Effect of a Mean Free Path on Nonlinear Losses of Trapped Vortices Driven by a RF Field

Manula Randhika Pathirana Walive Pathiranage
Old Dominion University, mwali003@odu.edu

Alex Gurevich
Old Dominion University, agurevic@odu.edu

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EFFECT OF MEAN FREE PATH ON NONLINEAR LOSSES OF TRAPPED VORTICES DRIVEN BY A RF FIELD*

W.P.M.R Pathirana†, Alex Gurevich
Center for Accelerator Science, Old Dominion University, Norfolk, USA

Abstract

We report extensive numerical simulations on nonlinear dynamics of a trapped elastic vortex under rf field, and its dependence on electron mean free path l_i. Our calculations of the field-dependent residual surface resistance $R_i(H)$ take into account the vortex line tension, the linear Bardeen-Stephen viscous drag and random distributions of pinning centers. We showed that $R_i(H)$ decreases significantly at small fields as the material gets dirtier while showing field independent behavior at higher fields for clean and dirty limit. At low frequencies $R_i(H)$ increases smoothly with the field amplitude at small H and levels off at higher fields. The mean free path dependency of viscosity and pinning strength can result in a nonmonotonic mean free path dependence of R_i, which decreases with l_i at higher fields and weak pinning strength.

INTRODUCTION

RF losses in SRF cavities are quantified by the quality factor Q_0 which is inversely proportional to the surface resistance R_s. The surface resistance consists of two parts, $R_s = R_{BCS} + R_i$, where $R_{BCS} \propto \omega^2 \exp(-\Delta/T)$ comes from thermally activated quasiparticles while R_i quantified a weakly-temperature dependent residual resistance. The temperature independent R_i can produce a large fraction of the total dissipation about $\approx$ 20% for Nb and $\approx$ 50% for Nb_3Sn at 2 K and 1-2 GHz [1]. So the dependence of R_i on the magnetic field H, frequency f and mean free path (l_i) is of much interest. The main contributions to R_i comes from trapped vortices generated during the cavity cool down through the critical temperature T_c at which the lower critical field $H_{c1}(T)$ vanishes [2–10]. In this case even small stray fields $H > H_{c1}(T)$ such as unscreened earth magnetic field can produce vortices in the cavity. During the subsequent cooldown to $T \simeq 2$ K some of these vortices exit the cavity but some get trapped by the material defects such as non-superconducting precipitates, network of dislocations or grain boundaries.

Low-field rf losses of pinned vortices have been calculated by many authors [3, 11–15]. Nonlinear quasi-static electromagnetic response of perpendicular vortices has been addressed both for weak collective pinning [1], and strong pinning [16, 17]. The extreme nonlinear dynamics of a vortex under a strong ac magnetic field at which $R_i(H)$ decreases with H because of the decrease of vortex viscosity with the velocity was addressed in [18]. The dissipation of vortices under a strong magnetic field in the cases of mesoscopic pinning has been calculated recently by [19]. The nonlinear dynamics of the trapped vortex and the field dependence R_i can also be tuned by nonmagnetic impurities. Yet, the mean free path dependency of the rf power generated by flexible oscillating vortex though a random pinning potential remains poorly understood. In this work, we calculate field dependent $R_i(H)$ and its dependencies on the mean free path, frequency and the pinning strength due to a trapped vortex line under rf magnetic field. Our calculation take into account the vortex line tension, pinning force, and Bardeen-Stephen viscous drag force.

