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HIGH-ENERGY EFFECTIVE ACTION FROM SCATTERING OF SHOCK WAVES IN QCD

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I demonstrate that the amplitude for high-energy scattering can be factorized as a convolution of the contributions due to fast and slow fields. The fast and slow fields interact by means of Wilson-line operators – infinite gauge factors ordered along the straight line. The resulting factorization formula gives a starting point for a new approach to the effective action for high-energy scattering in QCD.

1 Introduction.

It is well known that the power behavior of BFKL cross section violates the Froissart bound. The BFKL pomeron describes only the pre-asymptotic behavior at not very large energies and in order to find the true high-energy asymptotics in perturbative QCD we need to unitarize the BFKL pomeron.

One of the most popular ideas on solving this problem is to reduce the QCD at high energies to some sort of two-dimensional effective theory which will be simpler than the original QCD, maybe even to the extent of exact solvability. Some attempts in this direction were made starting from the work [2] but the matter is an open issue for the time being. Here I will describe the new approach to the effective action [3] based on the high-energy factorization in rapidity space[4].

2 Factorization for high-energy scattering.

Factorization in rapidity space means that the high-energy scattering amplitude can be represented as a convolution of contributions due to "fast" and "slow" fields. To be precise, we choose a certain rapidity η_0 to be a "rapidity divide" and we call fields with $\eta > \eta_0$ fast and fields with $\eta < \eta_0$ slow where η_0 lies in the region between spectator rapidity η_A and target rapidity η_B. [a]

[a] Instead of rapidity, we will often use the decomposition in Sudakov variables

$$p^\mu = \alpha_p p_1^\mu + \beta_p p_2^\mu + p_\perp^\mu \tag{1}$$

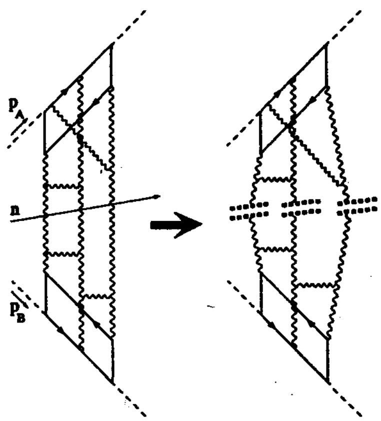

Figure 1. Structure of the factorization formula. Dashed, solid, and wavy lines denote photons, quarks, and gluons, respectively. Wilson-line operators are denoted by dotted lines and the vector n gives the direction of the "rapidity divide" between fast and slow fields.

The interpretation of this fields as fast and slow is literally true only in the rest frame of the target but we will use this terminology for any frame.

To explain what we mean by the factorization in rapidity space let us consider the classical example of high-energy scattering of virtual photons with virtualities $\sim -m^2$.

$$A(s,t) = -i\langle 0|T\{j(p_A)j(p'_A)j(p_B)j(p'_B)\}|0\rangle. \tag{2}$$

where $j(p)$ is the Fourier transform of electromagnetic current $j_\mu(x)$ multiplied by some suitable polarization $e^\mu(p)$.

The factorization formula for the amplitude (2) has the form [4] (see Fig. 1):

$$\begin{aligned} &\int \mathcal{D}A\mathcal{D}\bar{\Psi}\mathcal{D}\Psi e^{iS(A,\Psi)} j(p_A)j(p'_A)j(p_B)j(p'_B) = \\ &\int \mathcal{D}\mathcal{A}\mathcal{D}\bar{\psi}\mathcal{D}\psi e^{iS(\mathcal{A},\psi)} j(p_A)j(p'_A) \int \mathcal{D}\mathcal{B}\mathcal{D}\bar{\chi}\mathcal{D}\chi \\ &j(p_B)j(p'_B)e^{iS(\mathcal{B},\chi)} \exp\left\{i\int d^2x_\perp U^{ai}(x_\perp)V_i^a(x_\perp)\right\} \end{aligned} \tag{3}$$

where p_1^μ and p_2^μ are the light-like vectors close to p_A and p_B, respectively ($p_A^\mu = p_1^\mu - p_2^\mu p_A^2/s$, $p_B^\mu = p_2^\mu - p_1^\mu p_B^2/s$). Then the fields with $\alpha > \sigma$ are fast and those with $\alpha < \sigma$ are slow where where σ is defined in such a way that the corresponding rapidity is η_0. (In explicit form $\eta_0 = \ln\frac{\sigma}{\tilde{\sigma}}$ where $\tilde{\sigma} \equiv \frac{m^2}{s\sigma}$)

