1^{ai} - \mathcal{V}_2^{ai})_z[0, F_{\bullet i}, 0]_z^a + (\mathcal{U}_1^{ai} - \mathcal{U}_2^{ai})_z(0, F_{*i}, 0)_z^a\Big\}\Big) \end{aligned} \tag{7}$$

Taken separately, the sources $\sim \mathcal{U}_i$ create a shock wave $\mathcal{U}_{1i}\theta(x_*) + \mathcal{U}_{2i}\theta(-x_*)$ and those $\sim \mathcal{V}_i$ create $\mathcal{V}_{1i}\theta(x_\bullet) + \mathcal{V}_{2i}\theta(-x_\bullet)$ In QED, the two sources $\mathcal{U}_i$ and $\mathcal{V}_i$ do not interact (in the leading order in α) so the sum of the two shock waves

$$\bar{A}_i^{(0)} = \mathcal{U}_{1i}\theta(x_*) + \mathcal{U}_{2i}\theta(-x_*) + \mathcal{V}_{1i}\theta(x_\bullet) + \mathcal{V}_{2i}\theta(-x_\bullet)\,, \quad \bar{A}_\bullet^{(0)} = \bar{A}_*^{(0)} = 0 \tag{8}$$

is a classical solution. In QCD, the interaction between these two sources is described by the commutator $g[\mathcal{U}_i, \mathcal{V}_k]$ (the coupling constant g corresponds to the three-gluon vertex). We take the trial configuration in the form of a (improved) sum of the two shock waves (see Eq. (10) below) and expand the "deviation" of the full QCD solution from the QED-type ansatz (8) in powers of commutators $[U,V]$.

As demonstrated in Ref. 3, the gluon field at space (or time) infinity is a pure-gauge field which has the form of a sum of the shock waves plus a correction proportional to their commutator. Technically, for a pair of pure gauge fields $\mathcal{U}_i(x_\perp)$ and $\mathcal{V}_i(x_\perp)$ we define $\mathcal{W}_i(x_\perp)=\mathcal{U}_i(x_\perp)+\mathcal{V}_i(x_\perp)+gE_i(x_\perp;U,V)$ as a pure gauge field satisfying the equation $(i\partial_i+g[\mathcal{U}_i+\mathcal{V}_i,)E^i=0$. In the first order in $[U,V]$ this field has the form

$$E^a_i(U,V) \ = \ -(x_\perp|U\frac{p^k}{p_\perp^2}U^\dagger+V\frac{p^k}{p_\perp^2}V^\dagger-\frac{p^k}{p_\perp^2}|^{ab}[\mathcal{U}_i,\mathcal{V}_k]^b-i\leftrightarrow k) \tag{9}$$

where $[\mathcal{U}_i,\mathcal{V}_k]^a\equiv 2\mathrm{Tr}t^a[\mathcal{U}_i,\mathcal{V}_k]$.

The zero-order approximation for the solution of the classical equations for the functional integral (7) can be taken as a superposition of pure gauge fields in the forward, backward, left, and right quadrants of the space.

$$\begin{aligned}&\bar{A}^{(0)}_\bullet=\bar{A}^{(0)}_*=0,\qquad \bar{A}^{(0)i}=\mathcal{W}^i_F(x_\perp)\theta(x_*)\theta(x_\bullet) \\ &+\mathcal{W}^i_L(x_\perp)\theta(-x_*)\theta(x_\bullet)+\mathcal{W}^i_R(x_\perp)\theta(x_*)\theta(-x_\bullet)+\mathcal{W}^i_B(x_\perp)\theta(-x_*)\theta(-x_\bullet)\end{aligned} \tag{10}$$

where

$$\begin{aligned}\mathcal{W}^i_F&=\mathcal{U}^i_1+\mathcal{V}^i_1+E^i_F,\quad \mathcal{W}^i_L=\mathcal{U}^i_2+\mathcal{V}^i_1+E^i_L\\ \mathcal{W}^i_R&=\mathcal{U}^i_1+\mathcal{V}^i_2+E^i_R,\quad \mathcal{W}^i_B=\mathcal{U}^i_2+\mathcal{V}^i_2+E^i_B\end{aligned} \tag{11}$$

and $E^i_F(U_1,V_1)$, $E^i_L(U_2,V_1)$, $E^i_R(U_1,V_2)$, and $E^i_B(U_2,V_2)$ are given by Eq. (9). It is easy to demonstrate that the Lipatov vertex of gluon emission by the colliding shock waves is

$$L_i \ = \ 2(E^i_F-E^i_L-E^i_R+E^i_B) \ = \ 2(\mathcal{W}^i_F-\mathcal{W}^i_L-\mathcal{W}^i_R+\mathcal{W}^i_B) \tag{12}$$

(in the first order in $[U,V]$ expansion). As usually, the effective action in the leading LLA order is a product of two Lipatov vertices times the infinitesimal rapidity interval

$$S_{\rm eff}= \ \frac{\alpha_s\Delta\eta}{4}\int d^2z_\perp L^a_i(z_\perp)L^{ia}(z_\perp) \tag{13}$$