DYNAMIC EQUATIONS

Consider a single vortex pinned by materials defects as shown in the Fig. 1. Here the vortex is driven by the ac

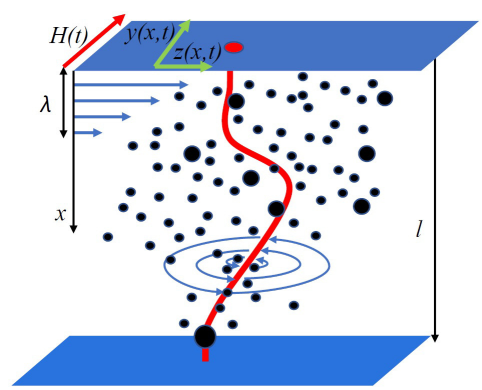

Figure 1: A flexible vortex shown by the read line driven by the rf surface current. The black dots represent pinning centers such as non-superconducting precipitates. Green arrows show vortex tip displacement on the YZ plane.

Meissner currents flowing in a thin layer of $\sim \lambda$ at the surface. The ac displacement of the vortex $\mathbf{R} = [Y(X,t), Z(X,t)]$ is mainly confined within the elastic skin depth [3] so that the vibrating vortex segment interacts only with a few pins while the rest of the vortex does not move. In this situation, the electromagnetic response of a perpendicular vortex becomes dependent on its position and the statistical distribution of random pinning potentials. For instance, Fig.1 shows a representative case of bulk pinning by small, randomly-distributed non-superconducting precipitates. The dynamic equation for trapped vortex shown in the Fig. 1 is given by:

$$M\frac{\partial^2 \mathbf{R}}{\partial t^2} + \eta\frac{\partial \mathbf{R}}{\partial t} = \epsilon\frac{\partial^2 \mathbf{R}}{\partial X^2} - \nabla U(X,\mathbf{R}) - \hat{y} f_L(X,t), \quad (1)$$

$$f_L(X,t) = (\phi_0 H/\lambda) e^{-X/\lambda}\sin\omega t, \quad (2)$$

* This work was supported by NSF under Grant 100614-010 and Grant 1734075.
† mwali003@odu.edu

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where H is the amplitude of the applied magnetic field with the frequency f, λ is the London penetration depth, M is the vortex mass per unit length, $\epsilon = \phi_0^2(\ln\kappa + 0.5)/4\pi\mu_0\lambda^2$ is the vortex line energy, $\kappa = \lambda/\xi$ is the Ginzburg-Landau (GL) parameter, ξ is the coherence length, and η is the viscous drag coefficient. Equations (1) and (2) represent a balance of local forces acting on a curvilinear vortex: the inertial and viscous drag forces in the left hand side are balanced by the elastic, pinning and Lorentz forces in the right hand side. It is assumed that: 1. The field is well below the superheating field [20–23] so that the London model is applicable. 2. The Magnus force causing a small Hall angle [24–26] is negligible. 3. The low frequency rf field ($\hbar\omega \ll \Delta$) does not produce quasiparticles, and the quasi-static London equations are applicable [27]. 4. Bending distortions of the vortex are small so the linear elasticity theory [11, 28] is applicable. We consider here the core pinning of vortices [11,28,29] represented by a sum of pinning centers modeled by the Lorentzian functions [30]:

$$U(X,\mathbf{R}) = -\sum_{n=1}^{N} \frac{U_n}{1 + [(X - X_n)^2 + |\mathbf{R} - \mathbf{R}_n|^2]/\xi^2}. \quad (3)$$

Here, X_n, Y_n and Z_n are the coordinates of the n-th pinning center, and U_n are determined by the gain in the condensation energy in the vortex core at the pin [11,28,29]. To take into account dependencies of superconducting parameters on the mean free path l_i in Eqs. (1)-(3), we used $\rho_n \propto l_i^{-1}$ and the conventional GL interpolation formulas $\lambda = \lambda_0\Gamma$, and $\xi = \xi_0/\Gamma$, where $\Gamma = (1 + \xi_0/l_i)^{1/2}$. As a result, we obtain the following dimensionless nonlinear partial differential equations for the local coordinates $y(x,t) = Y/\lambda_0$ and $z(x,t) = Z/\lambda_0$:

$$\gamma\dot{y} = y'' - \sum_{n=1}^{N} A_n(x,\mathbf{r})(y - y_n) + \beta_t e^{-x/\Gamma}, \quad (4)$$