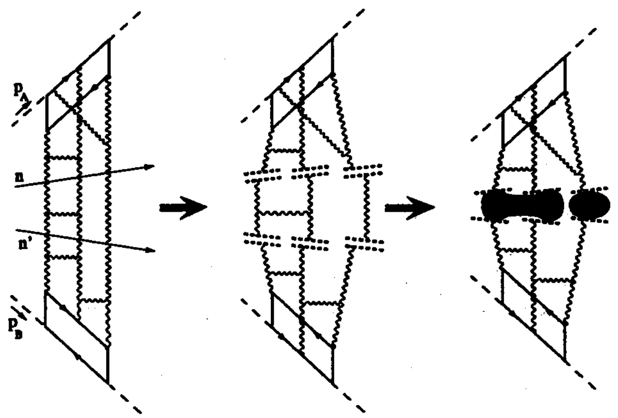

Figure 2. The effective action for the interval of rapidities $\eta_0 > \eta > \eta_0'$. The two vectors n and n' correspond to "rapidity divides" η_0 and η_0' bordering our chosen region of rapidities.

Here $U_i(x) = U^\dagger(x)\frac{i}{g}\frac{\partial}{\partial x_i}U(x)$ where the Wilson-line operator $U(x)$ is the infinite gauge link ordered along the straight line collinear to $n = \sigma p_1 + \tilde{\sigma} p_2$ corresponding to the "rapidity divide" η_0. In explicit form

$$U(x_\perp) = [\infty n + x_\perp, -\infty n + x_\perp] \tag{4}$$

Hereafter $[x, y]$ denotes the straight-line ordered gauge link suspended between the points x and y:

$$[x, y] \stackrel{\text{def}}{\equiv} P\exp\left(ig\int_0^1 du(x-y)^\mu A_\mu(ux + (1-u)y)\right) \tag{5}$$

The operator V_i is given by the same formula as operator U_i with the only difference that the gauge links are constructed from the fields B_μ.

The functional integrals over $\mathcal{A}$ fields give logarithms of the type $g^2 \ln 1/\sigma$ while the integrals over slow $\mathcal{B}$ fields give powers of $g^2\ln(\sigma s/m^2)$. With logarithmic accuracy, they add up to $g^2 \ln s/m^2$. Beyond the logarithmic accuracy, one should expect the corrections of order of g^2 to the effective action $\int dx_\perp U^i V_i$.).

3 Effective action for high-energy scattering

In order to define an effective action for a given interval in rapidity $\eta_0 > \eta > \eta_0'$ we use the master formula (3) two times as illustrated in Fig. 2. We obtain then

$$iA(s,t) = \int \mathcal{D}\mathcal{A} e^{iS(\mathcal{A})} j(p_A) j(p'_A) \int \mathcal{D}\mathcal{B} e^{iS(\mathcal{B})} j(p_B) j(p'_B) e^{iS_{\text{eff}}(V_i, Y_i; \frac{\sigma}{\sigma'})} \tag{6}$$

where S_{eff} for the rapidity interval between η and η' is defined as

$$e^{iS_{\text{eff}}(V_i, Y_i; \frac{\sigma}{\sigma'})} = \int \mathcal{D}\mathcal{C} e^{iS(\mathcal{C})} e^{i\int d^2x_\perp V^{ai}(x_\perp) U_i^a(x_\perp) + i\int d^2x_\perp W^{ai}(x_\perp) Y_i^a(x_\perp)} \tag{7}$$

[b]: In this formula $W_i(x_\perp) = W^\dagger i\partial_i W$ and $Y_i(x_\perp) = Y^\dagger i\partial_i Y$. Both Wilson-line operators W and Y have the form $[\infty n' + x_\perp, -\infty n' + x_\perp]$ but the operators W are made from $\mathcal{C}$ fields while Y's are made from $\mathcal{B}$ fields. (The direction $n' = \sigma' p_1 + \tilde{\sigma}' p_2$ corresponds to the rapidity η': as usual, $\ln \sigma'/\tilde{\sigma}' = \eta'$ where $\tilde{\sigma}' = m^2/s\sigma'$). Also, the operators U here are constructed from the $\mathcal{C}$ fields and the operators V are still made from $\mathcal{A}$ fields.