The corresponding functional integral for the finite rapidity interval has the form[3]

$$\begin{aligned}&e^{iS_{\rm eff}(U_1(x),U_2(x),V_1(x),V_2(x);\eta_1-\eta_2)}=\int DV_j(x,\eta)DU_j(x,\eta)\Big|^{V_j(x,\eta_1)=V_j(x)}_{U_j(x,\eta_2)=U_j(x)}\\ &\times\ e^{i\int d^2x[\mathcal{V}^a_{1i}(x)-\mathcal{V}^a_{2i}(x)][\mathcal{U}^{ai}_1(x,\eta_1)-\mathcal{U}^{ai}_2(x,\eta_1)]}\\ &\times\ e^{\int_{\eta_2}^{\eta_1}d\eta\int d^2x[-i(\mathcal{V}^a_{1i}-\mathcal{V}^a_{2i})\frac{\partial}{\partial\eta}(\mathcal{U}^{ai}_1-\mathcal{U}^{ai}_2)\ +\ \frac{\alpha_s}{4}L^a_i(V,U)L^{ai}(V,U)]}\end{aligned} \tag{14}$$

It can be demonstrated that the functional integral (14) reproduces the non-linear BK evolution[4] in the case of one small source (when either $\mathcal{U}_i$ or $\mathcal{V}_i$ is small).

3.3. *Gauge-invariant form of the effective action*

The product of two Lipatov vertices (13) can be rewritten it in the gauge-invariant "diamond" form of trace of four Wilson lines at $x_{\bullet,*} = \pm\infty$ as suggested in a recent paper [5](see also [6,7]):

$$\begin{aligned}
\frac{1}{8}L_i^a(U,V)L^{ai}(U,V) \;=\; & \mathrm{tr}\{[-\infty p_1, F_{\bullet i}, \infty p_1]_{\infty p_2}[\infty p_2, F_{*i}, -\infty p_2]_{\infty p_1} \\
& \times [\infty p_1, -\infty p_1]_{-\infty p_2}[-\infty p_2, \infty p_2]_{-\infty p_1} \;+\; \mathrm{tr}[-\infty p_1, \infty p_1]_{\infty p_2} \\
& \times [\infty p_2, F_{*i}, -\infty p_2]_{\infty p_1}[\infty p_1, F_{\bullet i}, -\infty p_1]_{-\infty p_2}[-\infty p_2, \infty p_2]_{-\infty p_1} \\
& + \mathrm{tr}[-\infty p_1, \infty p_1]_{\infty p_2}[\infty p_2, -\infty p_2]_{\infty p_1}[\infty p_1, F_{\bullet i}, \infty p_1]_{-\infty p_2} \\
& \times [-\infty p_2, F_{*i}, \infty p_2]_{-\infty p_1} \;+\; \mathrm{tr}[-\infty p_1, F_{\bullet i}, \infty p_1]_{\infty p_2}[\infty p_2, -\infty p_2]_{\infty p_1} \\
& \times [\infty p_1, -\infty p_1]_{-\infty p_2}[-\infty p_2, F_{*i}, \infty p_2]_{-\infty p_1}\} \qquad (15)
\end{aligned}$$

where the transverse arguments in all Wilson lines are $x_\perp$. The structure of the effective action in the functional integral (14) is presented in Fig. 4. Note that the two terms in the exponent in the effective action, shown in Fig. 4, are both local in $x_\perp$ but differ with respect to the longitudinal coordinates: the first (kinetic) term is made from the Wilson lines located at $x_+ = 0$ or $x_- = 0$ while the second term is made from the Wilson lines at $x_\pm = \pm\infty$. Unfortunately, the transition between these Wilson lines is nonlocal in $x_\perp$ (see Eq. (9)) and so the resulting effective action is a non-local function of the dynamical variables U and V.

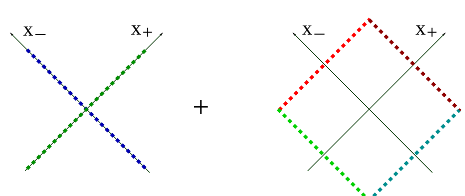

Fig. 4. Wilson-line structure of the effective action.

4. Conclusion

The functional integral (14) gives an example of the effective 2+1 theory for high-energy scattering in QCD. It is only a model - the genuine effective action for the $2+1$ high-energy theory of Wilson lines must include all the contributions $\sim [U,V]^n$. However, this model is correct in the case of weak projectile fields and strong target fields, and vice versa. In terms of Feynman diagrams, the effective action (14) includes both "up" and "down" fan ladders and the pomeron loops. In the dipole language, it describes both multiplication and recombination of dipoles (see the discussion in Refs.8, 9).

In conclusion I would like to emphasize that the effective action (14) summarizes all present knowledge about the high-energy evolution of Wilson lines in a way symmetric with respect to projectile and target and hence it may serve as a starting point for future analysis of high-energy scattering in QCD.

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