$$\gamma\dot{z} = z'' - \sum_{n=1}^{N} A_n(x,\mathbf{r})(z - z_n), \quad (5)$$

$$y'(0,t) = z'(0,t) = y'(l,t) = z'(l,t) = 0. \quad (6)$$

Here the prime and the dot imply differentiation over the dimensionless coordinate $x = X/\lambda_0$ and time $t = tf$, respectively, the vortex mass is neglected. $\mathbf{r} = [y(x,t), z(x,t)]$, and:

$$\gamma = \frac{g_0\Gamma^4 l_i f}{g\xi_0 f_0}, \qquad f_0 = \frac{H_{c10}\rho_{n0}}{H_{c20}\lambda_0^2\mu_0}, \quad (7)$$

$$\beta_t = \beta\sin(2\pi t), \qquad \beta = \frac{g_0\Gamma H}{gH_{c10}}, \quad (8)$$

$$A_n = \frac{g_0\Gamma^5\zeta_{n0}}{g\left[1 + \Gamma^2\kappa_0^2(x - x_n)^2 + \Gamma^2\kappa_0^2|\mathbf{r} - \mathbf{r}_n|^2\right]^2}, \quad (9)$$

$$\zeta_{n0} = 2\kappa_0^2 U_n/\epsilon_0, \quad (10)$$

$$g_0 = \ln\frac{\lambda_0}{\xi_0} + \frac{1}{2}, \qquad g = \ln\frac{\lambda_0\Gamma^2}{\xi_0} + \frac{1}{2}. \quad (11)$$

where $\lambda_0, \xi_0, \epsilon_0, \rho_{n0}, H_{c10} = (\phi_0/4\pi\mu_0\lambda_0^2)(\ln\kappa_0 + 0.5)$ and $H_{c20} = \phi_0/2\pi\mu_0\xi_0^2$ are the the penetration depth, coherence length, vortex line energy, normal-state resistivity, lower and upper critical fields in the clean limit, respectively. The amplitude U_n is related to the elementary pinning energy by $u_p = \pi\xi U_n$, so that $\zeta_n = 2\kappa^2 u_p/\pi\epsilon\xi = g_0\Gamma^5\zeta_{n0}/g$ as u_p is independent of mean free path [14].

We first estimate γ and ζ_n for a dirty Nb with $\lambda_0 = \xi_0 = 40$ nm and $U_n = 1.4$ meV/nm. Hence, $\gamma_0 = g_0 f/f_0 \approx 0.004$, and $\zeta_{n0} \simeq 0.04$ at $f = 1$ GHz. Another essential parameter is the decay length L_ω of oscillating bending disturbance along the vortex line induced by a weak rf current at the surface [3]

$$L_\omega = \sqrt{\frac{\epsilon}{\eta\omega}} = \frac{\lambda}{\sqrt{2\pi\gamma}}, \quad (12)$$

For Nb_3Sn, we have $L_\omega \simeq 5.15\lambda = 572$ nm at 1 GHz. In this case, dissipative oscillations of the elastic vortex extend well beyond the rf field penetration depth.

The power of rf losses is obtained by summing contributions of all vortices, $P = \sum_k \int \langle J(X,t)\partial_t Y_k(X,t)dX\rangle$, where $Y_k(X,t)$ describes the k−th vortex and $\langle .., \rangle$ means time averaging (see Ref. [19]). It is convenient to define a mean dimensionless power $p = P/P_0$ and the surface resistance r_i per vortex:

$$p = \frac{\gamma_0}{g_0\Gamma N_v}\sum_{k=1}^{N_v}\int_0^1 dt \int_0^l \beta_t e^{-x}\dot{y}_k(x,t)dx, \quad (13)$$