This formula gives a rigorous definition for the effective action for a given interval in rapidity (cf. ref. [5]). Next step would be to perform explicitly the integrations over the longitudinal momenta in the r.h.s. of Eq. (7) and obtain the answer for the integration over our rapidity region (from η to η') in terms of two-dimensional theory in the transverse coordinate space which hopefully would give us the unitarization of the BFKL pomeron. At present, it is not known how to do this. The effective action approach is probably not less difficult than the direct calculation of the many-pomeron exchanges in the perturbation theory but for the case of effective-action language we have some additional powerful methods such as semiclassical approach.

4 Effective action and collision of two shock waves

The functional integral (7) which defines the effective action is the usual QCD functional integral with two sources corresponding to the two colliding shock waves. The semiclassical approach is relevant when the coupling constant is relatively small but the characteristic fields are large (in other words, when $g^2 \ll 1$ but $gV_i \sim gY_i \sim 1$). In this case one can calculate the functional integral (7) by expansion around the new stationary point corresponding to the classical wave created by the collision of the shock waves. With leading log accuracy, we can replace the vector n by p_1 and the vector n' by p_2. Then the functional integral (7) takes the form:

$$e^{iS_{\text{eff}}(V_i, Y_i; \frac{\sigma}{\sigma'})} = \int \mathcal{D}A e^{iS_{QCD}(A)} e^{i\int d^2x_\perp V^{ai}(x_\perp) U_i^a(x_\perp) + i\int d^2x_\perp W^{ai} Y_i^a(x_\perp)} \tag{8}$$

[b]For brevity, we do not display the quark fields.

where now

$$U(x_\perp) = [\infty p_1 + x_\perp, -\infty p_1 + x_\perp] V(x_\perp) = [\infty p_2 + x_\perp, -\infty p_2 + x_\perp] \quad (9)$$

The classical equations for the wave created by the collision have the form[3]:

$$\begin{aligned}
&D^\mu F_{\mu i} = 0 \qquad (10)\\
&D^\mu F_{*\mu} = \delta(\tfrac{2}{s}x_\bullet)[\tfrac{2}{s}x_* p_1, -\infty p_1]_{x_\perp} \hat{\nabla}_i V^i(x_\perp)[-\infty p_1, \tfrac{2}{s}x_* p_1]_{x_\perp}\\
&D^\mu F_{\bullet\mu} = \delta(\tfrac{2}{s}x_*)[\tfrac{2}{s}x_\bullet p_2, -\infty p_2]_{x_\perp} \tilde{\nabla}_i Y^i(x_\perp)[-\infty p_2, \tfrac{2}{s}x_\bullet p_2]_{x_\perp}
\end{aligned}$$

where

$$\hat{\nabla}_i \mathcal{O}(x) \equiv \partial_i \mathcal{O}(x) - i[U_i(x_\perp), \mathcal{O}(x)], \qquad \tilde{\nabla}_i \mathcal{O}(x) \equiv \partial_i \mathcal{O}(x) - i[W_i(x_\perp), \mathcal{O}(x)]$$

[c]. Unfortunately, it is not clear how to solve these equations. One can start with the trial field which is a simple superposition of the two shock waves

$$A_*^{(0)} = A_\bullet^{(0)} = 0, \quad A_i^{(0)} = \Theta(x_\bullet)V_i + \Theta(x_*)Y_i \qquad (11)$$

and improve it by taking into account the interaction between the shock waves order by order. We use the notations $z_\bullet \equiv z_\mu p_1^\mu$ and $z_* \equiv z_\mu p_2^\mu$ which are essentially identical to the light-front coordinates: $z_* = z_+\sqrt{\frac{s}{2}}$, $z_\bullet = z_-\sqrt{\frac{s}{2}}$. (Note that we changed the name for the gluon fields in the integrand from $\mathcal{C}$ back to A). The parameter of this expansion is the commutator $g^2[Y_i, V_k]$. Moreover, it can be demonstrated that each extra commutator brings a factor $\ln\frac{\sigma}{\sigma'}$ and therefore this approach is a sort of leading logarithmic approximation. In the lowest nontrivial order one gets:

