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Comments on "Fluctuations in guiding center plasma in two dimensions"

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In a recent paper[1] Taylor and Thompson calculated autocorrelations in density for the electrostatic guiding center plasma in two dimensions by means of a variant of the random phase approximation. In their notation, they find, for the ensemble average $S_{\mathbf{k}}(\tau) \equiv \langle \rho_{-\mathbf{k}}(t)\rho_{\mathbf{k}}(t-\tau)\rangle$, the expression [their Eq. (23)] $S_{\mathbf{k}}(\tau) = q_k \cos\Omega_k\tau$, where [their Eq. (13)] $q_k \equiv n\lambda^2k^2/(1+\lambda^2k^2)$, and [their Eq. (24)]

$$\Omega_k{}^2 = -\frac{\alpha^2}{(2\pi)^2}\int d^2\mathbf{l}\,\frac{(\hat{\mathbf{b}}\cdot\mathbf{l}\times\mathbf{k})^2}{l^2}\left[\left(\frac{1}{k^2}-\frac{1}{l^2}\right)q_1 - \left(\frac{1}{k^2}-\frac{1}{(\mathbf{k}-\mathbf{l})^2}\right)q_{\mathbf{k}-\mathbf{l}}\right].$$

It is claimed that [their Eq. (27)] for small k, $\Omega_k{}^2 \to (KTc^2/6lB^2)k^4$.

The purpose of this comment is to remark that an error in evaluating the integral (24) has occurred, and that the correct small k limit of $\Omega_k{}^2$ should be $\Omega_k{}^2 \to \alpha^2nk^2/16$. Because the major "volume divergent" contribution to the integral, which is later assumed to represent the diffusion coefficient, comes from the region of small k, this "divergence" no longer results (e.g., in their last equation). This invalidates the principal result of the paper and leaves the diffusion coefficient in flat disagreement with that previously published by Taylor and McNamara.[2]

The error has occurred in ignoring the transformation of the domain of integration, when the second half of the integral in Eq. (24) is transformed to get Eq. (25). Moreover, the assumption which motivates the authors Reply (that there can be a meaningful distinction between an "interaction cutoff" and a "fluctuation cutoff") is incorrect. [It should also be noted that Eq. (1) of their Reply is incorrect. It results from an inconsistent use of the fluctuation cutoff in determining the integration limits.] For any thermal equilibrium Coulomb system (guiding center, finite gyroradius, or unmagnetized), a straightforward proportionality exists between the spectral density of the fluctuations and the Fourier transform of the two-body interaction. All that is required is a small plasma parameter expansion of the equilibrium BBGKY hierarchy derived from the Gibbs distribution.[3] This result is independent of the details of the force law between particles and is independent of dimensionality.[4] There is thus no ambiguity to the statement that in wave number space, the fluctuation spectrum vanishes at exactly those places where the interaction potential vanishes, and nowhere else.

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[1] J. B. Taylor and W. B. Thompson, Phys. Fluids **16**, 111 (1973). (We use their notation.)

[2] J. B. Taylor and B. McNamara, Phys. Fluids **14**, 1492 (1971).

[3] See, e.g., D. Montgomery in *Kinetic Theory*, edited by W. Brittin, A. O. Barut, and M. Guenin (Gordon and Breach, New York, 1967), p. 35.

[4] G. Vahala and D. Montgomery, J. Plasma Phys. **4**, 425 (1971).