$$r_i(\beta) = 2p(\beta)/\beta^2, \quad (14)$$

where $P_0 = \lambda_0 f_0 \epsilon_0$ and N_v is the number of vortices. The dimensionless r_i is related to the surface resistance R_i which defines the power losses per unit area $P = R_i H^2/2$ by $R_i = P_0 r_i n_\square/H_{c10}^2$. Here $n_\square = B_0/\phi_0$ is a vortex areal density producing a small induction $B_0 \ll B_{c1}$. Using here f_0 from Eq. (7) and $\epsilon_0 = \phi_0 H_{c10}$, we obtain:

$$R_i = \frac{\rho_{n0}B_0}{\lambda_0 B_{c20}} r_i. \quad (15)$$

NUMERICAL RESULTS

We solved Eqs. (4)-(6) numerically using COMSOL [31]. In our simulations, a straight vortex was initially put in a particular pinning potential, and after $\mathbf{r}(x,t)$ relaxes to a stable shape, the rf field was turned on. Then we run the program until $\mathbf{r}(x,t)$ reaches steady-state oscillations after a transient period $\delta t \lesssim 90/f$ and use this solution to calculate R_i. For the case of bulk pinning N identical pins were distributed randomly in a $l \times l_y \times l_z$ box and Eqs. (4)-(6) were solved for different mean free path, frequency, and rf field amplitudes, making sure that l_y and l_z are adjusted in such a way that the vortex always remains within the box during the rf period. The mean pin density $n_i = N/ll_yl_z$ was fixed through out the simulations.

Shown in Fig. 2 are the dependencies of the surface resistance $r_i(\beta)$ on the field amplitude $\beta = H/H_{c1}$ calculated for different mean free path values at $\kappa_0 = 2$. Here r_i is nearly

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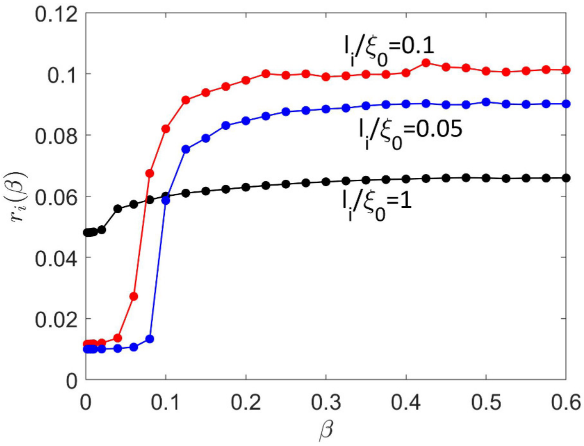

Figure 2: Field dependence of $r_i(\beta)$ calculated for different mean free path at $\kappa_0 = 2$ and $\zeta_{n0} = 0.04$. Other parameters are $l/\lambda_0 = 10$, $\gamma_0 = 0.004$, $n_i = 0.5\lambda_0^{-3}$.

independent of H at the higher field but develops a field dependence at smaller field values. As the mean free path is reduced $r_i(\beta)$ starts to decreases at small fields but increases as the field increases. This low field behavior is because L_ω decreases as l_i decreases, and the vortex interacts with small number of pins resulting in a lower surface resistance. The $r_i(\beta)$ at higher fields is mostly limited by the vortex drag, and the effect of pinning fluctuations weakens, resulting a field-independent behavior. As l_i decreases, the transition to flux flow regime from pinning regime occurs at a higher field because of higher pinning strength $\zeta_n \propto \Gamma^5$. For instance this transition occurs at $\beta \sim 0.1$ for $l_i/\xi_0 = 0.05$ but $\beta \sim 0.04$ for $l_i/\xi_0 = 1$. Curiously, $r_i(\beta)$ at $l_i/\xi = 0.05$ is slightly smaller than at $l_i/\xi_0 = 0.1$. Figure 3 shows the field depen-