$$\begin{aligned}
A_i^{(1)} &= -\frac{g}{4\pi^2}\int dz_\perp([Y_i(z_\perp), V_k(z_\perp)] - i \leftrightarrow k)\frac{(x-z)^k}{(x-z)_\perp^2}\\
&\quad \left\{\ln\left(1 - \frac{(x-z)_\perp^2}{x_\parallel^2 + i\epsilon}\right) + 2\pi i\theta(x_*)\theta(x_\bullet)\right\}\\
A_\bullet^{(1)} &= \frac{gs}{16\pi^2}\int dz_\perp \frac{1}{x_* + i\epsilon}\ln(-x_\parallel^2 + (x-z)_\perp^2 + i\epsilon)[Y_k(z_\perp), V^k(z_\perp)]\\
A_*^{(1)} &= -\frac{gs}{16\pi^2}\int dz_\perp \frac{1}{x_\bullet + i\epsilon}\ln(-x_\parallel^2 + (x-z)_\perp^2 + i\epsilon)[Y_k(z_\perp), V^k(z_\perp)] \qquad (12)
\end{aligned}$$

where $x_\parallel^2 \equiv \frac{4}{s}x_* x_\bullet$ is a longitudinal part of x^2. These fields are obtained in the background-Feynman gauge. Let us now find the effective action. In the trivial order the only non-zero field strength components are

[c]They are essentially equivalent to the classical equations describing the collision of two heavy nuclei in ref. [6].

$F^{(0)}_{\bullet i} = \delta(\frac{2}{s}x_*)Y_i(x_\perp)$ and $F^{(0)}_{*i} = \delta(\frac{2}{s}x_\bullet)V_i(x_\perp)$ so we get the familiar expression $S^{(0)} = \int d^2x_\perp V^{ai}Y^a_i$. In the next order one has

$$S^{(1)} = -\frac{2}{s}\int d^4x F^{(1)ai}_*(x)F^{(1)a}_{\bullet i}(x) + \int d^2x_\perp 2\mathrm{Tr}[Y^i, V_i]\Big([x_\perp, -\infty p_2 + x_\perp]^{(1)} - [x_\perp, -\infty p_1 + x_\perp]^{(1)}\Big) \qquad (13)$$

The first term contains the integral over $d^4x = \frac{2}{s}dx_\bullet dx_* d^2x_\perp$. In order to separate the longitudinal divergencies from the infrared divergencies in the transverse space we will work in the $d = 2 + 2\epsilon$ transverse dimensions. It is convenient to perform at first the integral over x_* which is determined by a residue in the point $x_* = 0$. The integration over remaining light-cone variable $x_\bullet$ factorizes then in the form $\int_0^\infty dx_\bullet/x_\bullet$ or $\int_{-\infty}^0 dx_\bullet/x_\bullet$. This integral reflects our usual longitudinal logarithmic divergencies which arise from the replacement of vectors n and n' in (7) by the light-like vectors p_1 and p_2. In the momentum space this logarithmical divergency has the form $\int d\alpha/\alpha$. It is clear that when α is close to σ (or σ') we can no longer approximate n by p_1 (or n' by p_2). Therefore, in the leading log approximation this divergency should be replaced by $\ln\frac{\sigma}{\sigma'}$:

$$\int_0^\infty dx_\bullet \frac{1}{x_\bullet} = \int_0^\infty d\alpha\frac{1}{\alpha} \rightarrow \int_\sigma^{\sigma'} d\alpha\frac{1}{\alpha} \;=\; \ln\frac{\sigma}{\sigma'} \qquad (14)$$

