Anchoring of Drifting Spiral and Scroll Waves to Impermeable Inclusions in Excitable Media

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Excitable media encompass a broad class of highly nonlinear, distributed nonequilibrium systems of physical, chemical, and biological systems \[1\], including nerve and cardiac tissues \[2\]. A distinct feature of excitable media is their ability to support self-sustained waves and high-frequency sources of repetitive excitation in the form of rotating spiral waves (in 2D) \[3,4\] and scroll waves (in 3D) \[5\].

Rotating waves are organized around vortexlike cores that are called filaments in the case of scroll waves \[6,7\]. Spiral waves behave in many ways like particles: they can drift \[2,8,9\], interact, and form dynamic bound states \[10–12\].

Drifting spiral and scroll waves can also anchor (pin) to localized heterogeneities, giving rise to sustained periodic or quasiperiodic activity \[2,13–16\]. Anchoring is believed to play a crucial role in maintaining abnormal high-frequency heart rhythms, including ventricular tachycardia and fibrillation, the main causes of sudden cardiac death \[2\]. The dynamics of anchored spirals, particularly methods of unpinning them, have been studied extensively with regard to cardiological applications \[17–19\].

In this study, we focus on impermeable inclusions, which in a cardiac context represent tissue that is electrically uncoupled from the excitable myocardium, e.g., blood vessels or scars. Unlike parametric heterogeneities, impermeable inclusions represent a different class of heterogeneities that reflect the topological connectivity of the excitable medium. Despite the importance of impermeable inclusions, their anchoring ability is much less understood. This is in part due to their fundamentally nonperturbative nature, which prevents the application of analytical tools developed for parametric heterogeneities \[16,20,21\].

We derive the equation of motion of a spiral wave core in the vicinity of an impermeable inclusion by adopting a particle-physics approach. We bombard the inclusions with spiral wave cores of different speeds and impact parameters. By analyzing perturbations of drift trajectories we derive forces controlling the dynamics of the core in the vicinity of inclusion. Such “scattering experiments” are carried out numerically using a generic reaction-diffusion model of an excitable medium with Barkley kinetics \[22\],

\[
\begin{align*}
\partial_t u &= \epsilon^{-1} u(1-u)(u-(v+b))/a + D\nabla^2 u + E \cdot \nabla u \\
\partial_t v &= u - v + \alpha E \cdot \nabla v,
\end{align*}
\]

where \(u\) is the activator variable, \(v\) is the inhibitor variable, \(D\) is the diffusivity, and \(E\) is an external field. The advection term \(E \cdot \nabla u\) was introduced to make spirals drift towards the inclusion. The parameter \(\alpha\) is used to switch advection of \(v\) on \((\alpha = 1)\) or off \((\alpha = 0)\).

Without loss of generality, we assumed that \(E\) is oriented along the \(y\) axis. No-flux boundary conditions were set both at the border of the inclusion and at external medium boundaries. The values of the kinetic parameters \(a, b,\) and \(\epsilon\) are given in the caption of Fig. 1; they were chosen such that in an unperturbed system \((E = 0)\), the spiral rotation was stationary and scroll wave filaments had positive tension \[7\]. We induced spiral and scroll waves as previously described \[12\]. To determine drift velocity, we tracked the movement of the spiral core, defined as the region with \(u < 0.99 u_{\text{max}}\) over one spiral rotation period \((T)\). We use the spiral wavelength \(\lambda\) as a natural unit of length.

Our numerical experiments show that the dynamics of a filament or spiral core approaching an inclusion can be complex and sometimes counterintuitive. For example, we found that in the same excitable medium, drifting filament or spiral cores anchored to small inclusions but always circumnavigated larger inclusions.

Figures 1(a) and 1(b) show an example of a 3D numerical experiment, in which a drifting filament has been set on...
inclusion is governed by the following simple equation:

\[
\frac{dx}{dt} = v_E + F(r', \rho)
\]

where \(x(t)\) is the center of the spiral wave core at time \(t\), \(v_E\) is the gradient-induced drift velocity, \(r'\) is the distance between the spiral core and the boundary of the inclusion, and \(F\) is the force that the inclusion exerts on the spiral core. \(F\) has a dimension of velocity, analogous to a Stokes force in a viscous medium; see also Refs. [16,20].

As in the case of localized parametric heterogeneities [16], \(F\) has a non-zero azimuthal component \(F_\phi\), which induces orbital drift of the spiral around the inclusion [see Fig. 2(c)].

\[
F = F_r + F_\phi = F_r e_r + F_\phi e_\phi.
\]

To validate Eqs. (3)–(5), we conducted a series of numerical experiments in which we systematically varied \(\Delta\), \(|E|\), and \(\rho\). Figure 3 shows two families of drift trajectories for \(|v_E| = 0.005\) and \(|v_E| = 0.01\) and \(\Delta\) ranging from \(-0.3\) to \(0.3\) \((\rho = 0.16)\). For \(|v_E| = 0.005\), spirals initiated far from the inclusion never anchor [see Fig. 3(a)]; they circumnavigate the inclusion either from the left (trajectories \(-5 \rightarrow -2\)) or from the right (trajectories \(1 \rightarrow 5\)). Increasing \(|v_E|\) makes anchoring possible [see panel (b)]. Trajectories \(-1 \rightarrow -2\), which were deflected from the anchor in panel (a), now anchor.