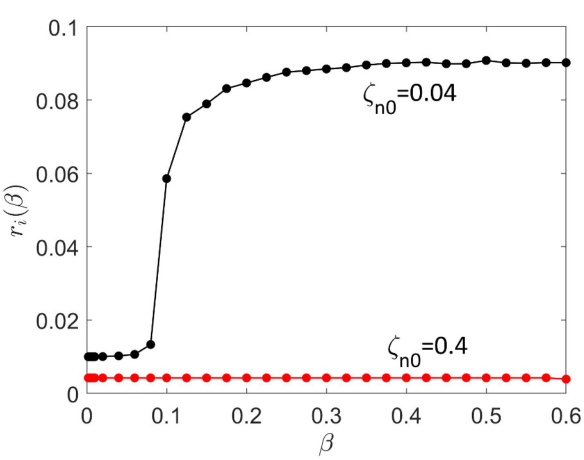

Figure 3: Surface resistance $r_i(\beta)$ calculated for different pinning strength $\zeta_{n0} = 0.04, 0.4$ at $l/\lambda_0 = 10$, $\gamma_0 = 0.004$, $n_i = 0.5\lambda_0^{-3}$, $l_i/\xi_0 = 0.05$ and $\kappa_0 = 2$.

dencies of $r_i(\beta)$ for bulk pinning calculated at two values of the pinning parameter ζ_n. The surface resistance $r_i(\beta)$ for weak pinning with $\zeta_n = 0.04$, increases sharply above $\beta \simeq 0.1$ due to the rapid transition from pinning regime to flux flow regime while strong pinning with $\zeta_n = 0.4$ stays approximately independent from the field as its depinning field $\beta_p(\zeta_{n0} = 0.4) >> \beta_p(\zeta_{n0} = 0.04)$ which can be calculated approximately using $\beta_p \simeq (\zeta_n\lambda/\kappa^2)\sqrt{n_i l} \sim 0.6$ [19] at $\zeta_{n0} = 0.4$ and $\beta_p \sim 0.06$ for $\zeta_{n0} = 0.04$. This restricts the motion of the vortex and results in a lower $r_i(\beta)$ at strong pinning.

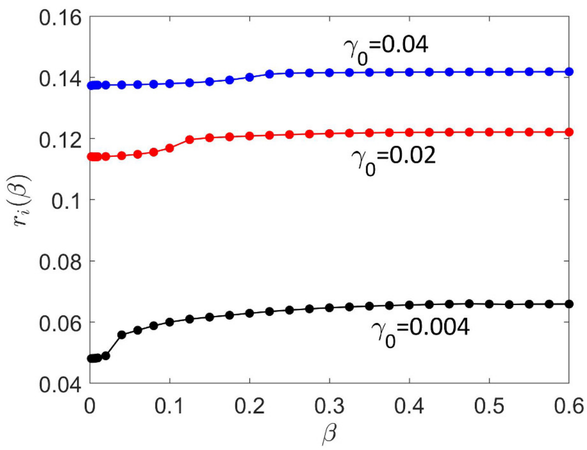

Figure 4: Surface resistance $r_i(\beta)$ calculated for different frequencies $\gamma_0 = 0.004, 0.02, 0.04$ at $l/\lambda_0 = 10$, $n_i = 0.5\lambda_0^{-3}$, $\kappa_0 = 2$, $l_i/\xi = 1$ and $\zeta_{n0} = 0.04$.

Now we turn to the effect of pinning on the frequency dependence on $r_i(\beta, \gamma)$ shown in Fig. 4. At a high frequency $\gamma = 0.4$ the surface resistance $r_i(\beta)$ is nearly independent of the field amplitude β because the rf losses are dominated by the linear vortex drag. As the frequency decreases, a linear dependence of $r_i(\beta)$ develops at small fields for which pinning reduces $r_i(\beta)$. This result is consistent with the calculations of $r_i(\beta)$ in a quasi-static limit [1].