The (first-order) gauge links in the second term in r.h.s. of Eq. (13) have the logarithmic divergence of the same origin which also should be replaced by $\ln\frac{\sigma}{\sigma'}$. Performing the remaining integration over $x_\perp$ in the first term in r.h.s. of Eq. (13) we obtain the the first-order classical action in the form[3]:

$$S^{(1)} = \qquad (15)$$
$$-\frac{ig^2}{8\pi^2}\ln\frac{\sigma}{\sigma'}\int d^2x_\perp d^2y_\perp \big(L^a_1(x_\perp)L^a_1(y_\perp) + L^a_2(x_\perp)L^a_2(y_\perp)\big)\frac{\Gamma(\epsilon)}{(x-y)^{2\epsilon}_\perp}$$

where

$$L^a_1 = if^{abc}Y^a_jV^{bj}, L^a_2 = if^{abc}\epsilon_{ik}Y^{bi}V^{ck} \qquad (16)$$

At $d = 2$ we have an infrared pole in $S^{(1)}$ which must be canceled by the corresponding divergency in the trajectory of the reggeized gluon. The gluon reggeization is not a classical effect in our approach - rather, it is a quantum correction coming from the loop corresponding to the determinant of the operator of second derivative of the action. The corresponding contribution to effective action is

$$S_{\mathrm{r}} = \frac{g^2 N_c}{8\pi^3} \ln \frac{\sigma}{\sigma'} \int d^2x_\perp d^2y_\perp \left(V_i^a(x_\perp) Y^{ai}(y_\perp) - V_i^a(x_\perp) Y^{ai}(x_\perp)\right) \frac{\Gamma^2(1+\epsilon)}{((x-y)_\perp^2)^{(1+2\epsilon)}} \tag{17}$$

The complete first-order ($\equiv$ one-log) expression for the effective action is the sum of $S^{(0)}$, $S^{(1)}$, and S_{r}. It may be demonstrated that in the leading log approximation the effective action can be represented by the functional integral over Wilson-line variables with the action given by $S^{(1)} + S_{\mathrm{r}}$ (the study is in progress).

5 Conclusion

In conclusion I would like to discuss the relation of this method to other approaches to the high-energy effective action discussed in the literature.

Historically, the idea how to reduce QCD at high energies to the two-dimensional effective theory was first suggested in ref.[2] where the leading term $S^{(0)} = \int dx_\perp U_i V_i$ was obtained. However, careful analysis of the assumptions made in this paper shows that the authors considered the fixed-angle limit of the theory ($s, t \to \infty$) rather than the Regge limit (where $\to \infty$ but t is fixed). It turns out that the leading term is the same for both limits, but the subsequent terms differ.

Careful analysis of the effective action in the Regge limit was performed in the papers by Lipatov and collaborators [7]. The definition of the effective action in these papers is close to eq. (6). However, the effective action is presented there in terms of the reggeons built from fast and slow gluons rather than from the corresponding Wilson-line operators V_i and Y_i. In the first order, when the reggeized gluon is inentical to the usual gluon the expressions for the effective action are equivalent. In general, the explicit formula for the relation of the reggeized gluons to usual gluon operators is not known and therefore it is not possible to compare the intermediate formulas for the higher terms in the effective action. Hoewever, since the physical results for the BFKL pomeron and the three-pomeron vertex coincide I think that the effective action obtained in refs. [7],[5] is equivalent to $S^{(1)} + S_{\mathrm{r}}$.

The most close in spirit to our semiclassical method is the renormalization-group approach to the high-energy scattering from the large nuclei advocated in the papers of L. McLerran and collaborators (see e.g. refs.[8],[9], [10]). In this approach, the small-x evolution of one strong shock wave is studied in the light-like gauge. (With such choice of gauge the second shock wave can be treated perturbatively at the very end of the evolution process).

The strong shock wave is created by the sources $\rho(x_\perp)$ so the evolution of the effective action $S(\rho)$ is obtained. In our terms, this amounts to the solution of classical eqs. (10) using the trial configuration $A_i = U_i\theta(x_*)$ (instead of starting point $A_i = U_i\theta(x_*) + V_i\theta(x_\bullet)$ taken in this paper). Unfortunately, due to different gauges adopted in our paper and refs. [8,10] the treatment of the boundary terms in the functional integral is different which leads to the different sources for the shock waves and makes it hard to compare the intermediate formulas. However, since again the BFKL results coincide I think these effective actions are essentially the same.

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