It is interesting that the spiral wave fails to anchor when on the central collision course with an inclusion \((\Delta = 0)\) but anchors when its trajectory is significantly off center (trajectories \(-1 \rightarrow -2\)). Significant asymmetry is also visible in Fig. 3(a) (the separatrix shown with a dashed line is shifted well to the left). This asymmetry is determined by the chirality of the spiral. In this particular experiment, the spiral rotated clockwise (CW). For a counterclockwise (CCW) rotating spiral, all trajectories are reflected at the
line parallel to $v_E$ and cutting through the center of the inclusion.

To demonstrate that Eq. (3) is correct, we did the following for a large variety of trajectories. We calculated $dx/dt$ at different points for each trajectory. Then, using Eqs. (3)–(5) we derived $F_r$ and $F_\phi$ for each point and corrected them for time lag, which gave us $F_r(r')$ and $F_\phi(r')$. In Figs. 4(a) and 4(b), we show $F_r(r')$ and $F_\phi(r')$ for different values of $|E|$ and $\alpha$ and $\Delta$. One can see that despite significant variations in $|E|$ and $\alpha$ [Fig. 4(a)] or $\Delta$ [Fig. 4(b)], the points from different data sets cluster around well-defined functions.

The function $F_r$ has a pronounced biphasic shape. At distances larger than the effective core radius ($r' > \lambda/2\pi$), $F_r$ is very small. As $r'$ decreases, $F_r$ grows (positive values correspond to repulsion). After reaching a maximum ($F_r^{\text{max}}$), $F_r$ declines, and then it changes sign and becomes attractive. Around $r' = 0.1$, the attraction grows very rapidly (we only show an extrapolation of $F_r$ for these small values of $r'$ because the attractive force grows rapidly as the core approaches the boundary of inclusion which makes its accurate assessment in our simulations difficult).

The function $F_\phi$ is biphasic as well; however, the positive values of $F_\phi$, indicating CCW orbital drift around the inclusion, are very small. The zero crossing (transition to CW motion) occurs around $r' = 0.15$. It is important to note that the dense clustering of the experimental points from different trajectories in Figs. 4(a) and 4(b) not only demonstrates the validity of Eqs. (3) and (4) but also has an important practical implication: for a given $\rho$, both $F_r$ and $F_\phi$ can be reconstructed from a single anchoring trajectory.

Our next step was to explore the dependence of $F_r$ and $F_\phi$ on the inclusion size $\rho$. We found that while $F_r$ and $F_\phi$ do depend on $\rho$, this dependence can be separated from that on $r'$ by

$$F_r(r', \rho) = \tilde{F}_r(r') F_r^{\text{max}}(\rho)$$

(6)

$$F_\phi(r', \rho) = \tilde{F}_\phi(r') F_\phi^{\text{max}}(\rho).$$

(7)

The functions $\tilde{F}_r$ and $\tilde{F}_\phi$ are shown in Fig. 4(c) for three different values of $\rho$, ranging from $0.08\lambda$ to $0.32\lambda$. The fact that data points overlap for different values of $\rho$ shows that the variables $r'$ and $\rho$ can indeed be separated as Eqs. (6) and (7) suggest.

We determined $F_r^{\text{max}}$ for different values of $\rho$ [see Fig. 4(d)]. One can see that $F_r^{\text{max}}(\rho)$ appears to be a simple exponential function

$$F_r^{\text{max}}(\rho) = F_r^{\text{max}}(\infty)(1 - \exp(-\beta\rho)).$$

(8)

where $F_r^{\text{max}}(\infty)$ is the maximum of $F_r$ at $\rho = \infty$. Note that to find $\beta$ and $F_r^{\text{max}}(\infty)$ and thus to fully define $F_r^{\text{max}}(\rho)$, one needs to determine $F_r^{\text{max}}$ for only two values of $\rho$. It is interesting that the empirically derived value of $\beta = 0.177\lambda$ is close to effective core radius $\lambda/2\pi = 0.159\lambda$, which means that the effect of obstacles with the radius larger than the effective core size exert the force that is close to the one produced by an infinite wall.

The shape of $F_r$ and $F_\phi$ and their dependence on $\rho$ fully explains the complex dynamic behavior illustrated in...
The limitation of our method is the insufficient resolution at very small distances from the obstacles. In principle, a better resolution can be achieved by using much smaller inclusion sizes and respectively much smaller \( E \) and drift speeds. Such simulations involve significant reduction of discretization steps and increase of computational costs which will require a dedicated effort.

In summary, our study has two main results. (i) We have introduced and tested a novel phenomenological approach for measuring forces exerted by an inclusion onto a spiral in an excitable medium. Our approach does not require the knowledge of dynamical equations of the system and can be applied to any experimental system with any type of inclusions or parametric heterogeneities. The independence of \( F_r \) and \( F_{\phi} \) of \( E \) as well as the simple scaling rule in Eqs. (6) and (7) suggest that the only experimental information required to fully define Eq. (3) are two anchoring trajectories for different inclusion sizes. (ii) We have advanced the understanding of chiral forces that favor anchoring during off-center collisions. These findings improve the understanding of the interaction of drifting scroll waves with impermeable inclusions and represent an important step toward understanding scroll wave dynamics in the heart.

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**Fig. 5.** Drift trajectories calculated using Eqs. (3) and (4) and reconstructed \( F_r \) and \( F_{\phi} \). Predicted trajectories (gray) versus true trajectories (black) for \( |v_E| = 6.8 \times 10^{-3} \lambda / T \) (a) and \( |v_E| = 9.1 \times 10^{-3} \lambda / T \) (b). Trajectory labels (–2, …, 2) indicate the \( \Delta \) of the trajectory in units of 0.12 \( \lambda \).

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