Shown in Fig. 5 are the dependencies of the surface resistance $r_i(l_i)$ on the mean free path calculated at two different filed amplitude and κ_0. The peak in $r_i(l_i)$ shown in Fig. 5 (a) results from the interplay of the decrease of the vortex viscosity $\eta(l_i)$ and increase of pinning strength ζ_n as the vortex line gets softer in the dirty limit. Such a bell-shaped dependence of $r_i(l_i)$ has been observed experimentally [14, 15]. As the rf field amplitude β increases, the peak shifts to a lower mean path value. However, the opposite situation occurs at $\kappa_0 = 10$ shown in Fig. 5 (b). Here the dip in $r_i(l_i)$ occurs because the pinning strength parameter ζ_n increases significantly as l_i decreases, resulting in a lower r_i at small l_i. At higher field $\beta = 0.1$ which exerts larger Lorentz forces, pinning becomes less effective $r_i(l_i)$ shown in Fig. 5 (b) becomes similar to $r_i(l_i)$ shown in Fig. 5 (a) at $\kappa_0 = 2$.

CONCLUSION

We presented the numerical simulations of nonlinear dynamics of a single vortex moving in random pinning potentials under rf magnetic field. The power dissipated by an

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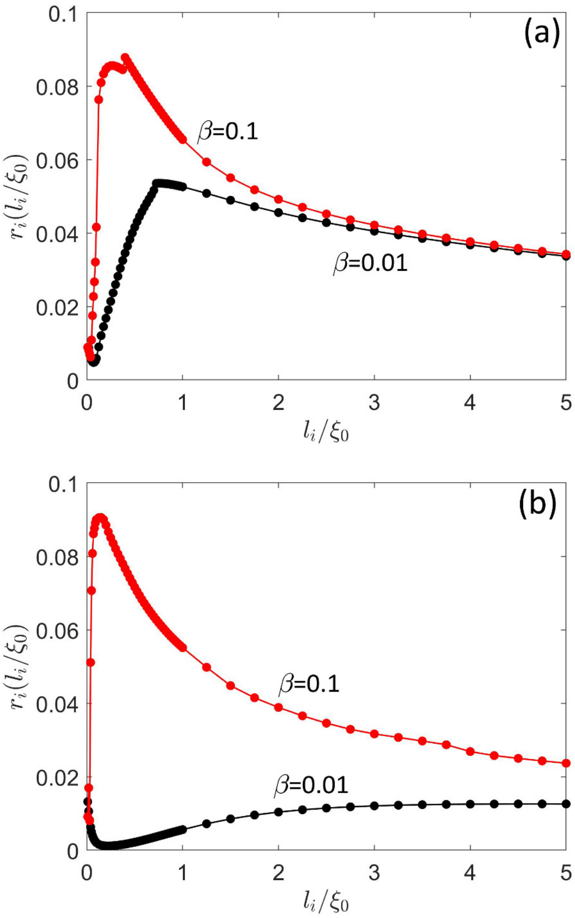

Figure 5: Mean free path dependencies of $r_i(l_i)$ calculated at $l/\lambda_0 = 10$, $n_i = 1.67\lambda_0^{-3}$, $\gamma_0 = 0.004$, $\beta = 0.01$ and $\beta = 0.1$, (a) $\kappa_0 = 2$, $\zeta_{n0} = 0.04$ (b) $\kappa_0 = 10$, $\zeta_{n0} = 1$.

oscillating vortex segment was calculated considering the line tension of the vortex, Bardeen-Stephen viscous drag force, and random pinning force with constant mean pin density at different rf fields amplitudes, mean free path, pinning strength and frequency. At low frequencies $R_i(H)$ gradually increases with the field at a small field, but as the frequency increases $R_i(H)$ becomes field independent. The field-dependent residual surface resistance decreases significantly at the small field in dirty material but shows a field-independent behavior at a higher field. We obtained a bell-shaped dependence of the surface resistance on the mean free path due to the interplay between the pinning and viscous forces.

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